Geometrical contribution to neutrino mass matrix

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Abstract

It is well known that the dynamics of fermions on curved spacetime requires a spin connection, which contains a part called contorsion, related to torsion of the spacetime. This is an auxiliary field, without independent dynamics but fully expressible in terms of the axial current density of fermions. Its effect is the appearance of a quartic interaction of all fermions in the action, leading to a nonlinear Dirac equation involving all fermions present. Noting that the coupling of contorsion to fermions is not protected by any symmetry, thus allowing for different couplings to left and right-chiral fermions, we show that all fermions gain an effective mass when propagating through fermionic matter. This may have an observable effect on neutrino oscillations. In particular we find that different neutrino flavors can mix even if they have zero rest mass in vacuum.

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I. INTRODUCTION

The origin of neutrino mass is a mystery [1, 2]. The Standard Model of particle physics, so called because it encompasses all the known elementary particles and the interactions between them, explains the masses of elementary particles in terms of electroweak symmetry breaking and the vacuum expectation value of the Higgs field. The Higgs field in the electroweak theory is a complex doublet whose potential reaches a local minimum for a continuous range of configurations of the field, corresponding to a non-vanishing vacuum expectation value (vev) of the neutral scalar Higgs field. This is the phenomenon of spontaneous symmetry breaking, which means that the vacuum is not invariant under the symmetry transformations of the classical Lagrangian and the quantum Higgs field consists of fluctuations around this vev.

In the Standard Model, it is the $SU(2) \times U(1)$ electroweak symmetry which is spontaneously broken in the vacuum. For the fermions, this symmetry is a chiral one. Let us focus on the leptons, but what we say can be generalized to quarks quite easily. Left-handed components of leptons pair up into doublets of weak isospin $\Psi_{eL} = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ while a right-handed component has never been observed for the neutrino and thus the right-handed electron e_R is by necessity a singlet. The Higgs doublet field $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ couples left-handed doublets to the right-handed singlet via the Yukawa-type interaction

$$- h_e \left(\bar{\Psi}_{eL} \Phi e_R + \bar{e_R} \Phi^\dagger \Psi_{eL} \right) . \tag{1.1}$$

For quantization, the Higgs doublet is expanded around its vev as $\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\zeta) \end{pmatrix}$, with ϕ^+ , H , ζ being quantum fields, i.e. fields with vanishing vevs. Then the Yukawa terms can be written as

$$- h_e \left[\frac{v}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L) + \bar{\nu}_{eL} e_R \phi^+ + \bar{e}_R \nu_{eL} \phi^- + \frac{1}{\sqrt{2}} (\bar{e} e H + i \, \bar{e} \gamma_5 e \zeta) \right] . \tag{1.2}$$

The first term, which provides the mass of electrons, thus owes its existence to spontaneous symmetry breaking. Since the Standard Model does not include a right handed component for the neutrino, a mass term for the neutrino is not generated by the Standard Model interactions.

But this is not completely true, as was first noted by Wolfenstein [3]. Interactions with a medium results in effective masses for the neutrinos belonging to different lepton families, leading to mixing and oscillations between the different neutrinos, an effect that has been used to explain the solar neutrino problem, as well as the shortfall of electron-antineutrinos coming from reactors. Neutrino oscillations occur because the mass eigenstates of the neutrinos are not identical with their flavour eigenstates. But the neutrino masses must be non-vanishing as well as close to one another for this argument to explain neutrino oscillations in vacuum. In material media the effective mass of the neutrino is significantly modified because of interactions. The change is different for the electron neutrino ν_e , which has both charged-current and neutral-current interactions with the electrons in the medium, compared to ν_μ and ν_τ which have only the second kind of interaction. This can enhance neutrino oscillations significantly, depending on the distance travelled in matter by the neutrinos.

Let us first see what happens to neutrinos propagating in vacuum [9–11]. If the neutrinos are all massless and thus degenerate eigenstates of the Hamiltonian, there will be no oscillation. Suppose however that the neutrinos have mass, different masses for different species, and further that the mass eigenstates are not identical with the flavor eigenstates [1]. Then there will be mixing among neutrino eigenstates, which can be parametrized by a unitary matrix. The neutrino field ν_l which appears in a doublet with a lepton l is related to the field ν_α whose excitations are mass eigenstates by this matrix U as [24]

$$|\nu_{lL}\rangle = \sum_\alpha U_{l\alpha}|\nu_{\alpha L}\rangle \, . \tag{1.3}$$

At time t , the flavor eigenstates are related to the mass eigenstates by

$$|\nu_{lL}\rangle = \sum_\alpha e^{-iE_\alpha t} U_{l\alpha}|\nu_{\alpha L}\rangle \, . \tag{1.4}$$

Then the probability of finding a $\nu_{l'}$ at time t in a beam that had started out as ν_l is given by

$$\begin{aligned} P_{\nu_{l'}\nu_l}(t) &= |\langle \nu_{l'}|\nu_l(t)\rangle|^2 \\ &= \sum_{\alpha\,,\beta} \left| U^*_{l'\alpha} U_{l\alpha} U^*_{l\beta} U_{l'\beta} \right| \cos\left((E_\alpha - E_\beta)t - \phi_{ll'\alpha\beta}\right) \, , \end{aligned} \tag{1.5}$$

[1] This assumption represents departure from the Standard Model. It is justified *a posteriori* by the observation of neutrino oscillations [4–8]. The price of this assumption is the introduction of additional dynamical degrees of freedom, at scales beyond current limits of experimental observation, to protect electroweak gauge symmetry.

where $\phi_{ll'\alpha\beta} = \arg\left(U^*_{l'\alpha}U_{l\alpha}U^*_{l\beta}U_{l'\beta}\right)$. The neutrinos are ultrarelativistic and start with the same spatial momenta, so we can write their energies as $E_\alpha \simeq E + \frac{m_\alpha^2}{2E}$. We can also replace the time of travel t by the distance of travel x and write

$$P_{\nu_{l'}\nu_l}(t) = \sum_{\alpha,\beta} \left|U^*_{l'\alpha}U_{l\alpha}U^*_{l\beta}U_{l'\beta}\right| \cos\left(\frac{(m_\alpha^2 - m_\beta^2)x}{2E} - \phi_{ll'\alpha\beta}\right) . \tag{1.6}$$

Clearly there will be no mixing and no oscillation if the neutrinos have vanishing mass in the vacuum. On a curved spacetime however, geometry provides a contribution to the Hamiltonian, changing this conclusion.

II. FERMIONS IN CURVED SPACE

The dynamics of fermions in curved spacetime requires a spin connection, which specifies how the covariant derivative operator acts on spinors. The spin connection involves the Dirac matrices γ^I defined on an "internal" flat space, isomorphic to the tangent space at each point. In the presence of spinors it is convenient to describe gravity in terms of the spin connection A_μ^{IJ} and tetrads e_μ^I in a formulation that sometimes goes by the name of Einstein-Cartan-Kibble-Sciama (ECKS) gravity [12–15]. The Greek indices correspond to spacetime and Latin indices belong to the internal space, the tetrads relating the metric $g_{\mu\nu}$ of spacetime with the Minkowski metric $\eta_{IJ}\,(-+++)$ of the local tangent space through the relations $\eta_{IJ}e^I_\mu e^J_\nu = g_{\mu\nu}$.

In terms of these variables, the action of gravity plus (one species of) fermion can be written as

$$S = \int |e| d^4x \left[\frac{1}{2\kappa}F^{IJ}_{\mu\nu}e^\mu_I e^\nu_J + \frac{i}{2}\left(\bar\psi\gamma^K e^\mu_K\ D^f_\mu\psi - (\bar\psi\gamma^K e^\mu_K\ D^f_\mu\psi)^\dagger\right) + im\bar\psi\psi\right] . \tag{2.1}$$

where F is the curvature of the connection $D \equiv d + A$, while the covariant derivative of a fermion has been written as $D^f_\mu\psi = \partial_\mu\psi - \frac{i}{4}A^{IJ}_\mu\sigma_{IJ}\psi$. The cotetrad e^μ_I is defined as the inverse of the tetrad e^I_μ and satisfies $e^\mu_I e^I_\nu = \delta^\mu_\nu$. The tetrads and the spin connection are taken to be independent fields. Extremizing the action with respect to the spin connection and performing some index manipulations we find an expression for the spin connection,

$$A^{IJ}_\mu = \omega^{IJ}_\mu + \frac{\kappa}{8}e^K_\mu\bar\psi\left[\gamma_K, \sigma^{IJ}\right]_+\psi . \tag{2.2}$$

Here ω^{IJ}_μ is the part of the spin connection built purely out of tetrads,

$$\omega^{IJ}_\mu = \frac{1}{2}\left[e^K_\mu e_{\beta K}e^{\alpha[J}\partial_\alpha e^{J]\beta} + e^{[I}_\beta\partial_\mu e^{J]\beta} + e^{\alpha[I}e^{J]}_\beta e_{\mu K}\partial_\alpha e^{\beta K}\right] . \tag{2.3}$$

In the absence of spinorial matter the spin connection is fully described on shell by ω_μ^{IJ}, which in the metric formulation corresponds to the Levi-Civita (unique torsion-free metric-compatible) connection and is related to the Christoffel symbols as $\Gamma^\sigma_{\mu\nu} = e^\sigma_I \partial_\mu e^I_\nu + e^\sigma_I e_{\nu J} \omega_\mu^{IJ}$. The spin connection is not affected by the presence of bosonic fields if they are minimally coupled to gravity.

If we now extremize the action with respect to the tetrads, we will get an equation into which we insert the solution for A_μ^{IJ} obtained above and contract with tetrads to produce Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} \,, \tag{2.4}$$

where $R_{\mu\nu}$ and R are the Ricci tensor and scalar calculated using ω_μ^{IJ} (or alternatively from the metric), while the stress-energy tensor $T_{\mu\nu}$ is now quartic in the fermionic field,

$$\begin{aligned} T_{\mu\nu}(\psi, \bar\psi) = \frac{i}{4} \left(\partial_\mu \bar\psi \gamma_I \psi e^I_\nu - \bar\psi \gamma_I \partial_\mu \psi e^I_\nu + \frac{i}{4} \omega_\mu^{IJ} e^K_\nu \bar\psi \left[\gamma_K, \sigma_{IJ}\right]_+ \psi + (\mu \leftrightarrow \nu) \right) \\ + i m g_{\mu\nu} \bar\psi \psi - \frac{3\kappa}{16} g_{\mu\nu} \left(\bar\psi \gamma^I \gamma_5 \psi \right)^2 \,. \end{aligned} \tag{2.5}$$

In writing the last term we have used the identity $[\gamma_K, \sigma_{IJ}]_+ = 2\epsilon_{IJKL} \gamma^L \gamma^5$. The Dirac equation in the presence of gravity is thus

$$2\gamma^K e^\mu_K \partial_\mu \psi + e^\alpha_I \partial_\mu e^I_\alpha \, \gamma^K e^\mu_K \psi + \partial_\mu e^\mu_K \gamma^K \psi + 2m\psi - \frac{i}{4} A_\mu^{IJ} e^{\mu K} \left[\gamma_K, \sigma_{IJ}\right]_+ \psi = 0 \,. \tag{2.6}$$

Inserting the expression for A_μ^{IJ} into this equation, we can write it as

$$\gamma^K e^\mu_K \partial_\mu \psi - \frac{i}{4} \omega_\mu^{IJ} e^{\mu K} \gamma_K \sigma_{IJ} \psi + m\psi + \frac{3i\kappa}{8} \left(\bar\psi \gamma^I \gamma^5 \psi \right) \gamma_I \gamma^5 \psi = 0 \,. \tag{2.7}$$

This is the nonlinear Dirac equation that governs the motion of a fermion in curved spacetime. This equation has been known for a long time in various contexts for spacetimes with torsion [16–19]. Often this equation is written in "Planck units" in which Planck mass and Planck length are the units of mass and length respectively, so the κ in the nonlinear term is replaced by unity.

However, one important point often gets overlooked or at least is not explicitly mentioned, which is the fact that every fermion field must be included in the matter action and therefore all fermions will be present in the expression for spin connection,

$$A_\mu^{IJ} = \omega_\mu^{IJ} + \frac{\kappa}{8} e^K_\mu \sum_f \bar\psi_f \left[\gamma_K, \sigma^{IJ}\right]_+ \psi_f \,, \tag{2.8}$$

where the sum is over all species of fermions present in the universe. This term will also appear in the nonlinear Dirac equation for each type of fermion,

$$\gamma^K e^\mu_K \partial_\mu \psi_i - \frac{i}{4}\omega^{IJ}_\mu e^{\mu K}\gamma_K \sigma_{IJ}\psi_i + m\psi_i + \frac{3i\kappa}{8}\left(\sum_f \bar{\psi}_f \gamma^I \gamma^5 \psi_f\right)\gamma_I \gamma^5 \psi_i = 0\,. \tag{2.9}$$

It is instructive to derive this equation from the perspective of a spacetime with torsion. If we start from a spin connection written as $A^{IJ}_\mu = \omega^{IJ}_\mu + \Lambda^{IJ}_\mu$, we find that

$$F^{IJ}_{\mu\nu}(A) = F^{IJ}_{\mu\nu}(\omega) + \partial_{[\mu}\Lambda^{IJ}_{\nu]} + \left[\omega_{[\mu}, \Lambda_{\nu]}\right] + \eta_{KL}\Lambda^{IK}_{[\mu}\Lambda^{LJ}_{\nu]}\,. \tag{2.10}$$

This Λ is known as contorsion, and by extremizing the action with respect to it we find that the only nonvanishing variations come from the fermionic part of the action and the last term of $F^{IJ}_{\mu\nu}(A)$, so that the equation of motion for Λ is

$$\Lambda^{IJ}_\mu = \frac{\kappa}{8} e^K_\mu \sum_f \bar{\psi}_f \left[\gamma_K, \sigma^{IJ}\right]_+ \psi_f\,. \tag{2.11}$$

We can insert this solution for Λ into the Einstein equations and the Dirac equation, which are then exactly the same as we have found above. Furthermore, if we substitute this expression in the action, the resulting Einstein equations and the Dirac equation are also exactly the same as found above. In general, inserting a solution into the action gives incorrect results. In this case however, the antisymmetrized covariant derivative of Λ contribute to a total derivative in the action, so Λ is an auxiliary field (see Appendix).

The action of gravity with fermions is thus

$$\begin{aligned} S = \int |e| d^4x \Bigg[&\frac{1}{2\kappa} F^{IJ}_{\mu\nu}(\omega) e^\mu_I e^\nu_J + \frac{i}{2}\sum_f \left(\bar{\psi}_f \gamma^K e^\mu_K\ \hat{D}^f_\mu \psi_f - (\bar{\psi}_f \gamma^K e^\mu_K\ \hat{D}^f_\mu \psi_f)^\dagger + 2m_f \bar{\psi}_f \psi_f\right) \\ &+ \frac{1}{2\kappa}\eta_{KL}\Lambda^{IK}_{[\mu}\Lambda^{LJ}_{\nu]} e^\mu_I e^\nu_J + \frac{1}{8}\sum_f e^\mu_K \Lambda^{IJ}_\mu \bar{\psi}_f \left[\gamma^K, \sigma_{IJ}\right]_+ \psi_f \Bigg]\,, \end{aligned} \tag{2.12}$$

where we have written $\hat{D}^f_\mu \psi = \partial_\mu \psi - \frac{i}{4}\omega^{IJ}_\mu \sigma_{IJ}\psi$. Thus what we have is nothing more than general relativity with fermions. The contorsion Λ is an auxiliary field which enforces the interaction of spacetime geometry with fermionic fields but does not propagate. In the absence of fermions Λ vanishes, irrespective of any bosonic fields present as long as they are minimally coupled to gravity. Again this is all very well known, but writing the action in this form draws attention to another aspect which seems to have been overlooked.

The invariance of this action under local Lorentz transfomations means that Λ transforms homogeneously under them. In particular, the last term of the above action is invariant on its own. Since Λ does not transform inhomogeneously, the coupling of Λ to fermions is not like the coupling of a gauge field to fermions. The transformation of fermions does not affect that of Λ, so their coupling is not protected by any invariance. This way it is more analogous to the coupling of a real scalar field to fermions – the coefficient of $\bar{\psi}\phi\psi$ can be freely set by hand. But unlike a scalar field, Λ can couple chirally to fermions – it couples to the left-handed neutrinos irrespective of whether or not there are right-handed neutrinos in the universe. So there is no reason why different species of fermions cannot be coupled to Λ with different coupling strengths, analogous to the Yukawa coupling of fermions to a scalar field.

Therfore we propose that the generic form of the action of fermions coupled to gravity must be, not Eq. (2.12), but

$$S = \int |e| d^4x \left[\frac{1}{2\kappa} F_{\mu\nu}^{IJ}(\omega) e_I^\mu e_J^\nu + \frac{i}{2} \sum_f \left(\bar{\psi}_f \gamma^K e_K^\mu \ \hat{D}_\mu^f \psi_f - (\bar{\psi}_f \gamma^K e_K^\mu \ \hat{D}_\mu^f \psi_f)^\dagger + 2m_f \bar{\psi}_f \psi_f \right) \right.$$
$$\left. + \frac{1}{2\kappa} \eta_{KL} \Lambda_{[\mu}^{IK} \Lambda_{\nu]}^{LJ} e_I^\mu e_J^\nu + \frac{1}{8} \sum_f \Lambda_\mu^{IJ} e_K^\mu \left(\lambda_{fL} \bar{\psi}_{fL} \left[\gamma^K, \sigma_{IJ} \right]_+ \psi_{fL} + \lambda_{fR} \bar{\psi}_{fR} \left[\gamma^K, \sigma_{IJ} \right]_+ \psi_{fR} \right) \right] \tag{2.13}$$

where we have taken into account the possibility that the tensor currents due to left and right-handed fermions, which transform independently under local Lorentz transformations, may couple to Λ with different coupling constants λ_{fL} and λ_{fR}, respectively. Even though in this form the action appears to be a philosophical departure from how fermions have always been treated in general relativity, it is in fact a generic form which must inevitably appear when fermions are put in curved spacetime, unless the coupling constants λ_f are set to zero by fiat. Furthermore, since Λ leads to a torsion

$$C^\alpha{}_{\mu\nu} \equiv \Lambda_{[\mu}^{IJ} e_{\nu]J} e_I^\alpha = \frac{\kappa}{2} \epsilon^{IJKL} e_I^\alpha e_{\mu J} e_{\nu K} \sum_f \bar{\psi}_f \gamma_L \gamma_5 \psi_f \,, \tag{2.14}$$

which is totally antisymmetric and thus does not affect geodesics, all particles fall at the same rate in a gravitational field and the principle of equivalence is not violated by these coupling constants.

Solving for Λ and inserting the solution back into the action as before, we get

$$S = \int |e| d^4x \left[\frac{1}{2\kappa} F^{IJ}_{\mu\nu}(\omega) e^\mu_I e^\nu_J + \frac{i}{2} \sum_f \left(\bar{\psi}_f \gamma^K e^\mu_K \; \hat{D}^f_\mu \psi_f - (\bar{\psi}_f \gamma^K e^\mu_K \; \hat{D}^f_\mu \psi_f)^\dagger + 2 m_f \bar{\psi}_f \psi_f \right) \right.$$
$$\left. - \frac{3\kappa}{16} \left(\sum_f \left(\lambda_{fL} \bar{\psi}_{fL} \gamma_I \gamma^5 \psi_{fL} + \lambda_{fR} \bar{\psi}_{fR} \gamma_I \gamma^5 \psi_{fR} \right) \right)^2 \right] . \tag{2.15}$$

This is the generic form of the action of fermions in curved spacetime, which we will use as the starting point of further calculations below. It is in fact not meaningful to work with a Dirac equation containing Λ , because Λ must always equal its on-shell value. Furthermore, the quartic term is independent of the background metric, but must be included as long as there is gravity in the universe. The only ways this term can be absent from the action are if gravity is turned off ($\kappa \to 0$), or if the quartic couplings λ_f are assumed to be zero. This term is suppressed by two powers of Planck mass compared to the mass term, but it could still help avert gravitational singularities [20–23]. We will see that it can also in principle allow neutrino oscillations even when the neutrinos are massless.

III. NEUTRINO OSCILLATIONS

In considering the propagation of neutrinos through normal matter, i.e. solar or stellar cores or nuclear reactors, we need to take into account only the effects due to electrons and three colors each of up and down quarks in addition to the quartic self-interaction of the neutrinos. Weak interactions will be present of course, we will come back to the effect of that in a while. Let us also restrict to only two types of neutrinos as before. The quartic term relevant to our purpose is

$$\mathscr{L}_{(\bar{\psi}\psi)^2} = -\frac{3\kappa}{16} \left[\sum_{\alpha,\beta} \lambda_{\nu_\alpha} \lambda_{\nu_\beta} (\bar{\nu}_\alpha \gamma_I \nu_\alpha)(\bar{\nu}_\beta \gamma^I \nu_\beta) \right.$$
$$\left. -2 \sum_{\alpha,f} \lambda_{\nu_\alpha} (\bar{\nu}_\alpha \gamma_I \nu_\alpha) \left(-\lambda_{fV} \bar{\psi}_f \gamma^I \psi_f + \lambda_{fA} \bar{\psi}_f \gamma^I \gamma^5 \psi_f \right) \right] + \cdots \tag{3.1}$$

where we have used the fact the neutrinos are left-handed, written $\lambda_V = \frac{1}{2}(\lambda_L - \lambda_R)\,, \lambda_A = \frac{1}{2}(\lambda_L + \lambda_R)$ for the other fermions, and indicated by dots the terms which do not involve neutrinos. Let us now suppose that the ν_α which appear in the above expression, i.e. those which couple to Λ in Eq. (2.13), are different from the flavor neutrinos. This is our point of departure from the Standard Model, similar to assuming that mass eigenstates of neutrinos are different from their flavor eigenstates [2].

Following Wolfenstein [3] we calculate the forward scattering amplitude of the α-type neutrinos,

$$\mathcal{M} = -\frac{3\kappa}{8}\left(\bar{\nu}_\alpha \gamma_I \nu_\alpha\right)\lambda_{\nu_\alpha}\left\langle \sum_\beta \lambda_{\nu_\beta}\bar{\nu}_\beta\gamma^I\nu_\beta + \sum_{f=e,p,n}\left(\lambda_{fV}\bar{\psi}_f\gamma^I\psi_f - \lambda_{fA}\bar{\psi}_f\gamma^I\gamma^5\psi_f\right)\right\rangle , \tag{3.2}$$

where the average is taken over the background. In the second sum, the spatial components of the axial current average to spin in the nonrelativistic limit, which for normal matter is negligible. The axial charge is also negligible. Similarly, the spatial components of the vector current average to the spatial momentum of the background, which can also be neglected. Since neutrinos are ultrarelativistic, their density inside a finite volume such as a star is bounded by the rate of production times the average density of the region, i.e. several orders of magnitudes smaller than the density of electrons or baryons. Thus the average of the neutrino term can also be neglected.

So what we are left with is the average of the temporal component of the vector current of fermions, which is nothing but the number density of the fermions [3], $\langle\bar{\psi}\gamma^0\psi\rangle = -\langle\psi_f^\dagger\psi_f\rangle = -n_f$. The contribution of the forward scattering amplitude to the effective Hamiltonian density is therefore

$$\delta\mathscr{H}_{\rm eff} = \left(\sum_{f=e,p,n}\lambda_f n_f\right)\sum_\alpha \lambda_{\nu_\alpha}\nu_\alpha^\dagger\nu_\alpha, \tag{3.3}$$

where we have now dropped the subscript V and absorbed a factor of $\sqrt{\frac{3\kappa}{8}}$ in the definition of each of the λ.

This term acts as an effective mass term for the neutrinos, with $m_\alpha = \lambda_{\nu_\alpha}\rho$, where $\rho = \sum\lambda_f n_f$ is a weighted density of fermions that is the same for all neutrinos. With two species of neutrinos, we find $|m_2^2 - m_1^2| = \rho^2|\lambda_{\nu_2}^2 - \lambda_{\nu_1}^2|$. The mixing matrix takes the form

[2] It also appears to violate gauge invariance. We note however that the symmetry violating terms are suppressed by factors of Planck mass and disappear in the flat space limit $\kappa \to 0$; we can expect that any potential problem due to symmetry violation will appear when these terms, and thus the energy momentum density, are of order unity in Planck units. So we will ignore this issue for the moment as we are interested in energy densities far below that.

[3] We are being a bit sloppy here – the "density" of the fermion field is the time component of $j^\mu \equiv e_I^\mu\bar{\psi}\gamma^I\psi$. If the spacetime allows a 3+1 decomposition of the background metric as $g_{\mu\nu} = (-\lambda^2 + h_{ij})$, the volume measures can be related as $\sqrt{-g} = \lambda\sqrt{h}$, and $e_I^0 \doteq \lambda^{-1}\delta_I^0$, where δ_I^μ is the Kronecker delta. In this case $j^0 = -\lambda^{-1}\psi^\dagger\psi$ which is integrated over three spatial dimensions against the volume measure $\lambda\sqrt{h}$. We have assumed this decomposition.

$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, so the probability of conversion of one particular flavor of neutrino into the other becomes

$$P_{\rm conv} = \sin^2 2\theta \sin^2 \left(\frac{\rho^2 \Delta\lambda^2}{4E} x \right) , \tag{3.4}$$

where $\Delta\lambda^2 = |\lambda^2_{\nu_2} - \lambda^2_{\nu_1}|$.

This result is qualitatively different from the usual formula for neutrino oscillations in vacuum. We have not included a mass term for the neutrino in vacuum – all contributions to neutrino mass comes from the quartic interaction of the neutrino with fermions including itself. The actual background geometry of the spacetime does not contribute to the effective mass, at least for small curvatures, for which the leading order result of the forward scattering amplitude is sufficient. Thus a neutrino propagating through vacuum does not oscillate into different flavors. Oscillation occurs only in the region where there is a fermion density and stops when the neutrino leaves that region. This is exactly like what happens for oscillation due to weak interactions, except for the fact that leptons and baryons all contribute to the effective mass of neutrinos. The coupling constants λ are in principle measurable by looking at different media, such as stars with different baryon densities, or nuclear reactor cores, and measuring the corresponding oscillation lengths of neutrinos.

A non-vanishing λ_V for any fermion requires that the left-handed component of the fermion does not couple to torsion with the same strength as the right-handed component. Thus chiral symmetry is broken by torsion, or alternatively, by the quartic term which has its origin in spacetime geometry. Not only neutrinos, but all fermions get a contribution to their masses from this geometrical mechanism. Even if we assume that the contribution to effective mass is of the same order for all fermions in the same background matter density, the mass of very dense stars can be significantly larger than what is calculated from their baryon count. This can be expected to affect stellar models, dark matter estimates, and cosmology.

IV. WEAK INTERACTIONS

Neutrinos passing through matter will also interact with it via electroweak gauge fields. In this case, if we look at the effective four-fermion interaction at lowest order, only the interactions with electrons are relevant. This is because the weak interaction couples flavor eigenstates of the neutrinos with other fields; ν_e couples to electrons via both charged and neutral currents, while ν_μ couples to electrons only via the neutral current. The modification of the mixing angle due to weak interactions in normal matter is straightforward to calculate [24], as we show in outline below. The effective Lagrangian due to the charged current interaction can be written as

$$\mathscr{L}_{\rm cc} = -\frac{G_F}{\sqrt{2}} \left(\bar{\psi}_e \gamma^I (1-\gamma^5)\psi_e\right) \left(\bar{\nu}_e \gamma_I (1-\gamma^5)\nu_e\right) \,, \tag{4.1}$$

where a Fierz identity has been used. The (elastic) forward scattering amplitude provides the contribution to the Hamiltonian, $\sqrt{2}G_F \left\langle \bar{\psi}_e \gamma^I (1-\gamma^5)\psi_e \right\rangle (\bar{\nu}_{eL}\gamma_I \nu_{eL}) \simeq \sqrt{2} G_F n_e \nu^\dagger_{eL}\nu_{eL}\,.$ Normal matter does not contain muons, so ν_μ does not have a charged current interaction.

Both flavors of neutrinos have the same neutral current interactions, so that the contribution appears as a common term to the Hamiltonian,

$$V_{\rm nc} = \sqrt{2}G_F \sum_{f=e,p,n} n_f \left[I^f_{3L} - 2\sin^2\theta_W Q^f \right] \,, \tag{4.2}$$

where I^f_{3L} is the third component of weak isospin for the left-handed component of the fermion f and Q_f is its charge. For electrically neutral normal matter, the electron and proton contributions cancel each other and we are left with only the neutron contribution, equal to $-\sqrt{2}G_F n_n/2$ for both types of neutrinos. The Hamiltonian, diagonal in the space of mass eigenstates, can thus be written in flavor space as

$$H = H_c \mathbb{I} + \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} + \begin{pmatrix} \sqrt{2}G_F n_e & 0 \\ 0 & 0 \end{pmatrix} \,. \tag{4.3}$$

Here we have written H_c for the common terms in the Hamiltonian, and $\Delta m^2 = |m_2^2 - m_1^2| = \rho^2 \left| \lambda^2_{\nu_2} - \lambda^2_{\nu_1} \right|$. The effective mixing angle $\tilde{\theta}$, including the effects of both the geometric and weak contributions, is thus given by

$$\tan 2\tilde{\theta} = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2\sqrt{2}G_F n_e E} \,. \tag{4.4}$$

This formula is for ultrarelativistic neutrinos, and thus valid only in regions where matter density is not too high. For regions with low matter density and $n_e \simeq n_p \geq n_n$ and $n_e \to 0$, we find that the right hand side is proportional to n_e. For three generations of leptons we can make similar substitutions into the standard formula for neutrino oscillations. For neutrinos passing through regions where the matter density is not constant (MSW effect [3, 25, 26]), nonlinearity introduces additional complications particularly for very large matter densities, since effective masses of neutrinos and thus Δm^2, can vary greatly in such situations. We will not attempt to do that calculation here.

We conclude by making a couple of final remarks. The effective fermion action appears to be nonrenormalizable by power counting because of the quartic term. We have not worried about this issue here because the quartic couplings contain in them a factor of $\sqrt{\kappa}$ and thus must vanish in the flat space limit. So the counterterms in curved spacetime will have to involve curvature, so the question of renormalizability cannot be addressed without a theory of quantum gravity [27]. The second point is about the size of the quartic term. Neutrino masses are extremely small, but does the factor of $\sqrt{\kappa}$ which we have absorbed in the λs make them too small? We think that this question cannot be answered purely theoretically. Unlike in the case of weak interactions, where the energy required to create W-boson pairs from the vacuum sets the scale of the four-fermion interaction (and the oscillation formula can be calculated directly from quantum field theory [28]), here the scale is not related to the quantum dynamics of Λ, which does not in fact have any dynamics. Therefore the coupling constants λ are free and can be set only by comparison with experimental data, not from any theoretical argument.

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Appendix A: Calculation for Eq.(2.12)

Remembering that Λ_μ^{IJ} and ω_μ^{IJ} are antisymmetric in the internal indices, we can write the relevant portion of the action as

$$\begin{aligned} I &= \int d^4x\,|e|\left(\partial_{[\mu}\Lambda_{\nu]}^{IJ} + \left[\omega_{[\mu}, \Lambda_{\nu]}\right]\right)\, e_I^\mu e_J^\nu \\ &= \int d^4x\,|e|\left(\partial_\mu\Lambda_\nu^{IJ} - \partial_\nu\Lambda_\mu^{IJ} + \omega_{\mu K}^I\Lambda_\nu^{KJ} - \omega_{\nu K}^I\Lambda_\mu^{KJ} + \Lambda_{\mu K}^I\omega_\nu^{KJ} - \Lambda_{\nu K}^I\omega_\mu^{KJ}\right)\, e_I^\mu e_J^\nu\,. \end{aligned} \quad \text{(A1)}$$

Let us write the first, third and sixth terms of Eq. (A1) together as

$$\begin{aligned}\mathbb{A} &= e^{\mu}_{I} e^{\nu}_{J} \left(\partial_\mu \Lambda^{IJ}_{\nu} + \omega^{I}_{\mu K}\Lambda^{KJ}_{\nu} - \Lambda^{I}_{\nu K}\omega^{KJ}_{\mu}\right) \\ &= \partial_\mu \left(e^{\mu}_{I} e^{\nu}_{J}\Lambda^{IJ}_{\nu}\right) - \Lambda^{IJ}_{\nu} e^{\mu}_{I}\partial_\mu e^{\nu}_{J} - \Lambda^{IJ}_{\nu} e^{\nu}_{J}\partial_\mu e^{\mu}_{I} + e^{\mu}_{I} e^{\nu}_{J}\omega^{I}_{\mu K}\Lambda^{KJ}_{\nu} - e^{\mu}_{I} e^{\nu}_{J}\Lambda^{I}_{\nu K}\omega^{KJ}_{\mu}\,. \end{aligned} \tag{A2}$$

Let us call $e^{\mu}_{I} e^{\nu}_{J}\Lambda^{IJ}_{\nu} \equiv \Lambda^{\mu}$. Then the first, third and fourth terms of Eq. (A2) can be written together as

$$\begin{aligned}\mathbb{A}_1 &= \partial_\mu\Lambda^\mu - \Lambda^{IJ}_{\nu} e^{\nu}_{J}\partial_\mu e^{\mu}_{I} + e^{\mu}_{I} e^{\nu}_{J}\omega^{I}_{\mu K}\Lambda^{KJ}_{\nu} \\ &= \partial_\mu\Lambda^\mu - \Lambda^{KJ}_{\nu} e^{\nu}_{J} e^{\alpha}_{K} e^{I}_{\alpha}\partial_\mu e^{\mu}_{I} + e^{\mu}_{I} e^{\nu}_{J}\omega^{I}_{\mu K} e^{K}_{\alpha} e^{\alpha}_{L}\Lambda^{LJ}_{\nu} \\ &= \partial_\mu\Lambda^\mu + \Lambda^\alpha (e^{\mu}_{I}\partial_\mu e^{I}_{\alpha} + \omega^{I}_{\mu K} e^{K}_{\alpha} e^{\mu}_{I}) \\ &= \partial_\mu\Lambda^\mu + \Lambda^\alpha\Gamma^{\mu}_{\mu\alpha} = \nabla_\mu\Lambda^\mu\,. \end{aligned} \tag{A3}$$

The remaining terms of $\mathbb{A}$ are

$$\begin{aligned}\mathbb{A}_2 &= -\Lambda^{IJ}_{\nu} e^{\mu}_{I}\partial_\mu e^{\nu}_{J} - e^{\mu}_{I} e^{\nu}_{J}\Lambda^{I}_{\nu K}\omega^{KJ}_{\mu} \\ &= -\Lambda^{IK}_{\nu} e^{\mu}_{I} e^{\alpha}_{K} e^{J}_{\alpha}\partial_\mu e^{\nu}_{J} - e^{\mu}_{I} e^{\nu}_{J}\Lambda^{I}_{\nu K} e_{\alpha L} e^{\alpha K}\omega^{LJ}_{\mu} \\ &= \Lambda^{IK}_{\nu} e^{\mu}_{I} e^{\alpha}_{K} e^{\nu}_{J}\partial_\mu e^{J}_{\alpha} + \Lambda^{IK}_{\nu} e^{\mu}_{I} e^{\alpha}_{K}\omega^{J}_{\mu L} e^{L}_{\alpha} e^{\nu}_{J} \\ &= \Lambda^{IK}_{\nu} e^{\mu}_{I} e^{\alpha}_{K}\left(e^{\nu}_{J}\partial_\mu e^{J}_{\alpha} + \omega^{J}_{\mu L} e^{L}_{\alpha} e^{\nu}_{J}\right) \\ &= \Lambda^{IK}_{\nu} e^{\mu}_{I} e^{\alpha}_{K}\Gamma^{\nu}_{\mu\alpha} = 0\,. \end{aligned}$$

In the last equality we have used the fact that Λ and ω are antisymmetric in the internal indices while Γ is symmetric in its lower indices. Denoting the remaining terms of the integral I together as $\mathbb{B}$ and proceeding the same way as above, we find that

$$\mathbb{B} = \nabla_\mu\Lambda^\mu\,. \tag{A4}$$

Thus we can write Eq. (A1) as

$$I = 2\int d^4x\,|e|\,\nabla_\mu\Lambda^\mu\,. \qquad \text{Q.E.D.} \tag{A5}$$