

Gravity-induced geometric spin Hall effect as a probe of universality of free fall of quantum waves

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We present a novel fundamental effect that for the matter waves the space-averaging free-fall point of quantum particles undergoes a spin-dependent transverse shift in the gravitational field of Earth. This effect is similar to the geometric spin Hall effect (GSHE) [Aiello et al., Phys. Rev. Lett. 103, 100401 (2009)] and can be called gravity-induced GSHE. This effect suggests that there might be violations of the universality of free fall (UFF) or weak equivalence principle (WEP) in the quantum domain.

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The universality of free fall (UFF) is tested as one of the so-called weak form of the Einstein equivalence principle which is the most important guide on establishing Einstein's general relativity. The classical tests of UFF with macroscopic masses have achieved a high precision of about 10^{-13} [1, 2] and no violations are found so far. Since 1960s, to extend the domain of the test body, the verifications of the UFF based on microscopic particles in the quantum regime have been studied theoretically and experimentally (for a recent review see [3]). Recently, WEP test experiments using atom interferometers are proposed to reach the level of 10^{-15} [4, 5]. The increasing interest of using quantum systems is not only to pursue high precision tests based on quantum systems offering a varied properties, e.g. charge [6], properties of anti-matter [7–9], spin [10–12] and internal structure, but also to test possible violations of equivalence principle allowed by theoretic work, such as spin-gravity coupling [13–16], spin-torsion coupling [17–19], extended or modified theories of gravity, and almost all of tentative theories to reconcile or unify the general relativity and the standard model of particle physics [20, 21].

In fact, quantum particles, which are different from classical point-like particles, are extended in space and display wave-like features. Thus, the notion of quantum WEP should be different from the conventional WEP for classical particles [22–24]. In this paper, considering the properties of matter waves, we present a remarkable phenomenon that the space-averaging free-fall point of quantum particles allows a spin-dependent transverse split in the gravitational field. Since such effect is similar to the result of geometric spin Hall effect, we call it gravity-induced GSHE.

The geometric spin Hall effect of light [25, 26] was proposed in 2009, which states that a spin-dependent transverse displacement of a light intensity centroid is observed in a plane tilted with respect to the propagation of the light beam. Unlike the conventional spin Hall

effect of light as a result of light-matter interaction [27–29], the GSHE of light is of purely geometric nature. On the other hand, the gravity-induced GSHE differs from the gravitational Hall effect (or similar effects) presented in the literatures [30–32]. For light beams, the so-called gravitational Hall effect describes a helicity-dependent local deviation from the photon geodesic in general relativity. Since the gravitational deflection of light is small in terrestrial experiments, in the following discussion we discuss the gravity-induced GSHE with polarized electron beam. It can be extends to other matter waves such as polarized neutron and atom beams, etc.

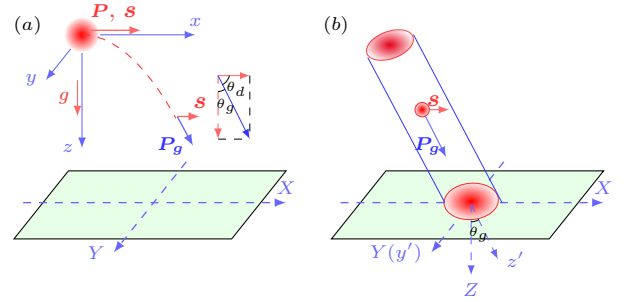


FIG. 1. (a): A sketch of a longitudinally polarized electron beam originally propagating in the x direction and the detection plane $X - Y$ in the horizontal direction. θ_d is the deflection angular of the electron beam in the action of the Earth's gravity and $\theta_g = \pi/2 - \theta_d$. (b): The equivalent schematic diagram of geometric spin Hall effect occurring without the Earth's gravitational effect. The electron beam moving towards the horizontal detection plane with titled angular θ_g , energy ε_g and momentum p_g and spin in the horizontal direction approximately for every electron. The laboratory frame is denoted by XYZ frame, and the beam frame is $x'y'z'$ frame.

The structure of this paper is as follows. We first derive our result of gravity-induced GSHE in a simple method without knowledge of the detailed wavefunction of the particle beam. Next, we discuss other possible case of gravity-induced GSHE further. Finally, we give our conclusions.

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Specifically, let us first consider a system of a longitudinally polarized electron beam originally propagating in the horizontal direction (x -axis) at a certain height h_g and the detection plane sitting in the horizontal plane in the action of the Earth's gravity [see Fig. 1(a)]. Our results show that the electron's space-averaging point of free fall will be shifted by $\delta \propto \lambda_g/(2\pi)\sigma \cos^{-1}\theta_g$ along the y axis with respect to its classical point of free fall. Here $\sigma = \pm 1/2$ is the original spin of electron in the propagation direction of the electron beam and the sign of σ denotes its handedness with respect to the original direction of propagation. This displacement's magnitude is of the order of the original wavelength λ of the beam. This effect implies that electrons or other quantum particles in different spin orientations follow "different paths", which suggests possible violations of the UFF to some extent.

To understand the gravity-induced GSHE, we just transform it to the case of GSHE without gravitational effect. Here we adopt two approximations. First, the spin rotation or precession of the electron due to the action of the Earth's gravity can be omitted. Second, the movement of the electron's wave-packet-center can be approximately replaced by its classical trajectory. These two approximations are acceptable and for related discussions please refer to Refs [33–35]. Based on such approximations, our question can be treated as a pure question of GSHE. Now consider an electron beam moving towards the horizontal detection plane with titled angular θ_g , energy ε_g , momentum p_g and spin in the horizontal direction approximately for every electron, and the gravitational effect no longer appears [see Fig. 1(b)]. As long as we treat approximately the electron as a classical particle, the titled angular θ_g , energy ε_g , momentum p_g can be calculated by Newton's law of gravitation, all of which are determined by the original wavelength λ (or momentum) of the electron beam and the height h_g .

To get the space-averaging free-fall point of electron, what we should evaluate is the spatial distribution of the intensity of electron beam in the horizontal detection plane. In fact, the energy-momentum (E-M) tensor $T^{\mu\nu}$ of field, which bands together the energy density, momentum density, energy flux density and momentum flux density, is a very convenient tool to analyze the properties of field. Following the article of Aiello et al. [25], here we also use the energy flux to represent the intensity of electron beam in the horizontal detection plane. Thus, the space-averaging free-fall point of electron can be determined by the barycenter of the energy flux T^{z0} across the horizontal detection plane:

$$\langle y \rangle_{gk} = \int y \bar{T}^{z0} dx dy / \int \bar{T}^{z0} dx dy. \quad (1)$$

For simplicity, we transform from the laboratory frame to the beam frame. The beam frame and laboratory frame are connected by a rotation transformation i.e. $x^\mu = \Lambda^\mu_\nu x'^\nu$ [or $x'^\mu = (\Lambda^{-1})^\mu_\nu x^\nu$]. Here the rotation

transformation matrix is

$$\Lambda^\mu_\nu = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_g & 0 & \sin\theta_g \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\theta_g & 0 & \cos\theta_g \end{bmatrix}, \quad (2)$$

and so we get the related transformation of energy flux in these two frames

$$T^{z0}(x) = \Lambda^z_\alpha \Lambda^0_\beta T'^{\alpha\beta}(x') = T'^{z0} \cos\theta_g - T'^{x0} \sin\theta_g. \quad (3)$$

Then we obtain

$$\langle y \rangle_{gk} = \frac{\int y' (\bar{T}'^{z0} \cos\theta_g - \bar{T}'^{x0} \sin\theta_g) dx' dy'}{\int (\bar{T}'^{z0} \cos\theta_g - \bar{T}'^{x0} \sin\theta_g) dx' dy'}. \quad (4)$$

For electron field as Dirac field, we can take the familiar symmetric E-M tensor [36]:

$$T_{\text{sym}}^{\mu\nu} = \frac{i}{4} \bar{\psi} (\gamma^\mu \partial^\nu + \gamma^\nu \partial^\mu) \psi + h.c. \quad (5)$$

where $+h.c.$ indicates the addition of the Hermitian conjugate of the foregoing terms. One important use of the symmetric E-M tensor is to construct a conserved angular momentum tensor in a fully orbital-like form:

$$M_{\text{sym}}^{\lambda\mu\nu} = x^\mu T_{\text{sym}}^{\lambda\nu} - x^\nu T_{\text{sym}}^{\lambda\mu}. \quad (6)$$

In the beam frame, T'^{x0} can be ignored compared to T'^{z0} because the beam mainly carries energy along the propagation direction. Thus, the denominator in Eq. (4) leaves only the term T'^{z0} , and we have

$$\int \bar{T}'^{z0} dx' dy' = K'_z \simeq n \varepsilon_g, \quad (7)$$

where n is the electron number per unit time across the plane $x' - y'$, namely the electron number flux. Moreover, for the symmetric E-M tensor, we have the following angular momentum sum rules:

$$\begin{aligned} \int y' \bar{T}'^{z0} dx' dy' &= \frac{1}{2} \int (y' \bar{T}'^{z0}_{\text{sym}} - z' \bar{T}'^{y0}_{\text{sym}}) dx' dy' \\ &= \frac{1}{2} N \sigma_{x'} \hbar = \frac{1}{2} N \sigma \cos\theta_g \hbar, \end{aligned} \quad (8)$$

$$\begin{aligned} \int -y' \bar{T}'^{x0} dx' dy' &= \frac{1}{2} \int (x' \bar{T}'^{y0}_{\text{sym}} - y' \bar{T}'^{x0}_{\text{sym}}) dx' dy' \\ &= \frac{1}{2} N \sigma_{z'} \hbar = \frac{1}{2} N \sigma \sin\theta_g \hbar. \end{aligned} \quad (9)$$

Here N is the electron number per unit length along the direction of propagation and we have $n = v_g N = N p_g / \varepsilon_g$, where v_g is the velocity of the electron. Hence, for a beam with spin polarization we obtain the barycenter of energy flux of the symmetric E-M tensor:

$$\langle y_{\text{sym}} \rangle_{gk} = \frac{N \sigma (\cos^2\theta_g + \sin^2\theta_g) \hbar}{2n \varepsilon_g \cos\theta_g} = \frac{\lambda_g}{4\pi} \sigma / \cos\theta_g. \quad (10)$$

Here the angular-depended relation in Eq. (10) is $\cos^{-1}\theta_g$, which is different from $\tan\theta$ in Eq. (12) of Ref. [25]. This is because the electron is no longer in longitudinal polarization at the detection plane. We can give a simple explanation of result (10). The denominator in Eq. (1) is related to the component of electron's energy flux or momentum normal to the detection plane, $p_g \cos\theta_g$; the nominator in Eq. (1) corresponds to the projection of electron's spin in the detection plane, $\sigma\hbar$. Hence, we get immediately the result $\langle y \rangle_{gk} \propto \lambda_g \sigma / (2\pi) \cos^{-1}\theta_g$. This means that the position of the barycenter of the beam changes with its original spin orientation.

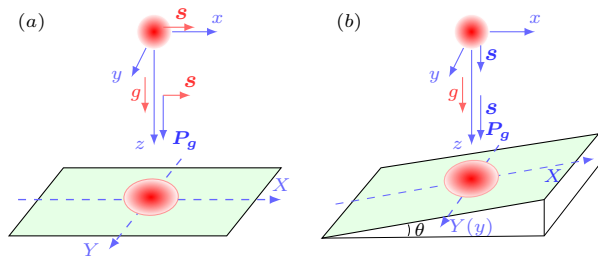


FIG. 2. (a): A sketch of static electrons freely falling to the detection plane $X-Y$ in the horizontal direction and carrying spin along the x direction. (b): A sketch of static electrons freely falling to the detection plane $X-Y$ tilted by an angle θ with respect to the horizontal direction and carrying spin along the z direction. The laboratory frame is denoted by XYZ frame, and the beam frame is xyz frame.

There are other possible configurations for the gravity-induced GSHE of matter particles. Here we consider two other simple systems. In both cases, depicted in Fig. 2, the electrons are first static and then released into free fall. Differences between the two cases are the spin polarizations of electron and the angles between the detection planes and the horizontal direction. Following the above analysis, we can easily get the result about the gravity-

induced GSHE for the case indicated by Fig. 2(a)

$$\langle y_{\text{sym}} \rangle_{gk}^a = \frac{N\sigma\hbar}{2n\varepsilon_g} = \frac{\lambda_g}{4\pi}\sigma, \quad (11)$$

and that for the case indicated by Fig. 2(b)

$$\langle y_{\text{sym}} \rangle_{gk}^b = \frac{N\sigma\hbar \sin\theta}{2n\varepsilon_g \cos\theta} = \frac{\lambda_g}{4\pi}\sigma \tan\theta. \quad (12)$$

Both Eq. (11) and (12) show that the free-fall points of electron yield asymmetric patterns in the detection planes. For the free-falling electrons with transverse spin polarization from the same height, they display the asymmetric pattern in the horizontal detection plane; for the free-falling electrons with longitudinal spin polarization they display the asymmetric pattern in the tilted detection plane, and moreover, they reach the detection plane with different fall distance and time. All of these results imply that the electrons experience “different path”. Especially, if the electrons have different spin polarizations, they yield “different path structure”.

In conclusion, we have demonstrated the existence of gravity-induced GSHE. The “free-fall points” of quantum particles vary with the polarization of spin, which originates from the wave nature of quantum particles. The measurement of this effect will be of great interest and importance, for it implying possible observation of the violation of universality of free fall in the quantum realm. In comparison, the existing tests of universality of free fall using atom interferometers mainly analyse the phase information of matter wave by interference to extract the so-called acceleration caused by gravitational field. We encourage experimentalists to test the gravity-induced GSHE as a new probe of universality of free fall of quantum particles, so as to clarify the notion of quantum WEP.

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