

How does the photon's spin affect Gravitational Wave measurements?

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Abstract

We study the effect of the polarization of light beams on the time delay measured in Gravitational Wave experiments. To this end, we consider the Mathisson-Papapetrou-Dixon equations in a gravitational wave background, with two of the possible supplementary conditions: by Frenkel-Pirani, or by Tulczyjew. According to the literature, no spin effect is seen in the first case for photons, while we show that the second case is richer. The result is a tiny effect on the measured time delay of photons depending on their polarization state.

1 Introduction

Gravitational wave detection in interferometers such as the Laser Interferometer Gravitational-Wave Observatory (LIGO) and the Virgo observatory involves laser beams travelling through an inhomogeneous gravitational field. The gravitational wave profile is reconstructed by measuring the difference of time of flight of the laser beams in two perpendicular arms. To compute theoretically the time of flight deviations of the laser induced by a gravitational field, the beam is treated as a collection of photons, with each photon subject to the geodesic equation of General Relativity. In this paper, light is still considered as a localized massless particle, a photon, but now without neglecting its spin, which is absent from the geodesic equation.

Spinning localized (extended) test particles are introduced in General Relativity by considering the Mathisson-Papapetrou-Dixon (MPD) equations [1, 2, 3]. These are very general equations obtained when considering the dipole moment, either angular momentum or intrinsic spin, of the test particles. With this method, when considering only the monopole moment of a test particle, the geodesic equation is recovered, while when also considering its dipole moment, the MPD equations are obtained. Note that Souriau also finds these equations in a purely geometrical way when also neglecting the quadrupole moment and higher ones [4].

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With X, P, S denoting respectively the position, 4-momentum, and spin tensor of the test particle, the Mathisson-Papapetrou-Dixon equations are given by,

$$\dot{P}^\mu = -\frac{1}{2}R^\mu{}_{\rho\alpha\beta}S^{\alpha\beta}\dot{X}^\rho, \quad (1.1)$$

$$\dot{S}^{\mu\nu} = P^\mu\dot{X}^\nu - P^\nu\dot{X}^\mu, \quad (1.2)$$

where the dot over the trajectory X denotes the ordinary derivative with respect to its affine parameter, $\dot{X} = dX/d\tau$, while the dot over P and S denotes the covariant derivative with respect to that same parameter.

Note that this system is not deterministic, we lack an equation for \dot{X} . Supplementary conditions, or equations of state, are required to constrain the equations. There are two main choices considered in the literature. Indeed, for a massless particle in Minkowski, we have $P = \dot{X}$ and $S^\mu{}_\nu P^\nu = 0$ holds, as Souriau shows this leads to the Maxwell equations after geometric quantization [5]. Now, when there is curvature in the MPD equations, the velocity \dot{X} and the momentum P may not be parallel anymore. Hence the two choices: whether we consider the 4-velocity or the 4-momentum to be in the kernel of the spin tensor. The first choice is the Frenkel-Pirani constraint $S^\mu{}_\nu\dot{X}^\nu = 0$ [6, 7] where the particle is characterized by its conserved mass $\tilde{m} \equiv P^\mu\dot{X}_\mu$. The second choice is the Tulczyjew constraint $S^\mu{}_\nu P^\nu = 0$ [8] together with its conserved mass $m^2 \equiv P^\mu P_\mu$. In both cases, the particle is also characterized by its conserved longitudinal spin, which we will define later. For an extended body, the Tulczyjew condition is natural and generally accepted, as it allows to parametrize uniquely the worldline of the center of mass [3], while that choice is not unique for the Pirani constraint. Now, for an elementary particle, especially the photon, we lose that argument of the uniqueness of the worldline of the center of mass, and there seems to be no canonical way to choose an equation of state. Thus, the debate between the Frenkel-Pirani constraint or the Tulczyjew constraint, or maybe another one, remains open.

The way to differentiate the two equations of states for elementary particles seems to be only through experiment. In general, elementary test particles behave in a different way depending on which equation of state they obey, as we will see in an example below. This is true both in a gravitational field, but also in an electromagnetic field, which can also be included in the MPD equations [3, 4]. For example, for massive and charged elementary particles, both equations of state recover the spin precession equation of the Bargmann-Michel-Telegdi (BMT) equations [9] from the MPD equations, in the weak field limit. Recall that the BMT equations describe the spin precession of a particle in a constant electromagnetic field, and are used to measure experimentally with very high precision the gyromagnetic moment g of the test particle [10]. While both equations of state lead to the spin precession equation, they both feature an extra anomalous velocity [4, 11, 12, 13] for the test particle, not present in the original BMT equations. Theoretical differences of the trajectory between the two equations of states are not lacking, though the expected deviations seem extremely small in any case considered, whether we study a particle in a gravitational or electromagnetic field.

The MPD equations with the Frenkel-Pirani constraint are not continuous in the limit $\tilde{m} \rightarrow 0$. The equations for massless particles seem to be unrelated to the massive ones [14, 15]. Notice that massive spinning particles with the Frenkel-Pirani constraint are expected to deviate from the usual geodesics in a non trivial way, even in Minkowski spacetime [15]. Obtaining

equations of motion for the massless particle does not seem to be straightforward. Nevertheless, authors who worked on the subject [16, 17] agree to say that the Frenkel-Pirani constraint implies that massless particles follow null geodesics, regardless of the gravitational background.

So, if we accept the Frenkel-Pirani constraint, we can sleep peacefully, since there would be no birefringence effect of light due to spin-curvature coupling, thus no effect on gravitational waves detection.

The Tulczyjew constraint was originally justified for extended test particles [8, 3]. Since then, it has been extended to the case of point particles by some authors. In the massless case, the Tulczyjew constraint was used in recent years [18, 19, 20, 21] to correctly describe the anomalous velocity present in the Spin Hall Effect of Light in optical media, predicted by Fedorov and Imbert [22, 23], and measured experimentally in 2008 [24, 25]. See also [26] in the electromagnetic field. Moreover, while the equations of motion of spinning photons become degenerate in flat spacetime with this equation of state, the description becomes the one of a plane wave travelling at the speed of light, and Souriau showed [5] that geometric quantization of this system leads to the Maxwell equations.

Note that the technical study of massless spinning particles is rather different from that of massive spinning particles, for both equations of state, as subtleties arise. For an example of a massive particle in an exact gravitational wave with the Tulczyjew constraint, see [27].

Gravitational birefringence was already considered experimentally in 1974 [28], resulting in an upper bound to the gravitational birefringence effect in gravitational lensing, but the results are somewhat inconclusive, since we can expect such effects to be much weaker than the precision of the experiment [29, 30]. Experimental bounds can also be found for birefringence in other theories, for example with Lorentz violating effects, thanks to the high sensitivity of the interferometry experiments in LIGO and Virgo [31, 32].

The first short section 2 will be used to introduce the notations we use in this paper. Then, we will justify in section 3 the main trick of this paper which is to consider a photon in a gravitational wave background as the limit of an ultrarelativist particle travelling in one direction. The next section 4 contains the computations needed to obtain the equations of motion, followed by an analysis and conclusions in section 5.

2 Notations

First, let us introduce our notations. The metric has signature $(-, -, -, +)$. The components of the Riemann curvature tensor are defined by the convention $R^\mu{}_{\nu\alpha\beta} = \partial_\alpha \Gamma^\mu_{\beta\nu} - \partial_\beta \Gamma^\mu_{\alpha\nu} + \dots$. In this paper, we often suppress indices by considering linear maps instead of 2-tensors. For instance, we use the linear map $S = (S^\mu{}_\nu)$ and likewise for the shorthand notation $R(S)$, with $R(S)^\mu{}_\nu = R^\mu{}_{\nu\alpha\beta} S^{\alpha\beta}$. In the same way, we have the vector P and the associated covector $\overline{P} = (\overline{P}_\mu)$ where indices are lowered with the metric. Another shorthand notation will be $R(S)(S) = R_{\mu\nu\alpha\beta} S^{\mu\nu} S^{\alpha\beta}$.

For a skewsymmetric linear map F , the operator Pf gives its Pfaffian $\text{Pf}(F)$. With the fully skewsymmetric Levi-Civita tensor $\epsilon_{\mu\nu\rho\sigma}$, with $\epsilon_{1234} = 1$, we have the expression $\text{Pf}(F) = -\frac{1}{8} \sqrt{-\det(g_{\alpha\beta})} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$. We have the relation $\text{Pf}(F)^2 = \det(F)$. Indeed, the determinant of a skewsymmetric matrix can always be written as a perfect square.

Classical elementary particles are characterized by two conserved numbers, emanating from the co-adjoint representation of the Poincaré group: its mass m and its longitudinal spin s . We have seen already that the mass m is given by $m^2 = P^2$ in the case of the Tulczyjew constraint. The longitudinal spin, sometimes also called “scalar spin”, is defined by

$$-\frac{1}{2}\text{Tr}(S^2) = s^2 \quad (2.1)$$

For a photon, we have $s = \pm\hbar$, where $\text{sign}(s)$ is the helicity of the photon. This corresponds to circular polarization of light, either with left or with right handedness.

3 Photons as a limit of ultrarelativistic particles

The so called Souriau-Saturnini equations are the combination of the MPD equations, together with the Tulczyjew constraint $SP = 0$, and applied to the case of the photon. For massless particles, the momentum is such that $P^2 = 0$, and so, for $R(S)(S) \neq 0$, we have the equations (Souriau[4] and Saturnini[33] in French, see [34] for the proof in English),

$$\dot{X} = P + \frac{2}{R(S)(S)}SR(S)P, \quad (3.1)$$

$$\dot{P} = -s \frac{\text{Pf}(R(S))}{R(S)(S)} P, \quad (3.2)$$

$$\dot{S} = P\overline{\dot{X}} - \dot{X}\overline{P}. \quad (3.3)$$

The Souriau-Saturnini equations describe the trajectory of a massless photon with spin in a gravitational field. While they work rather well in a Robertson-Walker background [34], or in the proximity of a star [33, 30], they break down when the curvature of the gravitational background vanishes. This is due to the lonely term $R(S)(S)$ in the denominator of (3.1). When the curvature vanishes, the equations become those of a plane wave travelling at the speed of light. Indeed, massless and chargeless particles cannot be localized in flat spacetime with this approach. It becomes a problem for a metric of gravitational waves, as they are usually computed as a perturbation around flat spacetime.

This time, for massive particles, $P^2 = m^2 \neq 0$, and we have similar equations [35, 15],

$$\dot{X} = P - \frac{2SR(S)P}{4P^2 - R(S)(S)}, \quad (3.4)$$

$$\dot{P} = -\frac{1}{2}R(S)\dot{X}, \quad (3.5)$$

$$\dot{S} = P\overline{\dot{X}} - \dot{X}\overline{P}. \quad (3.6)$$

Notice that we recover the Souriau-Saturnini equations in the limit $P^2 \rightarrow 0$, which is not, a priori, trivial. For example, this is not the case with the Pirani constraint.

Now, for massive particles, the denominator of (3.4) behaves in a nicer way. When the Riemann tensor goes to zero, or when $m^2 \gg R(S)(S)$, we recover the usual geodesic equation.

To be sure the denominator doesn't vanish in the massive case, we should have $4m^2 > R(S)(S)$. We thus have a lower bound on the mass of the test particle. With f the frequency of the gravitational wave and c the speed of light, that requirement becomes

$$m^2 > \frac{\epsilon \pi^2 f^2 \hbar^2}{c^4} \quad (3.7)$$

Note that this depends on the amplitude ϵ of the gravitational waves. As this amplitude goes to zero, the mass restriction reduces to $m > 0$. In the case of gravitational wave detections, the frequency of gravitational waves is typically around $f = 50\text{Hz}$, and the amplitude around $\epsilon = 10^{-20}$. This gives

$$m > 10^{-59} \text{ kg}, \quad (3.8)$$

to have a consistent set of equations describing a massive particle with spin in a typical background with gravitational waves.

The main idea to compute the time delay due to the photon's spin in a background of gravitational waves is to only compute the effect in the direction defined by the momentum. Indeed, the photon goes back and forth in one direction of propagation, so here we are not interested in the full trajectory in space of the photon/particle. Therefore, to compute the delay, we can compute the effect of spin on a massive particle, though with a mass much smaller than its momentum. Since we only compute the time delay in the direction defined by the momentum, and since (3.4) reduces to (3.1) in the limit $P^2 \rightarrow 0$, the mass will drop out of the equations when compared to the momentum, thus giving us the expected time of flight delay for a photon.

Notice that, in any case, the best experimental measurements on the mass of a photon give us an upper bound for the mass of about 10^{-50} kg to 10^{-54} kg depending on the type of measurements and assumptions [36, 37]. These upper bounds are a few orders of magnitude higher than the constraint on the mass of the photon (3.8) in the massive equations.

4 Equations of motion for the ultrarelativistic photon

Using Cartesian coordinates (x_1, x_2, x_3, t) , we linearize the gravitational field equations with the metric,

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu} + \mathcal{O}(\epsilon^2) \quad (4.1)$$

where $(\eta_{\mu\nu}) = \text{diag}(-1, -1, -1, 1)$ is the flat Minkowski metric, $h_{\mu\nu}$ the linear deviation of the metric to flat spacetime, and $\epsilon \ll 1$ a small parameter encoding the amplitude of the gravitational wave.

Linearizing the Einstein field equations in ϵ , and considering a gravitational wave propagating in the direction of the z axis, leads to the well-known solution for the perturbation $h_{\mu\nu}$,

$$(h_{\mu\nu}) = \begin{pmatrix} f_+(t-x_3) & f_\times(t-x_3) & 0 & 0 \\ f_\times(t-x_3) & -f_+(t-x_3) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (4.2)$$

with f_+ and f_\times two functions describing the two polarization states of the gravitational waves.

For concreteness, take $f_+(t - x_3) = \cos(\omega(t - x_3))$ and $f_\times(t - x_3) = 0$ with $c = 1$. The linearized metric thus takes the form,

$$(g_{\mu\nu}) = \begin{pmatrix} -1 + \epsilon \cos(\omega(t - x_3)) & 0 & 0 & 0 \\ 0 & -1 - \epsilon \cos(\omega(t - x_3)) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \mathcal{O}(\epsilon^2) \quad (4.3)$$

Up to linear order in ϵ , we have $R^3_{131} = -R^3_{141} = -R^3_{232} = R^3_{242} = R^4_{131} = -R^4_{141} = -R^4_{232} = R^4_{242} = -\frac{1}{2}\omega^2\epsilon \cos(\omega(t - x_3))$.

Now, to alleviate notations, we write $c \equiv \cos(\omega(t - x_3))$. We can write the 4-momentum of a massive particle as,

$$(P^\mu) = \begin{pmatrix} p_1 \left(1 + \frac{\epsilon}{2}c\right) \\ p_2 \left(1 - \frac{\epsilon}{2}c\right) \\ p_3 \\ \sqrt{m^2 + p_1^2 + p_2^2 + p_3^2} \end{pmatrix} + \mathcal{O}(\epsilon^2) \quad (4.4)$$

such that $P^2 = m^2$.

The spin tensor is, up to linear order in ϵ ,

$$(S^\mu{}_\nu) = \begin{pmatrix} 0 & -s_3(1 + \epsilon c) & s_2(1 + \frac{\epsilon}{2}c) & \frac{(p_2 s_3 - p_3 s_2)}{\sqrt{m^2 + \|\mathbf{p}\|^2}}(1 + \frac{\epsilon}{2}c) \\ s_3(1 - \epsilon c) & 0 & -s_1(1 - \frac{\epsilon}{2}c) & \frac{(p_3 s_1 - p_2 s_3)}{\sqrt{m^2 + \|\mathbf{p}\|^2}}(1 - \frac{\epsilon}{2}c) \\ -s_2(1 - \frac{\epsilon}{2}c) & s_1(1 + \frac{\epsilon}{2}c) & 0 & \frac{(p_1 s_2 - p_2 s_1)}{\sqrt{m^2 + \|\mathbf{p}\|^2}} \\ \frac{(p_2 s_3 - p_3 s_2)}{\sqrt{m^2 + \|\mathbf{p}\|^2}}(1 - \frac{\epsilon}{2}c) & \frac{(p_3 s_1 - p_2 s_3)}{\sqrt{m^2 + \|\mathbf{p}\|^2}}(1 + \frac{\epsilon}{2}c) & \frac{(p_1 s_2 - p_2 s_1)}{\sqrt{m^2 + \|\mathbf{p}\|^2}} & 0 \end{pmatrix} \quad (4.5)$$

such that S is skewsymmetric, and still up to linear order,

$$SP = 0 \quad \text{and} \quad -\frac{1}{2}\text{Tr}(S^2) = j^2 \quad (4.6)$$

with

$$j^2 = \frac{(\mathbf{s} \cdot \mathbf{p})^2 + m^2 \|\mathbf{s}\|^2}{\|\mathbf{p}\|^2 + m^2} \quad (4.7)$$

Next, we have,

$$\text{Pf}(R(S)) = \mathcal{O}(\epsilon^2). \quad (4.8)$$

See Appendix A for the expressions of $R(S)(S)$ and $S R(S) P$.

We then have the equations of motion for the position of the massive particle (3.4),

$$\dot{X} = P - \frac{2SR(S)P}{4P^2 - R(S)(S)}, \quad (4.9)$$

So, we get the equations of motion on 3d-space, with respect to the time coordinate t , in the 3+1 splitting (\mathbf{x}, t) , as

$$\frac{d\mathbf{x}}{dt} = \frac{(2m^2 - \frac{1}{2}R(S)(S)) \mathbf{P} - \mathbf{S}R(\mathbf{S})\mathbf{P}}{(2m^2 - \frac{1}{2}R(S)(S)) P_4 - SR(S)P_4} \quad (4.10)$$

At this point, the mass terms allow us to take the limit $\epsilon \rightarrow 0$. Hence the development in linear order of ϵ , for a massive particle with momentum $\mathbf{p} = (0, p_2, 0)$,

$$\frac{dx_2}{dt} = \frac{p_2}{\sqrt{m^2 + p_2^2}} - \frac{\epsilon}{2} \frac{p_2(m^2 + p_2^2) + \omega^2 \left(p_2(s_1^2 - s_3^2) - \sqrt{m^2 + p_2^2} s_2 s_3 \right)}{(m^2 + p_2^2)^{3/2}} \cos(\omega(t - x_3)) + \mathcal{O}(\epsilon^2) \quad (4.11)$$

Hence, when $p_2^2 \gg m^2$, we have

$$\frac{dx_2}{dt} = 1 - \frac{\epsilon}{2} \cos(\omega(t - x_3)) - \frac{\epsilon}{2} \frac{\lambda_\gamma^2}{\lambda_{\text{GW}}^2} \frac{(s_1^2 - s_3^2 - s_2 s_3)}{\hbar^2} \cos(\omega(t - x_3)) + \mathcal{O}(\epsilon^2) \quad (4.12)$$

with λ_γ the wavelength associated to the photon, and $\lambda_{\text{GW}} = 2\pi/\omega$ is the wavelength of the gravitational wave. With values taken from LIGO/Virgo, $\lambda_\gamma = 1064\text{nm}$,

$$\frac{\epsilon}{2} \frac{\lambda_\gamma^2}{\lambda_{\text{GW}}^2} \sim 10^{-46}$$

This means that geodesic effects of order $\epsilon^2 \sim 10^{-40}$ would be seen before observing any spin effect in LIGO/Virgo type detectors.

From (3.6), we see that ds/dt is at most of order ϵ , so for the short scales involved in a detector, we assume that the spin vector is constant. Thus, the maximum effect is when photons are polarized such that $\mathbf{s} = (0, \hbar, \hbar)$, at least in the classical limit. In that case, the measured time delay is decreased from $\Delta\tau$ to

$$\widetilde{\Delta\tau} = \Delta\tau \left(1 - 2 \frac{\lambda_\gamma^2}{\lambda_{\text{GW}}^2} \right) \quad (4.13)$$

A corollary is that two photons of different polarization will have different times of flight. Thus, a beam made up of photons of random polarization will introduce a noise due to spin curvature effects. A way to eliminate this noise is to polarize the beams of light before sending them into the arms. However, the amplitude of the noise created by this birefringence is of the relative order of 10^{-46} in LIGO/Virgo, which is much below the current sensitivity in LIGO and Virgo experiments.

5 Conclusions

To take into consideration the possible effects of the photon's spin on its trajectory in curved space, we used the very general Mathisson-Papapetrou-Dixon equations for spinning test particles, together with two possible supplementary conditions for photons, by Frenkel-Pirani, or

by Tulczyjew. While for an extended body, such as a spinning star, either choice of supplementary condition does not have much of a practical impact on the observable trajectory, though Tulczyjew is usually preferred in that case, for elementary particles the choice has more consequences.

The Frenkel-Pirani equation of state for a massless particle leads to a trajectory along a null geodesic, regardless of the gravitational background. In that case, there would be no change to the geodesic trajectory of photons in a background of gravitational waves. Keep in mind, however, that with this supplementary condition, massive spinning particles have a non trivial trajectory even in flat spacetime, and there is no correspondence between the massive and massless case.

The Tulczyjew equation of state for a massless particle predicts a very small effect due to the polarization of the light on its trajectory. Since the massive equations with this supplementary condition lead to the massless equations in the limit $m \rightarrow 0$, and because of the instability of the localization of the test particle in the equations near zero curvature, the photon is treated in this paper as an ultrarelativistic massive particle. This mass, which can be both large compared to the spin-curvature coupling term $R(S)(S)$ and extremely small compared to the momentum of the photon allows for convenient limits to be taken in the equations. The geodesic equations in a gravitational wave background are recovered, together with a new term depending on the spin polarization of the photon. This means that with this supplementary condition, the time of flight of a photon in a detector depends on its polarization state. This dependence is, however, many order of magnitudes lower than the first order effects of gravitational waves on the time of flight. But, if we achieve that kind of precision, polarizing the laser beam in a specific way would be an easy way to reduce the noise introduced by birefringence. With enough precision, this could even potentially be a way to discriminate between the two possible equations of state.

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A Appendix: Expressions of $R(S)(S)$ and $SR(S)P$

From the expression of the Riemann tensor, of the spin tensor (4.5), $c \equiv \cos(\omega(t - x_3))$, and $R(S)(S) = R_{\mu\nu\lambda\sigma}S^{\mu\nu}S^{\lambda\sigma}$, we get,

$$R(S)(S) = \frac{2\omega^2\epsilon c}{m^2 + \|\mathbf{p}\|^2} \left[2(p_1s_1 - p_2s_2)s_3 \left(p_3 - \sqrt{m^2 + \|\mathbf{p}\|^2} \right) - (p_1^2 - p_2^2) s_3^2 + \right. \\ \left. - (s_1^2 - s_2^2) \left(p_3 \left(p_3 - 2\sqrt{m^2 + \|\mathbf{p}\|^2} \right) + (m^2 + \|\mathbf{p}\|^2) \right) \right] + \mathcal{O}(\epsilon^2). \quad (\text{A.1})$$

Similarly, we obtain, with $SR(S)P^\mu = R^\mu{}_{\nu\lambda\sigma}P^\nu S^{\lambda\sigma}$,

$$SR(S)P = \begin{pmatrix} SR(S)P_1 \\ SR(S)P_2 \\ SR(S)P_3 \\ SR(S)P_4 \end{pmatrix}, \quad (\text{A.2})$$

with,

$$\begin{aligned}
SR(S)P_1 = & K \left(s_3 (m^2 + \|\mathbf{p}\|^2) \left(\sqrt{m^2 + \|\mathbf{p}\|^2} - p_3 \right) \left(s_1 \left(\sqrt{m^2 + p^2} - p_3 \right) + p_1 s_3 \right) + \right. \\
& - \left(s_2 \sqrt{m^2 + \|\mathbf{p}\|^2} \left(\sqrt{m^2 + \|\mathbf{p}\|^2} - p_3 \right) + p_2 s_3 \right) \times \\
& \left. \times \left(\left(\sqrt{m^2 + \|\mathbf{p}\|^2} - p_3 \right) (p_2 s_1 + p_1 s_2) + 2p_1 p_2 s_3 \right) \right) + \mathcal{O}(\epsilon^2), \tag{A.3}
\end{aligned}$$

$$\begin{aligned}
SR(S)P_2 = & K \left(s_3 (m^2 + \|\mathbf{p}\|^2) \left(p_3 - \sqrt{m^2 + \|\mathbf{p}\|^2} \right) \left(s_2 \left(\sqrt{m^2 + \|\mathbf{p}\|^2} - p_3 \right) + p_2 s_3 \right) + \right. \\
& + \left(s_1 \sqrt{m^2 + \|\mathbf{p}\|^2} \left(\sqrt{m^2 + \|\mathbf{p}\|^2} - p_3 \right) + p_1 s_3 \right) \times \\
& \left. \times \left(\left(\sqrt{m^2 + \|\mathbf{p}\|^2} - p_3 \right) (p_2 s_1 + p_1 s_2) + 2p_1 p_2 s_3 \right) \right) + \mathcal{O}(\epsilon^2), \tag{A.4}
\end{aligned}$$

$$\begin{aligned}
SR(S)P_3 = & K \sqrt{m^2 + \|\mathbf{p}\|^2} \left((s_2^2 - s_1^2) \sqrt{m^2 + \|\mathbf{p}\|^2} \left(\sqrt{m^2 + \|\mathbf{p}\|^2} - p_3 \right)^2 + \right. \\
& + \left(\sqrt{m^2 + \|\mathbf{p}\|^2} - p_3 \right) \left(-s_3 \sqrt{m^2 + v p^2} (p_1 s_1 - p_2 s_2) + p_2^2 s_1^2 - p_1^2 s_2^2 \right) + \\
& \left. + 2p_1 p_2 s_3 (p_2 s_1 - p_1 s_2) \right) + \mathcal{O}(\epsilon^2), \tag{A.5}
\end{aligned}$$

$$\begin{aligned}
SR(S)P_4 = & K \sqrt{m^2 + \|\mathbf{p}\|^2} \left(2p_3^2 \left(\sqrt{m^2 + \|\mathbf{p}\|^2} - p_3 \right) (s_1^2 - s_2^2) + s_3 (p_1^3 s_1 - p_2^3 s_2) + \right. \\
& + 3s_3 p_1 p_2 (p_2 s_1 - p_1 s_2) + \left(m^2 + 3p_3^2 - 3\sqrt{m^2 + \|\mathbf{p}\|^2} p_3 \right) s_3 (p_1 s_1 - p_2 s_2) + \\
& - \left(\sqrt{m^2 + \|\mathbf{p}\|^2} - 2p_3 \right) (p_1^2 s_2^2 - p_2^2 s_1^2) + \left(\sqrt{m^2 + \|\mathbf{p}\|^2} - p_3 \right) s_3^2 (p_1^2 - p_2^2) + \\
& \left. - p_3 (p_1^2 s_1^2 - p_2^2 s_2^2) - m^2 p_3 (s_1^2 - s_2^2) \right) + \mathcal{O}(\epsilon^2), \tag{A.6}
\end{aligned}$$

and,

$$K = \frac{\omega^2 \epsilon \cos(\omega(t - x_3))}{(m^2 + \|\mathbf{p}\|^2)^{3/2}}. \tag{A.7}$$