Evidence for Primordial Gravitational Waves

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Primordial gravitational waves have crucial implications for the origin of the universe and fundamental physics. Using currently available cosmic microwave background data from Planck, WMAP, ACT and SPT separately or their combinations with BK18 B-mode polarization and DESI observations, we find the evidence of primordial gravitational waves at beyond the $5\,\sigma$ confidence level.

Introduction. Inflation [1–8], an quasi-exponential expansion phase of the very early universe after the Big Bang singularity, is believed to be responsible for the large scale homogeneity of the universe and the origin of small scale density fluctuations in the cosmic web. It not only solves the horizon and flatness problems [5] but also leaves imprints in the anisotropies of cosmic microwave background (CMB) observations [9] and large scale matter distribution [10]. However, the nature of inflation together with dark matter and dark energy is still unknown so far [11, 12]. Besides generating the scalar perturbations that seed the cosmic structure, inflation can also produce the quantum tensor perturbations, i.e., the so-called primordial gravitational waves (PGWs), which carry extremely important information of the very early universe. Unlike the electromagnetic messenger CMB that only probes back to the recombination epoch, PGWs can explore the high energy physics at $\sim 10^{-34}$ s very close to the start of the universe, due to feeble interaction with matter density perturbations during the travel through large cosmological distances [13]. A successful detection of PGWs could provide the energy scale of inflation and even the first experimental demonstration of quantization of gravitational interactions [9].

Generally, there are two main approaches to detect the PGWs, namely direct and indirect experiments. Direct detectors such as the LIGO-Virgo gravitational wave (GW) observatories where laser interferometers are used to detect the perturbations of space from astrophysical or cosmological sources, are not sensitive enough to detect PGWs at high frequencies and can just place upper bounds on the amplitude of PGWs at current stage [14]. Indirect measurements are obtained by investigating the effect of PGWs on the polarization spectrum of CMB at low frequencies [15–17]. Currently, using a data combination of Planck CMB, baryon acoustic oscillations (BAO), BICEP2, Keck Array and BICEP3 CMB polarization experiments up to the 2018 observing season, the BICEP/Keck Collaboration gives the $2\,\sigma$ upper bound on the tensor-to-scalar ratio $r < 0.036$ [18], which is a clear improvement of the $2\,\sigma$ upper limit $r < 0.1$ [15, 16] from Planck alone. The combination of direct and indirect approaches can further compress the parameter space of tensor modes [16, 19]. Nonetheless, due to the limitation of experimental sensitivity and the excess foreground contamination of astrophysical processes, it seems to be difficult to probe PGWs for direct detectors, and consequently CMB polarization data with appropriate experimental settings have a great potential to settle the issue of whether PGWs exist [17]. It is worth noting that the local pulsar timing arrays (PTAs) [20, 21] confront the similar problem of diverse GW sources like LIGO-Virgo or future LISA [22], although they can constrain primordial tensor modes by identifying inflationary gravitational waves as the sole source.

The next-generation CMB experiments such as CMB-S4 [23], Simons Observatory [24], LiteBIRD [25] and CORE [26] aim at searching for the definite evidence of cosmic inflation and promise the detection of PGWs at the level of $\sigma(r) \sim 10^{-3}$. This predicted precision may help find the imprints of PGWs, but two important questions in light of current CMB observations should be correctly answered before that: (i) Have PGWs been detected? (ii) If not, what precision of r could be required to find out PGWs? For this purpose, one needs to reanalyze the available CMB data and check all the details when implementing cosmological constraints on PGWs. In doing so, via a logarithmic prior of r, we obtain the decisive evidences of PGWs using currently available CMB datasets and their combinations with B-mode polarization and BAO observations.

Model. Cosmic inflation is driven by a scalar field, *inflaton*, with a canonical kinetic term slowly rolling in the framework of general relativity. It can be characterized by a phenomenological fluid with a significantly negative pressure [1–8]. During the inflation epoch, comoving tensor fluctuations are amplified from quantum vacuum fluctuations to become highly squeezed states resembling classical states. After inflation, these tensor modes reenter the Hubble horizon and evolves over time, and finally act as one kind of source of stochastic gravitational wave background observed by local GW experiments. Same as the Planck 2018 analysis [15, 16], we take the following primordial tensor power spectrum (PTPS) for a single-field slow rolling inflation

$$\mathcal{P}_T(k) = A_t \left(\frac{k}{k_\star}\right)^{n_t + \frac{1}{2} n_{trun} \ln\left(\frac{k}{k_\star}\right)} , \qquad (1)$$

where k, $k_\star$, A_t, n_t and n_{trun} denote the comoving wavenumber, tensor pivot scale, amplitude of PTPS, tensor spectral index and running of tensor spectral index, respectively. $r \equiv A_t/A_s$, where A_s is the amplitude of primordial scalar power spectrum. Throughout this work, we use the next-order inflation consistency relation adopted by the Planck collaboration, i.e., $n_t = -r(2 - r/8 - n_s)/8$ and $n_{trun} = r(r/8 + n_s - 1)/8$ [15, 16, 27], where n_s is the scalar spectral index.

Data and methodology. To probe the primordial tensor parameter space, we use the following observational datasets:

- CMB. CMB observations have extremely important implications for cosmology and astrophysics. They have measured the matter components, the topology and the large scale structure of the universe. We adopt the Planck 2018 high-ℓ `plik` temperature (TT) likelihood at multipoles $30 \leqslant \ell \leqslant 2508$, polarization (EE) and their cross-correlation (TE) data at $30 \leqslant \ell \leqslant 1996$, and the low-$\ell$ TT `Commander` and `SimAll` EE likelihoods at $2 \leqslant \ell \leqslant 29$ [28]. We employ conservatively the Planck lensing likelihood [29] from `SMICA` maps at $8 \leqslant \ell \leqslant 400$. Prior to Planck, the WMAP satellite gives early-time measurements of various aspects of cosmological physics. We use the TT, TE and polarization power spectrum data from WMAP at $2 \leqslant \ell \leqslant 1200$, $2 \leqslant \ell \leqslant 800$ and $2 \leqslant \ell \leqslant 23$ [30], respectively. We also consider the Atacama Cosmology Telescope (ACT) DR4 TTTEEE data [31, 32] at $350 < \ell < 8000$ and the South Pole Telescope (SPT) TTTEEE likelihood [33, 34] at $750 \leqslant \ell < 3000$.

- BK18. To constrain the inflationary physics, we use the BICEP2, Keck Array and BICEP3 CMB polarization data up to and including the 2018 observing season [18]. This dataset, which aims at detecting the CMB B-modes, includes the additional Keck Array observations at 220 GHz and BICEP3 observations at 95 GHz relative to the previous 95/150/220 GHz dataset. We refer to this dataset as “BK18”.

- BAO. BAO [35, 36] are rather clean probes to explore the evolution of the universe over time, which are unaffected by the nonlinear physics at small scales. Measuring the positions of these oscillations in the matter power spectrum at different redshifts can give strong constraints on the cosmic expansion history. We adopt the latest 12 DESI BAO measurements specified in [37], including the BGS sample in the redshift range $0.1 < z < 0.4$, LRG samples in $0.4 < z < 0.6$ and $0.6 < z < 0.8$, combined LRG and ELG sample in $0.8 < z < 1.1$, ELG sample in $1.1 < z < 1.6$, quasar sample in $0.8 < z < 2.1$ and the Lyman-α Forest sample in $1.77 < z < 4.16$ [38, 39].

As previous analyses [15, 16, 18], the theory to be tested with data is still the Λ-cold dark matter (ΛCDM) model plus the tensor-to-scalar ratio r. The only difference is replacing the traditional uniform prior with logarithmic prior for r. This improvement can help better capture the detailed information of PGWs at the order of magnitude that is much lower than current 2σ upper bound $r \sim 0.04$ [18], and even give the realistic range of r.

In order to perform the Bayesian analysis, we use the publicly available Boltzmann solver `CAMB` [40] to calculate the theoretical power spectrum and employ the Monte Carlo Markov Chain (MCMC) method to infer the posterior distributions of model parameters via the online package `CosmoMC` [41, 42]. We analyze the MCMC chains using the public package `Getdist` [43]. The convergence rule of MCMC runs is the Gelman-Rubin diagnostic $R - 1 \lesssim 0.01$ [44]. We adopt the following uniform priors for model parameters: the baryon fraction $\Omega_b h^2 \in [0.005, 0.1]$, cold dark matter fraction $\Omega_c h^2 \in [0.001, 0.99]$, amplitude of primordial scalar power spectrum $\ln(10^{10} A_s) \in [2, 4]$, scalar spectral index $n_s \in [0.8, 1.2]$, acoustic angular scale at the recombination epoch $100\theta_{MC} \in [0.5, 10]$, optical depth due to reionization $\tau \in [0.01, 0.8]$ and the logarithmic tensor-to-scalar ratio $\log_{10} r \in [-30, 5]$. The pivot scale we use is $0.05\ \mathrm{Mpc}^{-1}$. For convenience, we refer to the data combinations of BK18+Planck+DESI, BK18+WMAP+DESI, BK18+ACT+DESI and BK18+SPT+DESI as “BPD”, “BWD”, “BAD” and “BSD”, respectively.

Results. In light of currently available CMB, B-modes and BAO observations, our numerical results are presented in Fig.1 and Tab.I. Overall, we find the key evidence of PGWs at beyond the $5\,\sigma$ confidence level (CL) using four CMB experiments separately and their combinations with BK18 CMB B-modes and the latest DESI BAO observations (see the supplementary material for $5\,\sigma$ contours). For the CMB-only cases, Planck provides the tightest constraint $\log_{10} r = -5.5 \pm 2.5$ at the $1\,\sigma$ CL. Note that Planck's $2\,\sigma$ constraint $\log_{10} r = -5.5 \pm 4.1$ gives the $2\,\sigma$ upper bound 0.04, which compresses the r range by $\sim 60\%$ relative to 0.1 reported by the Planck collaboration [15]. Although WMAP's constraint $\log_{10} r = -5.1 \pm 2.7$ is similar to Planck, its scalar spectral index n_s has a significant shift towards a larger value when $\log_{10} r$ is larger. ACT's constraint $\log_{10} r = -4.3 \pm 3.7$ gives a slightly larger $\log_{10} r$ value but with a larger error than Planck and WMAP. Interestingly, the fact that ACT prefers $n_s = 1$, namely the scale-invariance, still remains here. Even though another ground-based telescope SPT provides the loosest constraint $\log_{10} r = -5.2 \pm 3.7$, it favors the similar n_s value to two satellite experiments Planck and WMAP. It is noteworthy that n_s constraints from all four CMB facilities are very consistent with the original results reported by each collaboration [15, 30, 32, 34], since tensor fluctuations are insensitive to other physical processes. Our analyses reveals that, prior to Planck, WMAP gives the first detection of PGWs, while Planck provides the tightest constraint on PGWs so far.

Furthermore, the addition of BK18 and DESI data just slightly shrinks the parameter spaces of (n_s, $\log_{10} r$) for Planck and WMAP, but reduces clearly the parameter spaces for ACT and SPT. This is mainly because BK18 prefers a similar upper limit of $\log_{10} r$ to Planck when

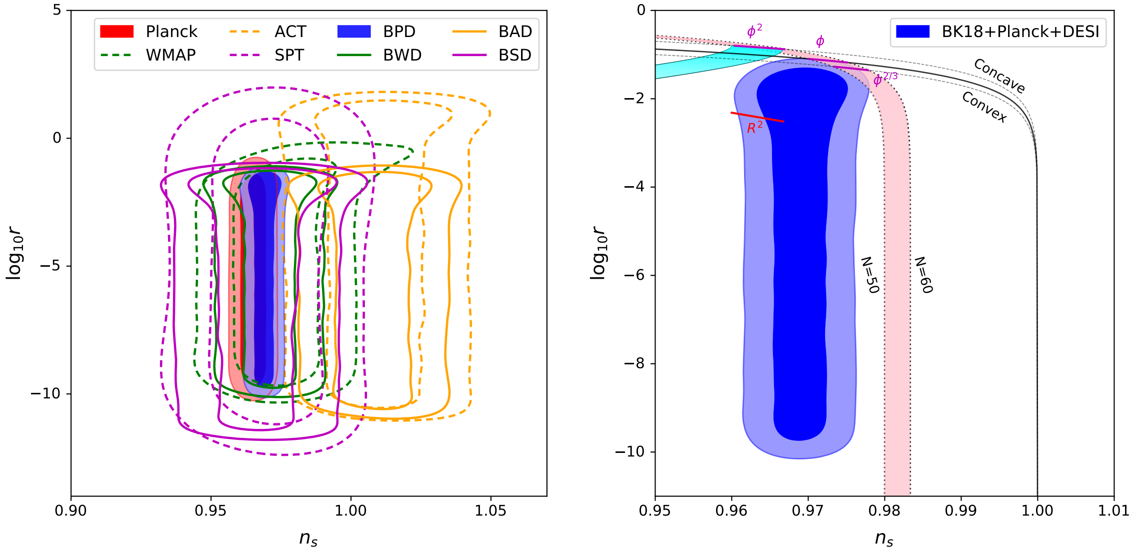

FIG. 1: *Left.* The two-dimensional marginalized posterior distributions of the parameter pair (n_s, $\log_{10} r$) from the Planck (red), WMAP (dashed green), ACT (dashed orange), SPT (dashed magenta), BPD (blue), BWD (solid green), BAD (solid orange) and BSD (solid magenta) observations, respectively. *Right.* The comparison between the allowed $1\,\sigma$ and $2\,\sigma$ (n_s, $\log_{10} r$) contours from the data combination of BK18+Planck+DESI (BPD) and the theoretical predictions of selected inflationary scenarios including R^2 inflation, power-law inflation and natural inflation. Here ϕ is the inflaton field and $N \equiv \ln a$ (a is scale factor) denotes the e-folding number.

TABLE I: The mean values and $1\,\sigma$ uncertainties of the logarithmic tensor-to-scalar ratio $\log_{10} r$ from different datasets.

Data	Planck	WMAP	ACT	SPT	BPD	BWD	BAD	BSD
$\log_{10} r$	-5.5 ± 2.5	-5.1 ± 2.7	-4.3 ± 3.7	-5.2 ± 3.7	$-5.1^{+3.7}_{-4.2}$	$-5.0^{+3.7}_{-4.4}$	$-5.6^{+4.2}_{-4.7}$	$-5.6^{+4.3}_{-5.7}$

assuming a fiducial cosmology and can not give a clear lower bound on $\log_{10} r$ (see supplementary material for details). Therefore, BPD, BWD, BAD and BSD give close $2\,\sigma$ upper limits in Fig.1. It is interesting to see that $1\,\sigma$ errors of $\log_{10} r$ in all four combinations are larger than CMB-only constraints. At first glance, this seems to be anomalous based on the experience that a combined dataset should give a tighter constraint than a single dataset. Nonetheless, these results are actually reasonable and subtle because the addition of BK18 data stretches the $1\,\sigma$ errors of $\log_{10} r$ to be very close to $2\,\sigma$ errors based on the fact that the one-dimensional posterior probability density distribution of $\log_{10} r$ from the BK18 data has a sharp peak around $r \sim 10^{-2}$ and no lower bound on r (see Fig.1). This means that $1\,\sigma$ $\log_{10} r$ errors given by BPD, BWD, BAD and BSD are very close to their $2\,\sigma$ uncertainties. Hence, these four combinations have tightened $2\,\sigma$ errors of $\log_{10} r$ compared to their corresponding CMB-only cases. Moreover, the reason why BPD prefers a larger n_s than Plank alone is that DESI BAO can increase the Hubble constant H_0 [37] when combined with the comoving sound horizon at the drag epoch from Planck [15].

Since all datasets have confirmed the detection of PGWs in single field slow-roll inflation, it is important and meaningful to compare current constraints with theoretical predictions of different inflation models. Specifically, we compare the predictions of R^2 inflation [2, 45, 46], power-law inflation [7, 47, 48] and natural inflation [49, 50] with the most stringent constraint from BPD. In the right panel of Fig.1, considering the e-folds range $N \in [50, 60]$, we find that natural inflation and models with concave potential are clearly ruled out. It is interesting that the 2σ contour from BPD has an overlap with the power-law inflation that has a potential $V(\phi) \propto \phi^n$ (roughly $n > \frac{2}{3}$). This means that such power-law inflationary scenarios can be revived within $2\,\sigma$ CL. Interestingly, when $N = 50$, R^2 inflation which is believed to the best model by the Planck collaboration [15] is now ruled out beyond the $2\,\sigma$ CL. However, it is still viable when $N = 60$. It is clear that there is still a large theory space needed to be explored in light of current constraints on PGWs.

Discussions and conclusions. During the almost two decades, the reason why PGWs are undetected is the usage of non-logarithmic prior of r. Employing the logarithmic prior, we confirm the detection of PGWs at beyond the $5\,\sigma$ CL in light of four available CMB datasets and their combinations with BK18 and DESI observations. The tightest constraint we obtain for the CMB-only cases is $\log_{10} r = -5.5 \pm 2.5$ from Planck. However, prior to Planck, WMAP is actually able to detect the CMB B-modes and gives the constraint $\log_{10} r = -5.1 \pm 2.7$. When comparing the theoretical predictions of selected inflationary scenarios with the most stringent constraint from BPD, we find that natural inflation and models with concave potential are obviously ruled out, some power-law inflation models can be revived within $2\,\sigma$ CL and R^2 inflation with a large size of very early universe (e.g., $N = 60$) is well favored. Our constraints could change the research status of inflationary cosmologies and inspire new explorations of inflation theories so as to explain currently allowed parameter space by data. For example, $r = 10^{-3} - 10^{-9}$ is urgent to be interpreted by new physics of the very early universe.

Interestingly, it seems that we can not conclude that the scale-invariance is completely ruled out, even if ACT's constraint on n_s is inconsistent with Planck, WMAP and SPT. This issue should be addressed by future high precision CMB observations such as CMB-S4 [23], Simons Observatory [24], LiteBIRD [25] and CORE [26].

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SUPPLEMENTARY MATERIAL

A. $5\,\sigma$ contours

The two-dimensional marginalized posterior distributions of the parameter pair (n_s, $\log_{10} r$) from Planck and BK18+Planck+DESI (BPD) are presented from $1\,\sigma$ to $5\,\sigma$ confidence levels in Fig.S1.

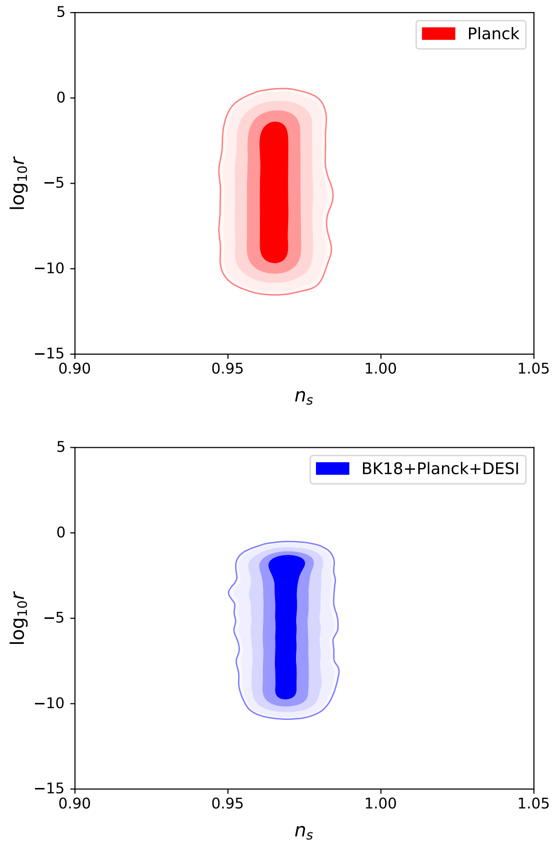

FIG. S1: The two-dimensional marginalized posterior distributions of the parameter pair (n_s, $\log_{10} r$) from the Planck (*top*) and BPD (*bottom*) observations within the 5σ confidence level.

B. BK18 constraints on $\log_{10} r$

The one-dimensional posterior probability density distributions of the logarithmic tensor-to-scalar ratio $\log_{10} r$ from four CMB experiments and BK18 B-mode polarization data are shown in Fig.S2. Note that for the BK18-only case, we have assumed the Planck 2018 fiducial cosmology. One can easily find that BK18-only constraint has two typical features: (i) a sharp peak around $\log_{10} r = -2$; (ii) no lower limit. These two features induces that the $1\,\sigma$ lower bound becomes smaller and $1\,\sigma$ upper bound is closer to its $2\,\sigma$ upper bound. Therefore, the anomalous $1\,\sigma$ errors of $\log_{10} r$ from BPD, BK18+Planck+DESI (BWD), BK18+ACT+DESI (BAD) and BK18+SPT+DESI (BSD) can be well explained. Actually, these four combinations have tightened $2\,\sigma$ uncertainties of $\log_{10} r$ compared to their corresponding CMB-only cases. We present the one-dimensional posterior probability density distributions of $\log_{10} r$ from BPD, BWD, BAD and BSD in the bottom panel of Fig.S2. It is easy to see that the shapes of one-dimensional posterior distributions from these four combinations are very consistent with the above-mentioned features from the BK18-only constraint. As a consequence, our constraining results are reasonable.

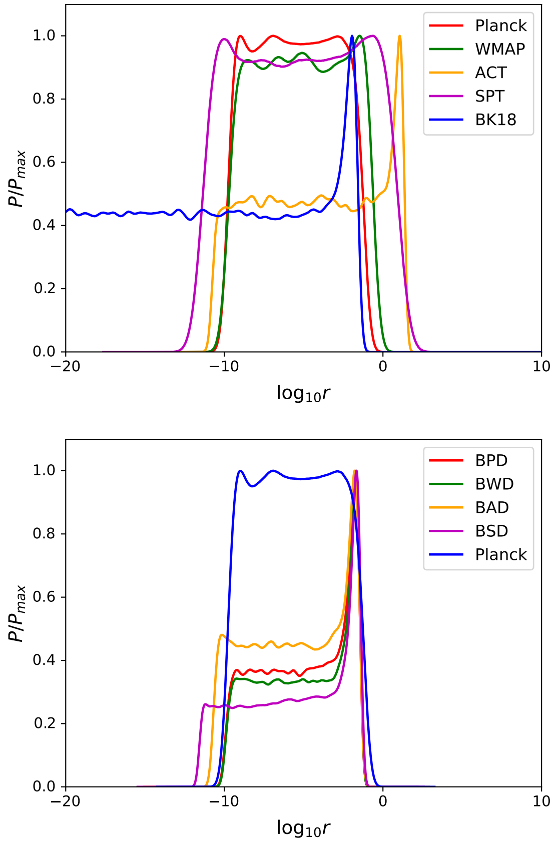

FIG. S2: The one-dimensional marginalized posterior distributions of $\log_{10} r$ from Planck, WMAP, ACT, SPT and BK18 datasets separately (*top*) and four data combinations: BPD, BWD, BAD and BSD (*bottom*).