

Detect Gravitational Waves Using Twisted Light - Dipole Interaction of Photons and Gravitational Waves

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(Dated: April 12, 2019)

Motivated by the next generation of gravitational wave (GW) detectors, we study the wave mechanics of a twisted light beam in the GW perturbed spacetime. We found a new gravitational dipole interaction of photons and gravitational waves. Physically, this interaction is due to coupling between the angular momentum of twisted light and the GW polarizations. We demonstrate that for the higher-order Laguerre-Gauss (LG) modes, this coupling effect makes photons undergoing dipole transitions between different orbital-angular-momentum(OAM) eigenstates, and leads to some measurable optical features in the 2-D intensity pattern. It offers an alternative way to realize precision measurements of the gravitational waves, and enables us to extract more information about the physical properties of gravitational waves than the current interferometry. With a well-designed optical setup, this dipole interaction is expected to be justified in laboratories.

PACS numbers: 04.80.Nn, 04.30.Nk, 03.50.De, 42.25.p

The direct detection of gravitational waves by the advanced LIGO interferometer [1] and the advanced Virgo interferometer [2] marks the beginning of the era of gravitational wave astronomy. These discoveries stimulate the great efforts devoted to develop more advanced optical detection technology. With upgrading to next generation of interferometer, it is expected to move to design sensitivity and observe ever-increasing numbers of GW sources. The next generation of GW laser-interferometric GW detector will be upgraded to have an unprecedented sensitivity by minimizing various technical noises. One limiting noise arises from mirror thermal noise, producing by Brownian motion of particles in coatings and substrates. Besides the physically cooling of mirror by choosing an appropriate optical material, an alternative solution is to change the mode shape of the laser beam inside the interferometer. One option is to resonate the higher-order Laguerre Gauss (LG_p^l) modes in the detector arm cavities to smooth out the thermal noise fluctuations over a bigger portion of the mirror surface[3]. A well-developed technology has made it possible to produce higher-order LG-modes with high power output and high mode purity as required in the GW detection[4]. Therefore, it would be crucial to understand the wave mechanics of the higher-order LG modes interacting with the passing gravitational wave.

For light propagation in a gravitational field, the curved space background can be transformed to a linear optical medium, whose optical properties characterized by the effective dielectric tensor are fully specified by the spacetime geometries[5]. A typical example is the deflection of light in gravitational fields, this phenomenon can be illustrated by the correspondence between the refractive index and the scalar gravitational potential. Again, a rotating gravitational field exhibits the optical activity attributed to the vortical dragging

vector. Basically, it can be also equivalently described by the coupling between the macroscopic rotation Ω and the internal spin \mathbf{s} of photon[6, 7]. This coupling has a typical form of $\sim \gamma_s \Omega \cdot \mathbf{s}$ (γ_s is introduced to quantify the coupling strength), which may give rise to an extra phase shift for circular polarized light[6, 8], and the helicity flip for spin-1/2 particles[9]. Moreover, we noted that there exists another classical phase shift referred as Sagnac factor[10], currently explained by the dragging effect of frame in general relativity[11], can be understood alternatively by the coupling between the rotation and the orbital angular momentum (OAM) of photons, $\sim \gamma_L \Omega \cdot \mathbf{L}$, γ_L is the coupling strength.

On the other hand, the study of rotational feature in light phenomena has a long history, even tracking back to nineteenth century. In 1898, Sadovsky [12] gave the first prediction that the light with circular and elliptic polarization can exert a rotatory action upon material objects in context of classical Maxwell theory, and the repeated discovery had been made by Poynting based on analysis of the energy flow of polarized light. Later on, a circulatory flow of light energy has been realized in the focused flied structure. Actually, this vortical properties of light is associated with the OAM of photons. The blooming of researches on the OAM of light has been until 1992, when the angular momentum of a paraxial beam with the Laguerre-Gaussian modes was explicitly worked out by Allen and co-workers[13]. Particularly, the twisted light propagating in an inhomogeneous optical medium has receive much attentions, leading to some interesting predictions including optical Magnus effect [14] or optical spin Hall effect[15] etc.

This paper does not focus on the technical aspects of higher-order LG-beams in optical cavity, but on how the high-mode twisted light propagating in the ripples of spacetime disturbed by the gravitational waves. To

formulate a possible coupling effect between angular momentum of photon and the GWs polarization, we will take a Schrödinger's view of the Maxwell theory in curved space, and explore how this coupling effect manifests its significance in future GW experiments.

Adopting the Hanni's 3+1 decomposition[16], and introducing a complex wavefunction defined by $|\Psi\rangle = \mathbf{D} + i\mathbf{B}$, the Maxwell equation in curved space $\{g_{\mu\nu}\}$ takes the Schrödinger-like form[6],

$$i\frac{\partial}{\partial t}|\Psi\rangle = \nabla \times \left(|\Psi\rangle/\sqrt{g^{00}} + i\mathbf{g} \times |\Psi\rangle \right) \quad (1)$$

together with the transverse condition $\nabla \cdot |\Psi\rangle = 0$. In Eq.(1), \mathbf{g} is the 3d dragging vector $g_i = g_{0i}, g^i = g^{i0}/g^{00}$ and the vectorial operations are performed in the 3d curvilinear space $\gamma_{ij} = -g_{ij}, \gamma^{ij} = -g^{ij} + g^{0i}g^{0j}/g^{00}$, and the electromagnetic vectors are identified by $D^i = F^{0i}/\sqrt{g^{00}}, B^i = \epsilon^{ijk}F_{jk}/2\sqrt{\gamma}$, γ is the determinant of γ_{ij} . It is easy to verify that, in a free space, the Hamiltonian in Eq.(1) reduces to $H = \mathbf{k} \cdot \mathbf{s}$, where $\mathbf{k} = -i\nabla$ is the momentum operator, \mathbf{s} is the spin-1 operator given by the adjoint representation of group SO(3), i.e., $\{s^i\}^{jk} = -i\epsilon^{ijk}$. Obviously, for a plane wave, it has two helicity states, the positive helicity with positive energy $\omega = |\mathbf{k}|$, and the negative one with $\omega = -|\mathbf{k}|$, corresponding the right and left polarization states of photon. The zero helicity state is eliminated by the transverse condition $\mathbf{k} \cdot |\Psi\rangle = 0$. The opposite helicity states are mutually complex conjugates.

Without loss of generality, suppose an incident light propagating in z-axis, and the wavefunction $|\Psi\rangle$ is decomposed into the transverse component $|\Psi\rangle_{\perp} = \Psi_x \mathbf{e}_x + \Psi_y \mathbf{e}_y$ with respect to the z-component $|\Psi\rangle_z$. We introduce a 'helicity basis' $\{\mathbf{e}_{\pm}, \mathbf{e}_z\}$, satisfying $(\mathbf{e}_z \cdot \mathbf{s})\mathbf{e}_{\pm} = \pm \mathbf{e}_{\pm}$, which is related to the 'Cartesian frame' explicitly by $\mathbf{e}_{\pm} = (\mathbf{e}_x \pm i\mathbf{e}_y)/\sqrt{2}$. In the 'helicity basis' (denoted by superscript c), we define the transverse component $|\Psi\rangle_{\perp}^h = \Psi_+ \mathbf{e}_+ + \Psi_- \mathbf{e}_-$, the differential operator is thus $\nabla_{\perp} = \nabla_+ \mathbf{e}_+ + \nabla_- \mathbf{e}_-$ with $\nabla_{\pm} = (\nabla_x \mp i\nabla_y)/\sqrt{2}$, accordingly. By these conventions, it is easy to write down the Maxwell equation in the following matrix form

$$i\frac{\partial}{\partial t}\Psi^h = \nabla \times (\mathbf{U}\Psi^h) \quad (2)$$

where \mathbf{U} is the 3×3 dielectric tensor specified by the space metric of the GW, under the weak field approximation, $\mathbf{g} = 0$, and $g_{ij} = \eta_{ij} + h_{ij}$,

$$\mathbf{U} = \begin{pmatrix} \mathbf{I} + \mathbf{Q} & \mathbf{q} \\ \mathbf{q}^\dagger & 1 + q_0 \end{pmatrix} \quad (3)$$

here \mathbf{I} is the 2×2 unit matrix, $\mathbf{q} = \{Q_{+1}, Q_{-1}\}^T$ is the dipole momentum with the components $Q_{\pm 1} = \frac{1}{\sqrt{2}}(h_{13} \mp ih_{23})$, $q_0 = h_{33}$, and \mathbf{Q} is the 2×2 matrix with the matrix

elements, the monopoles $Q_{11} = Q_{22} = \frac{1}{2}(h_{11} + h_{22})$, and the quadrupoles $Q_{12} = \frac{1}{2}(h_{11} - h_{22}) - ih_{12}$, $Q_{21} = Q_{12}^*$.

For simplicity, we consider the positive helicity state only, in this case, we take the wavefunction in the form of $|\Psi\rangle_{\perp}^h = \Psi_0^+ e^{-i\omega t + ikz} \mathbf{e}_+$, accordingly, the z-component can be derived from the transverse condition, $\Psi_z = \frac{i}{k} \nabla_{\perp} \cdot |\Psi\rangle_{\perp}$ where the paraxial approximation was used. Since GW's wavelength is always assumed to be much larger than light beam, the geometric-optics approximation is valid, and the leading terms at the lowest order gives the eigenstate equation, from which we have $\omega = (1 + Q_0)k$ and an extra phase shift along the optical path $\phi = k \int Q_0 dz$. Keeping the first order terms yields

$$i\frac{\partial \Psi_0^+}{\partial z} = -\frac{1}{k'} \nabla_+ \nabla_- \Psi_0^+ - (\mathbf{q} \cdot \frac{1}{i} \nabla_{\perp}) \Psi_0^+ \quad (4)$$

where $k' = k(1 + Q_0)/(1 + q_0)$ resulting from the gravitational frequency shift. In addition, the GW's wavelength is also assumed to be much larger than the optical length L , thus the spatial derivative of the GW fields $\{h_{ij}\}$ can be neglected. If we keep the zeroth order only, Eq.(4) reduces to the familiar paraxial equation in the flat space

$$\nabla_{\perp}^2 \Psi_{0+} + 2ik' \frac{\partial}{\partial z} \Psi_{0+} = 0 \quad (5)$$

which has a set of solutions of the Laguerre-Gaussian modes in the cylindrical coordinates

$$LG_n^l = \frac{a_n^l}{w(z)} \left(\frac{\sqrt{2}\rho}{w(z)} \right)^{|l|} L_n^{|l|} \left(\frac{2\rho^2}{w(z)^2} \right) e^{-\rho^2/w(z)^2} e^{ik\rho^2/2R} e^{i\ell\phi} e^{-i\varphi(z)} \quad (6)$$

where n and l are the radial and the azimuthal indices, the order of the mode is given by $N = 2n + |l|$, the constant $a_n^l = (2n!/\pi(n + |l|)!)^{1/2}$, $w(z)$ is the width of mode, R the radius of the wavefront curvature, and the Gouy phase factor $\varphi(z) = (N + 1) \tan^{-1}(z/z_R)$, $z_R = \frac{1}{2}kw_0^2$, here $w_0 = w(z = 0)$ is the beam waist.

The second term in Eq.(4) is the perturbation due to the passing GW, which appears as a dipole interaction,

$$H_I = -\mathbf{q} \cdot \mathbf{k}_{\perp} \quad (7)$$

where $\mathbf{k}_{\perp} = -i\nabla_{\perp}$ is the transverse momentum operator, and the dipole momentum \mathbf{q} is given by

$$\mathbf{q} = h^+ [\mathbf{e}_k - (\mathbf{e}_g \cdot \mathbf{e}_k) \mathbf{e}_g] + h^\times (\mathbf{e}_g \times \mathbf{e}_k) \quad (8)$$

where \mathbf{e}_g and \mathbf{e}_k denote for the unit vectors in the propagation directions of the gravitational wave and light beam respectively. Following Eq.(8), $\mathbf{q} \cdot \mathbf{e}_g = 0$, which implies the dipole momentum \mathbf{q} being in the polarization plane of the GW. If the incident GW is propagating parallelly with the light beam, the dipole is identically zero, $\mathbf{q} = 0$. This dipole interaction is physically due to the coupling of GW polarization and the transverse momentum of photons. The coupling term actually consists of the two contributions, one is $\propto h^+(\mathbf{e}_g \cdot \mathbf{k}_{\perp})$, another is

$\propto h^\times[\mathbf{e}_g \cdot (\mathbf{e}_k \times \mathbf{k}_\perp)]$. Formally, the former is from the GW polarization coupled with the OAM density flow of photons, and the latter from the spin density flow.

In the "helicity basis", the dipole interaction can be alternatively written by

$$H_I = i[Q_{-1}\nabla_+ + Q_{+1}\nabla_-] \quad (9)$$

where $Q_{\pm 1} = \pm i h e^{\pm i\beta}(\mathbf{e}_g \cdot \mathbf{e}_\mp)$, h and β are given by

$$h = \sqrt{h_+^2 \cos^2 \theta_g + h_\times^2} \quad \tan \beta = \frac{h_+}{h_\times} \cos \theta_g \quad (10)$$

The complex differential operators ∇_\pm in the cylindrical coordinates become

$$\nabla_\pm = \frac{1}{\sqrt{2}} e^{\mp i\phi} (\partial_\rho \pm \frac{1}{\rho} L_z) \quad (11)$$

where $L_z = -i\partial_\phi$ is the orbital angular momentum operator in z-direction. Obviously, ∇_\pm play a role similar to the creation and destruction operators. While operating on the LG modes. ∇_\pm will either lowers or raises the orbital angular momentum by one unit. It is not difficult to work out the ladder relation,

$$\nabla_\pm |n, \pm l\rangle_{LG} = k_w [\sqrt{n+l} |n, \pm(l-1)\rangle_{LG} + \sqrt{n+1} |n+1, \pm(l-1)\rangle_{LG}] \quad (12)$$

$$\nabla_\mp |n, \pm l\rangle_{LG} = -k_w [\sqrt{n+l+1} |n, \pm(l+1)\rangle_{LG} + \sqrt{n} |n-1, \pm(l+1)\rangle_{LG}] \quad (13)$$

where we define the wavenumber $k_w = 1/w_0$. Combining Eqs.(9) (12) and (13) states that, for a given LG mode $|n, l\rangle_{LG}$, the dipole interaction will make the initial mode converting into two modes with the OAM number of $l-1$ and $l+1$, the difference between them is 2. It can be easily understood by the polarization states of gravitational wave having helicity $h = \pm 2$. Because Q_{+1} and Q_{-1} are conjugates of each other, the total orbit angular momentum is conserved. It's noted that the system modeled by Eq.(7) is an analogous to Jaynes-Cummings model describing the interaction of a two-level atom with a single quantized mode of the radiation field.

Since the Laguerre-Gaussian modes form a complete and orthonormal set with respect to the mode indices n and l in the polar plane $\{\rho, \phi\}$, we can make a decomposition such that

$$\Psi_{0+} = \sum_{m,k} \xi_{m,k}(z) |m, k\rangle_{LG} \quad (14)$$

Inserting this expansion Eq.(14) into Eq.(4), we have

$$\frac{d\xi_{n,l}(z)}{dz} = \sum_{m,k} \langle n, l | Q_{-1}\nabla_+ + Q_{+1}\nabla_- | m, k \rangle \quad (15)$$

$\xi_{n,l}(z)$ can be obtained by direct integration over the summation in above equation.

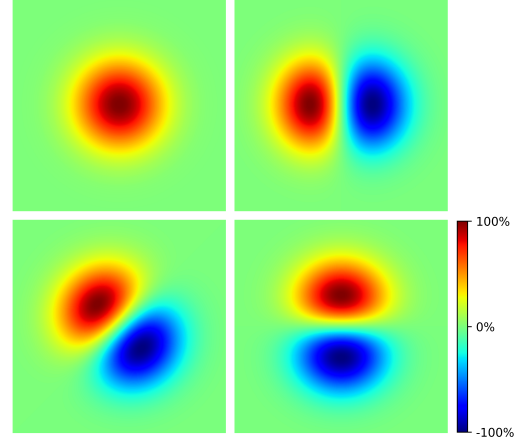


FIG. 1. The transverse self-interference intensity pattern for an incident Gaussian beam LG_0^0 (upper left) and the dipole structure with the rotation angles $\alpha = 0^\circ$, (upper right) $\alpha = 45^\circ$ (lower left) and $\alpha = 90^\circ$ (lower right), the color bar is labeled by the relative intensity.

In the following, we will present two typical ideal experiments to demonstrate the optical features induced by the dipole interaction. As a simplest case, let the incident beam is a Gaussian beam or LG_0^0 mode with zero OAM. Under the GW's perturbation, the dipole transition generates two splitting modes with the opposite unit OAMs, i.e., $l = \pm 1$,

$$|\text{out}\rangle = |\text{in}\rangle - Q_1 k_w L [e^{i\alpha} |0, +1\rangle_{LG} + e^{-i\alpha} |0, -1\rangle_{LG}] \quad (16)$$

where we have taken $Q_{\pm 1} = Q_1 e^{\pm i\alpha}$. The overall amplitude of $|0, \pm 1\rangle_{LG}$ is proportion to $Q_1 = \sqrt{h_{13}^2 + h_{23}^2}$. For the GW propagating in the $\{\theta_g, \phi_g\}$ direction, $Q_1 = \sin \theta_g h(\theta_g)$, and the rotation angle α , depending on the incident direction of the GW, is related to β by

$$\alpha + \phi_g = \frac{\pi}{2} + \beta \quad (17)$$

Eq.(16) indicates that, for the LG modes with the opposite OAM, the dipole interaction lead to extra opposite rotations by the angle α . This rotation should be visualized in the intensity pattern of light beam. In the first order of the GW strain $O(h)$, the readout intensity is

$$I \approx I_0 \left[1 - 4\sqrt{2} h k_w L \frac{\rho}{w(L)} \sin \theta_g \cos \varphi \cos(\phi + \alpha) \right] \quad (18)$$

where the strain h is given by Eq.(10), and $\varphi = \varphi(L)$ is the Gouy phase. Fig.(1) displays clearly the excess dipole components pointing to the direction α in the transverse intensity frame. In addition, it should be emphasized here that there exists an addition rotation freedom from the GW polarization around \mathbf{e}_g , with respect to which the polarizations $\{h_+, h_\times\}$ are defined. Performing an rotation in the transverse plane $\mathbf{e}'_\perp = R(\psi)\mathbf{e}_\perp$, (R

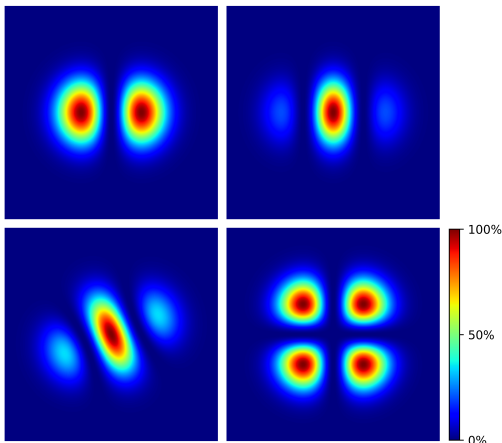


FIG. 2. The normalized transverse intensity pattern for an incident HG_{10} mode (upper left), and output excess mixed modes in different angles of $\alpha = 0^\circ$, (upper right) $\alpha = 45^\circ$ (lower left) and $\alpha = 90^\circ$ (lower right).

is a rotation transformation with an angle ψ), we have $\{h'_+, h'_\times\}^T = R(2\psi)\{h_+, h_\times\}^T$.

The second example illustrates an opposite case. Let us consider an incident beam of the Hermite-Gauss mode HG_{10} , which can be written by a superposition of two LG modes with the opposite OAM, $l = \pm 1$,

$$|\text{in}\rangle = HG_{10} = \frac{1}{\sqrt{2}}(|0, +1\rangle_{LG} + |0, -1\rangle_{LG}) \quad (19)$$

In the gravitational wave, there will appear extra four modes, we have

$$|\text{out}\rangle = |\text{in}\rangle + Q_1 k_w L \cos \alpha [|0, 0\rangle_{LG} + |1, 0\rangle_{LG}] \quad (20) \\ - \frac{1}{\sqrt{2}} Q_1 k_w L [e^{i\alpha} |0, +2\rangle_{LG} + e^{-i\alpha} |0, -2\rangle_{LG}]$$

In this numerical experiment, after subtracting the incident light intensity, we focus on the intensity pattern of the dipole induced modes only, which are thus quadratic in the strain $O(h^2)$. Clearly, one significant feature is brighten of the central intensity. For the $LG_0^{\pm 1}$ modes, it carries \hbar of OAM per photon and has a well-known donut-like intensity profile since the amplitude goes to zero at the center $\rho = 0$. Due to the dipole interaction, the induced mode involves one by lowering one unit of the OAM, which inevitably give rise to the vortex-free $|0, 0\rangle_{LG}$ mode, i.e., the Gaussian beam with the bright central spot. Fig.(2) demonstrates the normalized transverse intensity distribution in this experiment. Though this second order effect is quite small, it will be hopeful to detect it using the well-developed OAM sorting technique in future[17].

We summarize the paper and make some concluding remarks as follows. Taking a Schrödinger view of the Maxwell theory in curved space, we found a new gravitational dipole interaction between photons and GWs.

We demonstrate that this dipole interaction can produce some striking optical features in the 2-D intensity pattern, including (1) the induced dipole structure for the Gaussian beams, (2) a macroscopic rotation of the intensity pattern, depending not only on the GW incident direction, also on the polarization in the transverse plane as well as the ratio of cross and plus components of the GW, and (3) the central intensity brighten adding on the donut-like intensity profile for the lowest-order LG mode ($l = \pm 1$). Obviously, these features suggest an alternative way to measure GWs in the 2-D intensity space. Unlike the classical interference experiments for the GW detections[18], the dipole interaction makes the incident twisted light beam to form a multi-mode intensity profile in the image plane. In the first order of $O(h)$, the read-out signal is weaker than the current interference experiments by a factor of λ/w_0 , where λ is the wavelength of the laser beam, and w_0 the beam waist. This ratio may take a value of, e.g., $\sim O(10^{-1})$ or even less. However, the compensation for this decreasing signal strength can be made by extending the entire optical-path length of the 2-D coherent imaging. Most importantly, the benefit is clearly more physical information of the GWs inferred from the 2-D imaging. Finally, it is noted that the newly found dipole interaction, only existing beyond the plane wave approximation and having been unknown in the previous studies[18, 19], must be important for quantifying precisely the wave behaviors of the higher-order LG modes in optical cavity for the next generation of GW detectors.

Acknowledgments - FLL is supported by the Key Program of NFSC through grant 11733010 and 11333008, and the State Key Development Program for Basic Research of China (2015CB857000)

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