Do electromagnetic waves always propagate along null geodesics?

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We find exact solutions to Maxwell equations written in terms of four-vector potentials in non–rotating, as well as in Gödel and Kerr spacetimes. Exact electromagnetic waves solutions are written on given gravitational field backgrounds where they evolve. We find that in non–rotating spherical symmetric spacetimes, electromagnetic plane waves travel along null geodesics. However, electromagnetic plane waves on Gödel and Kerr spacetimes do not exhibit that behavior.

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Introduction.- Maxwell equations in curved spacetimes are $\nabla_\alpha F^{\alpha\beta} = 0$ and $\nabla_\alpha F^{*\alpha\beta} = 0$, where ∇_α stands for the covariant derivative defined for a metric $g_{\mu\nu}$, and $F^{\alpha\beta}$ is the antisymmetric electromagnetic field tensor, while $F^{*\alpha\beta}$ is its dual. If the electromagnetic field is written in terms of the four-vector potentials A_α, i.e., $F^{\alpha\beta} = g^{\alpha\mu}g^{\beta\nu}F_{\mu\nu}$ and $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$, then $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, where ∂_μ is a partial derivative. With these definitions, the equations $\nabla_\alpha F^{*\alpha\beta} = 0$ are identically satisfied. The equations to be solved are

$$\partial_\alpha \left[\sqrt{-g}\, g^{\alpha\mu} g^{\beta\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu)\right] = 0\,, \qquad (1)$$

where $g^{\mu\nu}$ is the inverse metric.

We deal with test electromagnetic fields which evolve on a given gravitational background field. Several exact solutions for Maxwell equations in curved spacetimes have been found [1–8]. One of the most interesting solutions of Eq. (1), due to their physical relevance, are electromagnetic plane waves [9, 10].

We take plane waves to be described by $A_\mu = a_\mu e^{i\mathcal{S}}$, where a_μ is the amplitude and $\mathcal{S}$ the phase of the wave. Both are real quantities and, in principle, both depend on space and time. The four-wavevector of the wave is defined by

$$K_\mu = \nabla_\mu \mathcal{S} = \partial_\mu \mathcal{S}\,, \qquad (2)$$

where K_0 is identified with the frequency of the wave, whereas K_i are the components of the (three dimensional) wavevector. The nature of the propagation of the electromagnetic wave is determined by a constraint satisfied by the four dimensional wave. For example, in flat-spacetime $K_\mu K^\mu = 0$, and electromagnetic radiation evolves along null geodesics, i.e., the wave travels with the speed of light.

Usually, Maxwell equations (1) are solved using the geometrical optics approximation [1, 4, 9–11], where the wavelength of the wave is considered much smaller than any characteristic length scale of the gravitational field in which the wave evolves. Using this approximation, Maxwell equations (1) are solved perturbatively, and in this approximation, the electromagnetic plane wave solutions are described by [9]

$$K_\mu K^\mu = 0\,, \quad a_\mu K^\mu = 0\,, \quad \nabla_\mu \left(K^\mu a^\alpha a_\alpha\right) = 0\,. \qquad (3)$$

The first equation implies that plane waves follow null geodesics, whereas the second one shows that is the wave is transverse. The third one is the photon number conservation equation. Despite the simplicity of solution (3), there is no guarantee that an exact solution to Eqs. (1) will satisfy the same conditions (3) for any given gravitational field.

The purpose is this work is to show that there exist particular (plane wave) solutions to Maxwell equations which do not evolve along null geodesics. Wave solutions have been studied previously [2–6] but these articles are not focused on plane waves. In several of these works [4–6], the analogy of the gravitational field with a medium (with its corresponding susceptibility and permeability) is exhibited in an explicit way. Therefore, it seems appropriate to find out whether electromagnetic plane waves can follow paths which are different from null geodesics.

Electromagnetic waves in non–rotating spherically symmetric spacetimes.- Before proceeding to find solutions propagating in Gödel and Kerr metrics, we show how the null geodesics propagation of light emerges from Maxwell equations (1) in non–rotating spherically symmetric spacetimes. Examples of these background gravitational fields are Schwarzschild, Reissner-Nordstrom [9], Friedmann–Robertson–Walker (FRW) [12] and wormholes [13], for instance. We consider a general symmetric metric in spherical coordinates ($t = x^0, r, \theta, \phi$), such that $g_{00} = f(t)q(r)$, $g_{rr} = h(t)b(r)$, $g_{\theta\theta} = h(t)r^2$, and $g_{\phi\phi} = h(t)r^2\sin^2\theta$, where $f(t), h(t), q(r)$ and $b(r)$ are (up to now) arbitrary functions of t and r respectively. All other metric components vanish.

A simple particular solution of Eqs. (1) can be found if the potential is chosen to be $A_\mu(t, r) = A_\phi(t, r)\delta_{\phi\mu}$,

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where δ is the Kronecker delta (a similar solution can be found for non–vanishing A_θ). Thus, Eqs. (1) simplify to

$$\sqrt{\frac{h}{f}}\partial_0\left(\sqrt{\frac{h}{f}}\partial_0 A_\phi\right)+\sqrt{\frac{q}{b}}\partial_r\left(\sqrt{\frac{q}{b}}\partial_r A_\phi\right)=0\,. \tag{4}$$

Defining new time and radial coordinates as $\tau = \int dt\sqrt{-f/h}$, and $\rho = \int dr\sqrt{b/q}$, the previous equation becomes the flat-spacetime wave equation $\partial_\tau^2 A_\phi - \partial_\rho^2 A_\phi = 0$, implying that A_ϕ may be written in a plane wave fashion as

$$A_\phi = \exp\left(i\omega\int dt\sqrt{-\frac{f}{h}}\pm i\omega\int dr\sqrt{\frac{b}{q}}\right)\,, \tag{5}$$

where ω is a constant. This solution defines the wavevectors $K_0=\omega\sqrt{-f/h}$, and $K_r=\pm\omega\sqrt{b/q}$, which satisfy

$$K_\mu K^\mu = g^{00}K_0^2+g^{rr}K_r^2=\frac{K_0^2}{fq}+\frac{K_r^2}{hb}=0\,. \tag{6}$$

Therefore, all non–rotating spherically symmetric spacetimes have electromagnetic plane wave solutions travelling along null geodesics which are transversal waves $A_\mu K^\mu = 0$. In general for this case, if the plane wave ansatz $A_\phi(t,r) = a(r)e^{itK_0+iS(r)}$ is made in Eq. (4), where a is the amplitude of the wave and $K_r=\partial_r S$, then one can obtain a solution for which the amplitude is constant, the wave follows null geodesics as in Eq. (6), and photon number is conserved [9].

Let us consider two very well known cases, namely, the Schwarzschild and FRW spacetimes. The null-geodesic behavior of the light (6), implies that for Schwarzschild spacetime the plane wave dispersion relation is

$$K_0=\pm\left(1-\frac{2M}{r}\right)K_r\,. \tag{7}$$

This solution corresponds to the well known gravitational redshift effect, where the plane wave is moving with group velocity $\partial K_0/\partial K_r = \pm(1-2GM/r)\neq\pm 1$. On the other hand, for the FRW spacetime, the wave disperses as

$$K_0=\pm\frac{\sqrt{1-kr^2}}{a}K_r\,, \tag{8}$$

which is the cosmological redshift for a Universe with curvature $k=-1,0,1$.

Electromagnetic waves in Gödel spacetime.- The Gödel metric describes a rotating Universe which features closed timelike curves. This metric is stationary [6, 14] and it is written in cartesian coordinates as $g_{00}=-1=-g_{xx}=-g_{zz}$, $g_{yy}=-2+4\exp(\sqrt{2}x\Omega)-\exp(2\sqrt{2}x\Omega)$, and $g_{0y}=\sqrt{2}[1-\exp(\sqrt{2}x\Omega)]$, where Ω is a constant related to the angular velocity of the rotating universe (which reaches the flat spacetime limit when $\Omega\to 0$). All other metric components vanish.

A particular solution to Eq. (1) can be found when the potential is chosen to be $A_\mu(t,x)=A_z(t,x)\delta_{\mu z}$. In this case, Maxwell equations reduce to

$$\partial_0^2 A_z+\frac{1}{\sqrt{-g}g^{00}}\partial_x\left(\sqrt{-g}\partial_x A_z\right)=0\,, \tag{9}$$

where $g=-\exp(2\sqrt{2}x\Omega)$ is the metric determinant and $g^{00}=1+2\exp(-2\sqrt{2}x\Omega)-4\exp(-\sqrt{2}x\Omega)$. To find the propagation of plane waves described by Eq. (9), one can define the variable $\xi=-e^{-\sqrt{2}x\Omega}/(\sqrt{2}\Omega)$, to rewrite the previous equation as $\partial_0^2 A_z+\beta(\xi)\partial_\xi^2 A_z=0$, where $\beta(\xi)=2\Omega^2\xi^2/(1+4\Omega^2\xi^2+4\sqrt{2}\Omega\xi)$. We can see that the wave equation cannot be cast in a flat spacetime analogue version, and thereby, the null geodesic behavior of the light in Gödel spacetimes is ruled out. To explicitly show this, let us go back to Eq. (9) and perform the plane wave ansatz $A_z(t,x)=a(x)\exp[i\omega t\pm iS(x)]$, where a is the wave amplitude and ω is a constant. Here, the wavevectors are $K_0=\omega$ and $K_x=\pm\partial_x S$, which allow us to describe a transversal electromagnetic plane wave $A_\mu K^\mu\equiv 0$. Using this ansatz in (9) we find

$$\begin{aligned} K_\mu K^\mu &= \frac{1}{\sqrt{-g}a}\partial_x\left(\sqrt{-g}\partial_x a\right)\,,\\ 0 &= \partial_x\left(\sqrt{-g}K_x a^2\right)\,, \end{aligned} \tag{10}$$

where $K_\mu K^\mu \equiv g^{00}K_0^2+K_x^2$. The first equation is the dispersion relation of the wave determining the behavior of light, and the second one is the photon number conservation. Interestingly, there is no solution of the previous system with constant amplitude. Also, there is no solution consistent with $K_\mu K^\mu = 0$. For such a case, Eqs. (10) become three different and inconsistent conditions for the two variables K_x and a.

From the last of Eqs. (10) one gets

$$a=\frac{1}{(-g)^{1/4}K_x^{1/2}}\,. \tag{11}$$

This solution can be used in the first of Eqs. (10) to get the dispersion relation

$$g^{00}\omega^2+K_x^2=-\frac{\Omega^2}{2}-\frac{K_x''}{2K_x}+\frac{3K_x'^2}{4K_x^2}\,, \tag{12}$$

where $'\equiv\partial_x$. From the previous exact dispersion relation, we can see that for Gödel spacetime a particular electromagnetic plane wave does not propagate in null geodesics. The exact geodesic behavior can be found by solving the differential equation (12) for K_x. This feature has its origin in the rotation of the spacetime, which modifies the path followed by photons (it is direct to obtain the correct null geodesic flat spacetime limit when Ω vanishes).

It is illustrative to calculate the solutions in the case $\Omega\, x \ll 1$. At second order in Ω, the solution to (12) is

$$K_x\approx\omega-2\Omega^2x^2\omega+\frac{3\Omega^2}{2\omega}\sin^2(\omega x)\,, \tag{13}$$

that allow us to recover the flat spacetime solution when $x = 0$ (and the metric becomes flat at that point). This solution implies that

$$K_\mu K^\mu = 3\Omega^2 \sin^2(\omega x)\,, \tag{14}$$

which is always positive. Thus, electromagnetic plane waves in a slowly rotating Gödel Universe propagates in space-like trajectories. This can be easily seen at small lenght scales $\omega x \ll 1$, such that solution (13) simplifies to

$$\frac{\omega}{K_x} \approx 1 + \frac{1}{2}\Omega^2 x^2 \geq 1\,, \tag{15}$$

and the plane waves propagate with superluminal velocity for $0 \leq \Omega x \ll 1$.

This somewhat surprising behavior of an electromagnetic plane wave solution stems from the non-stationary character of the spacetime under consideration. Electromagnetic waves in Gödel spacetimes have been studied by Mashoon [6] in a general formalism. However, no explicit plane wave solutions were presented in his article. Because of this new result regarding wave propagation in Gödel spacetimes, it is natural to inquire whether propagation of electromagnetic plane waves in Kerr spacetime presents similar features.

Electromagnetic waves in Kerr spacetime.- The stationary Kerr metric describes a rotating black hole of mass M and effective angular momentum a. It has non-vanishing metric components $g_{\mu\mu} = g_{\mu\mu}(r, \theta)$ and $g_{\phi 0} = g_{0\phi} = g_{0\phi}(r, \theta)$, with $\mu = \{t = x^0, r, \theta, \phi\}$. Explicitly, the metric in Boyer-Lindquist coordinates is $g_{00} = -1 + 2Mr/\rho^2$, $g_{rr} = \rho^2/\Delta$, $g_{\theta\theta} = \rho^2$, $g_{\phi\phi} = r^2 + a^2 + 2Mra^2 \sin^2\theta/\rho^2$, and $g_{0\phi} = -4Mar\sin^2\theta/\rho^2$, where $\rho^2 = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 - 2Mr + a^2$. In contrast to the two previous cases, now the metric depends on two spatial variables. We show below that Maxwell equations (1) can be solved exactly for all of the potential components (without choosing a gauge) for the Kerr metric.

Write the four-vector potential components for this case as $A_\mu(t, r, \theta)$, with no ϕ-dependence, for simplicity. With this assumption, the time-component of Maxwell equation (1) can be understood as a vanishing curl statement and may thus be solved by introducing a new field $\chi = \chi(t, r, \theta)$ such that

$$\begin{aligned}
\partial_\theta \chi &= -\sqrt{-g}g^{rr}\left[g^{00}\left(\partial_r A_0 - \partial_0 A_r\right) + g^{0\phi}\partial_r A_\phi\right]\,, \\
\partial_r \chi &= \sqrt{-g}g^{\theta\theta}\left[g^{00}\left(\partial_\theta A_0 - \partial_0 A_\theta\right) + g^{0\phi}\partial_\theta A_\phi\right]\,,
\end{aligned} \tag{16}$$

which identically satisfies the time-component of Eq. (1). The introduction of the new field χ also allows us to solve the r and θ-components of Eq. (1). Both equations reduce to one equation, namely,

$$\partial_0 \chi = \sqrt{-g}g^{rr}g^{\theta\theta}\left(\partial_r A_\theta - \partial_\theta A_r\right)\,. \tag{17}$$

Finally, the ϕ-component of Maxwell equations may be written as

$$\begin{aligned}
0 &= \sqrt{-g}\frac{g^{00}}{g_{\phi\phi}}\partial_0^2 A_\phi + \partial_r\left(\sqrt{-g}\frac{g^{rr}}{g_{\phi\phi}}\partial_r A_\phi\right) \\
&\quad + \partial_\theta\left(\sqrt{-g}\frac{g^{\theta\theta}}{g_{\phi\phi}}\partial_\theta A_\phi\right) - \partial_r\beta\partial_\theta\chi + \partial_\theta\beta\partial_r\chi\,,
\end{aligned} \tag{18}$$

where $\beta = g^{0\phi}/g^{00} = -g_{0\phi}/g_{\phi\phi}$ is related to the rotation rate of the black hole. On the other hand, as the metric is time-independent, from Eqs. (16) and (17) we can find an evolution equation for the χ field

$$\begin{aligned}
0 &= \sqrt{-g}\frac{g^{00}}{g_{\phi\phi}}\partial_0^2 \chi + \partial_r\left(\sqrt{-g}\frac{g^{rr}}{g_{\phi\phi}}\partial_r \chi\right) \\
&\quad + \partial_\theta\left(\sqrt{-g}\frac{g^{\theta\theta}}{g_{\phi\phi}}\partial_\theta \chi\right) + \partial_r\beta\partial_\theta A_\phi - \partial_\theta\beta\partial_r A_\phi
\end{aligned} \tag{19}$$

Now, definining the complex potential $Z_\pm = A_\phi \pm i\chi$, Eqs. (18) and (19) can be merged to

$$\begin{aligned}
0 &= \sqrt{-g}\frac{g^{00}}{g_{\phi\phi}}\partial_0^2 Z_\pm + \partial_r\left(\sqrt{-g}\frac{g^{rr}}{g_{\phi\phi}}\partial_r Z_\pm\right) \\
&\quad + \partial_\theta\left(\sqrt{-g}\frac{g^{\theta\theta}}{g_{\phi\phi}}\partial_\theta Z_\pm\right) \pm i\partial_r\beta\partial_\theta Z_\pm \mp i\partial_\theta\beta\partial_r Z_\pm\,.
\end{aligned} \tag{20}$$

In this way, the problem of getting the solutions to the four Maxwell equations (1), are now reduced to solve the two uncoupled equations (20). The time derivatives of the fields $Z_\pm$ can be identified with the two polarizations of a wave propagating in the ϕ-direction

$$\begin{aligned}
\partial_0 Z_\pm &= \partial_0 A_\phi \mp i\sqrt{-g}g^{rr}g^{\theta\theta}\left(\partial_r A_\theta - \partial_\theta A_r\right) \\
&= F_{0\phi} \mp i\sqrt{-g}F^{r\theta} = E_\phi \mp i\sqrt{-g}g^{rr}g^{\theta\theta}B_\phi
\end{aligned} \tag{21}$$

where E_ϕ and B_ϕ are the electric and magnetic fields in the ϕ-direction.

Eqs. (20) take into account the polarization of the plane wave. This means that the right-handed and left-handed polarizations can couple to the black hole rotation through the derivatives of β. If the spacetime is static (as in Schwarzschild case) this effect does not appear. This feature has been previously envisaged [6], but no exact solution for a plane wave was presented there. On the other hand, the uncoupled equations (20) for the two polarizations are different from the renowned Teukolsky equation [2, 3] for Kerr spacetime, which is coordinate separable. The Teukolsky equation is obtained through the use of the Newman-Penrose formalism to obtain second order differential equations for some fields that correspond to projections of the electric and magnetic fields on the Kinnersley's null tetrad. To clarify the relationship of our work to the one which appears in [2], it is worth mentioning that Teukolsky's approach yields third order differential equations for the electromagnetic potentials while our approach deals with second order equations for the same fields.

Now, to find a plane wave solution, the polarization function is written as $Z_{\pm}(t,r,\theta)=\xi_{\pm}(r,\theta)e^{i\omega t+iS_{\pm}(r,\theta)}$, where ω is a constant and $\xi_{\pm}$ is the amplitude of the wave (in inverse length units). Anew, the four wavevector components are defined as $K_{0\pm}=\omega$, $K_{r\pm}=\partial_r S_{\pm}$, $K_{\theta\pm}=\partial_\theta S_{\pm}$, and $K_{\phi\pm}=0$. Also notice that $A_\mu K^\mu{}_\pm = g^{00}\omega A_0+g^{rr}K_{r\pm}A_r+g^{\theta\theta}K_{\theta\pm}A_\theta\neq 0$ in general for Kerr spacetime and, therefore, the wave is not transverse. Using this decomposition, Eq. (20) gives rise to the dispersion relation

$$\begin{aligned}\frac{\sqrt{-g}}{g_{\phi\phi}}K^\mu{}_\pm K_{\mu\pm} &= \mp\partial_r\beta K_{\theta\pm}\pm\partial_\theta\beta K_{r\pm}\\ &+\frac{1}{\xi_\pm}\partial_r\left(\sqrt{-g}\frac{g^{rr}}{g_{\phi\phi}}\partial_r\xi_\pm\right)+\frac{1}{\xi_\pm}\partial_\theta\left(\sqrt{-g}\frac{g^{\theta\theta}}{g_{\phi\phi}}\partial_\theta\xi_\pm\right)\end{aligned} \tag{22}$$

where $K^\mu{}_\pm K_{\mu\pm}\equiv g^{00}\omega^2+g^{rr}K_{r\pm}^2+g^{\theta\theta}K_{\theta\pm}^2$, and a generalized photon number conservation law for Kerr spacetime given by

$$\begin{aligned}0 &= \partial_r\left(\sqrt{-g}\frac{g^{rr}}{g_{\phi\phi}}K_{r\pm}\right)+\partial_\theta\left(\sqrt{-g}\frac{g^{\theta\theta}}{g_{\phi\phi}}K_{\theta\pm}\right)\\ &+\left(2\sqrt{-g}\frac{g^{rr}}{g_{\phi\phi}}K_{r\pm}\mp\partial_\theta\beta\right)\frac{\partial_r\xi_\pm}{\xi_\pm}\\ &+\left(2\sqrt{-g}\frac{g^{\theta\theta}}{g_{\phi\phi}}K_{\theta\pm}\pm\partial_r\beta\right)\frac{\partial_\theta\xi_\pm}{\xi_\pm},\end{aligned} \tag{23}$$

which can be cast in a more appealing fashion

$$\begin{aligned}0 &= \partial_r\left[\left(\sqrt{-g}\frac{g^{rr}}{g_{\phi\phi}}K_{r\pm}\mp\frac{1}{2}\partial_\theta\beta\right)\xi_\pm^2\right]\\ &+\partial_\theta\left[\left(\sqrt{-g}\frac{g^{\theta\theta}}{g_{\phi\phi}}K_{\theta\pm}\pm\frac{1}{2}\partial_r\beta\right)\xi_\pm^2\right].\end{aligned} \tag{24}$$

In principle, the value of $K^\mu K_\mu$ should be determined by the dynamical equations. A consistent solution of Eq. (23) [or (24)] should specify the behavior of a plane wave travelling on a Kerr background. With this in mind, Eq. (23) [or (24)] must be solved in order to yield an expression for the wave amplitude. A particular solution to Eq. (24) is

$$\begin{aligned}\sqrt{-g}\frac{g^{rr}}{g_{\phi\phi}}K_{r\pm}\mp\frac{1}{2}\partial_\theta\beta &= \partial_\theta\xi_\pm+\frac{\lambda(\theta)}{\xi_\pm^2},\\ \sqrt{-g}\frac{g^{\theta\theta}}{g_{\phi\phi}}K_{\theta\pm}\pm\frac{1}{2}\partial_r\beta &= -\partial_r\xi_\pm,\end{aligned} \tag{25}$$

where λ is an arbitrary function of θ. This solution has the correct Schwarzschild spacetime limit when the wave has constant amplitude $\xi=\xi_0$, $\beta\to 0$, $K_\theta\to 0$ and $\lambda=\omega\xi_0^2/\sin\theta$. From Eqs. (25) we are able to find a solution for the wave amplitude that can be obtained by manipulating the equations of that set. In general, from Eqs. (25), we get (for a non-constant amplitude)

$$\xi_\pm = \lambda^{1/3}\left[2\sqrt{-g}\frac{g^{\theta\theta}}{g_{\phi\phi}}K_{\theta\pm}\pm\partial_r\beta\right]^{1/3}\left[\partial_r\left(\sqrt{-g}\frac{g^{rr}}{g_{\phi\phi}}K_{r\pm}\right)+\partial_\theta\left(\sqrt{-g}\frac{g^{\theta\theta}}{g_{\phi\phi}}K_{\theta\pm}\right)\right]^{-1/3}, \tag{26}$$

that solves the photon number conservation equation in Kerr spacetime. The same result may be gotten using Eq. (23). The solutions (25) can be used in dispersion relation (22) to finally yield

$$\begin{aligned}\frac{\sqrt{-g}}{g_{\phi\phi}}K^\mu{}_\pm K_{\mu\pm} &= \mp\partial_r\beta K_{\theta\pm}\pm\partial_\theta\beta K_{r\pm}+\frac{1}{\xi_\pm}\left[\partial_\theta\left(\beta^2-\frac{g_{00}}{g_{\phi\phi}}\right)K_{r\pm}-\partial_r\left(\beta^2-\frac{g_{00}}{g_{\phi\phi}}\right)K_{\theta\pm}\right]\\ &\mp\frac{1}{\xi_\pm}\left[\partial_r\left(\frac{\sqrt{-g}g^{rr}}{2g_{\phi\phi}}\partial_r\beta\right)+\partial_\theta\left(\frac{\sqrt{-g}g^{\theta\theta}}{2g_{\phi\phi}}\partial_\theta\beta\right)\right]-\frac{\omega\xi_0^2}{\xi_\pm^3}\partial_\theta\left(\frac{\sqrt{-g}g^{\theta\theta}}{\sin\theta g_{\phi\phi}}\right)\mp\frac{\sqrt{-g}\omega\xi_0^2g^{\theta\theta}\partial_\theta\beta}{\sin\theta g_{\phi\phi}\xi_\pm^4}\\ &+\frac{2\omega\xi_0^2}{\sin\theta\xi_\pm^4}\left(\beta^2-\frac{g_{00}}{g_{\phi\phi}}\right)K_{r\pm}-\frac{2\sqrt{-g}g^{\theta\theta}\omega^2\xi_0^4}{\sin^2\theta g_{\phi\phi}\xi_\pm^6},\end{aligned} \tag{27}$$

where we have used $\lambda=\omega\xi_0^2/\sin\theta$ (to match the Schwarzschild limit), and where the terms $\xi_\pm$, $\xi_\pm^3$, $\xi_\pm^4$ and $\xi_\pm^6$ in (27) must be replaced using Eq. (26).

We can see from the above dispersion relation that the plane wave has $K^\mu{}_\pm K_{\mu\pm}\neq 0$, and consequently the electromagnetic plane wave does not travel along null geodesics. The proper behavior of the wave can be found by solving the differential equation (27) for $S_\pm$. It is worth noting that both evolution equations for Z_+ and Z_- couple differently to the derivatives of β so, the solutions for each polarization state are different, in general. This effect is a direct consequence of the coupling of light polarization and the rotation of the central mass. In this way, light can travel along different paths at different speeds depending on its polarization. This an effect previously suggested by Masshhon [6] and it is analogous to the well known Faraday rotation effect in plasmas [15]. On the other hand, this polarization-gravity coupling effect is intimately related with photon spin coupling to gravitional fields studied in Ref. [16] from a classical viewpoint.

Discussion.- The propagation of light along null geodesics is an exact result for plane waves propagating in vaccum flat spacetimes [17], or an approximate geometrical optics limit for light propagation in curved spacetimes. However, we have shown that in Gödel and Kerr spacetimes, this behavior changes, and the electromagnetic plane wave does not follow null geodesics. This is due to the rotational nature of the spacetime (i.e., non-diagonal components of the metric) that produce an effective anisotropic medium where the photons propagate.

Therefore, there appears to be *no such as thing as the speed of light*, but electromagnetic radiation propagates with different speeds (different $K_\mu K^\mu$ values) depending on the interaction of its polarization with the gravitational background. One may still define the speed of light which corresponds to waves which propagate obeying $K_\mu K^\mu = 0$.

This remarkable result could have strong implications in astrophysics, where accurate measurements of the speed of light are crucial . Yet, the more stricking new idea emerges from Eq. (27) for Kerr metric, where the importance of the light polarization in the wave propagation is explicitly shown. It is well known that the polarization of a wave affects its propagation properties in a medium [10, 15], but to the best of our knowledge, no such predictions have been reported for plane waves travelling in vaccum in curved spacetime. Nevertheless, the relevance of the polarization of an electromagnetic wave in flat spacetime has been put in manifest in experiments showing that structured light waves (not plane waves) in vaccum can travel slower than speed of light [18, 19].

In conclusion, the gravitational field can alter the path that an electromagnetic plane wave follows. This is relevant for the understanding of the evolution and interaction of electromagnetic fields with matter at large scales in the Universe, and it could give hints in the interpretation of the non-constancy evidence of some fundamental physical parameters [20].

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