Constraints on Lorentz invariance violation from gamma-ray burst GRB090510

Zhi Xiao and Bo-Qiang Ma*

School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

We obtain modified dispersion relations by requiring the vanishing of determinant of inverse of modified photon propagators in Lorentz invariance violation (LIV) theory. Inspired by these dispersion relations, we give a more general dispersion relation with less assumption and apply it to the recent observed gamma-ray burst GRB090510 to extract various constraints on LIV parameters. We find that the constraint on quantum gravity mass is slightly larger than the Planck mass but is consistent with other recent observations, so the corresponding LIV coefficient ξ_1 has reached the natural order ($o(1)$) as one expects. From our analysis, the linear LIV corrections to photon group velocity might be not excluded yet.

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I. INTRODUCTION

Lorentz invariance violation (LIV or LV) has been intensively investigated both theoretically and experimentally in recent years. The revival passion of relativity violation in theoretical construction originates from the attempt to compromise general relativity with quantum mechanics. On the other hand, the experimental searches may provide us with concrete evidence to sift a most hopeful candidate of quantum gravity from a vast number of theories.

From theoretical aspect, some theories expect LIV to happen at high energies. For example, spontaneous Lorentz symmetry breaking may happen in string theory as the perturbative string vacua is unstable, thus some tensor fields generate nonzero vacuum expectation values [1]. The breaking of Lorentz symmetry also happens in other frameworks, such as loop gravity [2], foamy structure of spacetime [3], torsion in general gravity [4], etc.. More recently, Hořava proposed a power counting renormalizable theory of gravity [5] with a "dynamical critical exponent" z to characterize the anisotropic scaling properties between space and time. While Lorentz symmetry is breaking at high energies, it restores when this dynamical critical exponent flows to $z = 1$ at low energies. There are also some other proposals, such as the so called double special relativity [7], which preserves relativity principle with a nonlinear realization of Lorentz group, thus conventional Lorentz symmetry is also broken. One striking consequence of LIV is that the photon propagation speed is no longer a unique constant, generally, it depends on energy and propagation direction.

These theoretical investigations have promoted various experiments to search for the deviation from conventional linear dispersion relation for photons [8]. However, as the possible violation effects for photons must be very tiny, the detection of these effects present a significant challenge to experimentalists. In addition to improve the precision of measurements to find any possible evidence of LIV, we should also take efforts on searching for certain accumulating processes to amplify these tiny effects. Such idea has already been proposed on the observation of certain astronomical objects such as pulsars, active galactic nuclei (AGN) [9] and gamma-ray bursts (GRB) [15], etc., and the tiny LIV effect could manifest itself through the observation of rotation of linear polarization (birefringence) [16] or time of flight lag [15] for photons with different energies.

The paper is organized as follows. In Section 2, we review certain modified photon dispersion relations derived from several LIV models, including standard model extension (SME) with power counting renormalizable operators [16], effective field theory with dimension 5 operators [19] and Hořava's anisotropic U(1) theory. In Section 3, we focus on time of flight analysis of GRB and try to extract some LIV parameters from the recent observation of GRB090510 [18]. We briefly discuss the time of flight analysis of photons from cosmological distant objects, then we give a general dispersion relation used conventionally in the astrophysical analysis of LIV [9]. This general dispersion relation contains those terms derived from the models in Section 2 as special cases. We then extract constraints to linear LIV parameters from GRB090510 to $o(0.1)$, improved by 1 or 2 order of magnitude than those in [34] and [9, 35]. From the analysis of the time-lag formula we find that it is hard to significantly improve the constraints from this simple and rough analysis, unless other time-lag effects (like source effect, which is a major uncertainty in the time of flight analysis) can be clarified or other methods will be used.

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* Corresponding author. Email address: mabq@phy.pku.edu.cn

II. PHOTON DISPERSION RELATIONS

A. Background tensor field induced LIV

A systematical treatment of LIV to incorporate particle standard model with power counting renormalizable Lagrangian, called standard model extension (SME), was proposed by Kostelecký and Colladay in Ref. [16], where the photon sector reads

$$\mathcal{L}_{\text{photon}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} + \frac{1}{2}(k_{AF})_{\kappa}\epsilon^{\kappa\lambda\mu\nu}A_{\lambda}F_{\mu\nu}. \tag{1}$$

From (1), we deduce the equation of motion below

$$\partial^{\alpha}F_{\mu\alpha} + (k_F)_{\mu\alpha\beta\gamma}\partial^{\alpha}F^{\beta\gamma} + (k_{AF})^{\alpha}\epsilon_{\mu\alpha\beta\gamma}F^{\beta\gamma} = 0. \tag{2}$$

By expressing (2) in terms of 4-vector potential A_{μ} and assuming that the Fourier decomposition

$$A_{\mu}(x) \equiv a_{\mu}(p)\exp(-i\ p \cdot x) \tag{3}$$

is still reliable, we express (2) in the momentum space as

$$M_{\mu\nu}(p)a^{\nu}(p) = 0, \tag{4}$$

where

$$M_{\mu\nu}(p) = \eta_{\mu\nu}\ p^2 - p_{\mu}\ p_{\nu} - 2(k_F)_{\mu\kappa\lambda\nu}p^{\kappa}\ p^{\lambda} - 2i(k_{AF})^{\kappa}\epsilon_{\mu\kappa\lambda\nu}p^{\lambda}. \tag{5}$$

We impose gauge fixing condition and require the determinant of the reduced matrix to vanish, then we can obtain an implicit function $p^0(\vec{p})$, which is an eighth order-polynomial in p^0. Otherwise one can verify that the determinant of $M_{\mu\nu}$ vanishes as a consequence of gauge invariance of equation (2). For example, we use Lorentz gauge

$$\partial_{\alpha}\ A^{\alpha} = 0, \tag{6}$$

in momentum space, i.e.,

$$p_{\alpha}\ a^{\alpha}(p) = 0. \tag{7}$$

So we have the gauge fixed reduced matrix

$$M_{\mu\nu}^{\text{gf}}(p) = \eta_{\mu\nu}\ p^2 - 2(k_F)_{\mu\kappa\lambda\nu}p^{\kappa}\ p^{\lambda} - 2i(k_{AF})^{\kappa}\epsilon_{\mu\kappa\lambda\nu}p^{\lambda}. \tag{8}$$

For our purpose, we just try to extract a simplified result by assuming that (particle) rotational invariance still holds regardless of the explicit violation of Lorentz symmetry, i.e., only $(k_{AF})^0$ and α (a combination of $(k_F)_{\kappa\lambda\mu\nu}$, for details, see Appendix or [20]) are nonzero. With this assumption, we have the following matrix:

$$M^{\text{red}}(p) =$$
$$\begin{pmatrix} p^2 - \alpha\vec{p}^2 & \alpha p^0 p^1 & \alpha p^0 p^2 & \alpha p^0 p^3 \\ \alpha p^0 p^1 & -(p^2 + \alpha((p^0)^2 + \vec{p}^2 - (p^1)^2)) & \alpha p^1 p^2 + 2ik_{AF}^0 p^3 & \alpha p^1 p^3 - 2ik_{AF}^0 p^2 \\ \alpha p^0 p^2 & \alpha p^1 p^2 - 2ik_{AF}^0 p^3 & -(p^2 + \alpha((p^0)^2 + \vec{p}^2 - (p^2)^2)) & \alpha p^2 p^3 + 2ik_{AF}^0 p^1 \\ \alpha p^0 p^3 & \alpha p^1 p^3 + 2ik_{AF}^0 p^2 & \alpha p^2 p^3 - 2ik_{AF}^0 p^1 & -(p^2 + \alpha(p^0)^2 + \vec{p}^2 - (p^3)^2)) \end{pmatrix}. \tag{9}$$

Its determinant reads

$$\det(M^{\text{red}}(p)) = \left\{ 4(k_{AF}^0)^2\vec{p}^2 - \left((1+\alpha)(p^0)^2 - (1-\alpha)\vec{p}^2\right)^2 \right\}(1+\alpha)(p^2)^2. \tag{10}$$

By requiring $\det(M^{\text{red}}(p)) = 0$ (otherwise there would be no solution for a photon field), we have two dispersion relations, one is the conventional $p^2 = 0$ and the other is

$$(p^0)^2 = \frac{1}{(1+\alpha)}\left((1-\alpha)\vec{p}^2 \pm 2k_{AF}|\vec{p}|\right). \tag{11}$$

We can also use another equivalent method to obtain these two dispersion relations. First, we rewrite (1) in an explicitly quadratic form in the photon field, i.e.,

$$\begin{aligned}
\mathcal{L}_{\text{photon}} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} + \frac{1}{2}(k_{AF})_{\kappa}\epsilon^{\kappa\lambda\mu\nu}A_{\lambda}F_{\mu\nu} - \frac{1}{2\xi}(\partial \cdot A)^2 \\
&= -\frac{1}{2}\,\partial_{\mu}A_{\nu}\left(F^{\mu\nu} + 2(k_F)^{\kappa\lambda\mu\nu}\partial_{\kappa}A_{\lambda} + \frac{\eta^{\mu\nu}}{\xi}\partial \cdot A - 2\epsilon^{\kappa\lambda\mu\nu}(k_{AF})_{\kappa}A_{\lambda}\right) \\
&= \text{total\ \ derivative} + \frac{1}{2}\,A_{\nu}(\mathrm{D_F^{-1}})^{\nu\lambda}A_{\lambda},
\end{aligned}\tag{12}$$

where we have added gauge fixing terms $-\frac{1}{2\xi}(\partial \cdot A)^2$ in (12) and

$$(\mathrm{D_F^{-1}})^{\nu\lambda} \equiv \left(\Box\eta^{\nu\lambda} - \partial^{\nu}\partial^{\lambda}(1 - \frac{1}{\xi}) - 2(k_F)^{\nu\mu\kappa\lambda}\partial^{\mu}\partial^{\kappa} - 2\epsilon^{\nu\mu\kappa\lambda}(k_{AF})_{\mu}\partial^{\kappa}\right).\tag{13}$$

Using the same Ansatz (3), we define a matrix Σ in momentum space

$$\Sigma(p)_{\nu\rho} = -\left(p^2\eta_{\nu\rho} - (1 - \frac{1}{\xi})p_{\nu}p_{\rho}\right) + 2(k_F)_{\nu\mu\kappa\rho}p^{\mu}p^{\kappa} - 2i\epsilon_{\nu\mu\kappa\rho}p^{\mu}(k_{AF})^{\kappa}.\tag{14}$$

From the conventional free field theory, the differential operator inside the two fields in the quadratic form of certain Lagrangian (e.g., (12)) is just the inverse of free field propagator in position space (see [21] or [22]), thus (14) is just the inverse of photon propagator expressed in momentum space. We know that generally the inverse of propagator is just the dispersion relation, from which one can find the pole of the corresponding particle, so we expect that the determinant of (14) in case of $\xi \to \infty$ (i.e., without gauge fixing) is zero. Similarly, we can find the explicit dispersion relation in a special gauge by choosing the corresponding specific value of ξ. For example, we find that $\Sigma(p)_{\nu\rho}|_{\xi=1} = -M^{\text{gf}}(p)_{\nu\rho}$ ($\xi = 1$ is just the Lorentz gauge (6) used to obtain (9), and this choice can avoid the inequivalent gauge choice comparison. As pointed out in [16], different gauge choices are inequivalent with each other in the LIV electrodynamics). So in the rotational invariant case, this matrix can also lead to (11) and the conventional dispersion relation. We mention here that similar method to obtain photon propagator in the SME framework has also been obtained recently in [23], with a more systematic and complete treatment.

The leading order nonrenormalizable LIV operators (dimension 5) were systematically studied in [19], where Myers and Pospelov also introduced explicitly a timelike four-vector n^a to take LIV into account, thus this theory can be regarded as a leading nonrenomalizable part of SME. Since we are only interested in the study of the consequence of LIV to the propagation of GRB, we focus our attention only on photon field there. The corresponding Lagrangian is

$$\delta\mathcal{L}_{\text{photon}} = \frac{\xi}{2M_{Pl}}\epsilon^{\mu\nu\kappa\rho}n^{\alpha}F_{\alpha\rho}n \cdot \partial(n_{\kappa}F_{\mu\nu}).\tag{15}$$

We write it in another equivalent form, i.e.

$$\mathcal{L}_{\text{photon}} = \frac{1}{2}\,A_{\nu}\left(\Box\eta^{\nu\rho} - \frac{2\xi}{M_{Pl}}n \cdot \partial(n \cdot \partial\,n_{\kappa}\partial_{\mu}\epsilon^{\nu\mu\kappa\rho} + n_{\kappa}\partial_{\mu}\partial_{\alpha}\epsilon^{\nu\mu\kappa\alpha}n^{\rho})\right)A_{\rho} + \text{total\ \ derivative},\tag{16}$$

where we have added the Lorentz gauge fixing term. Then by performing the same procedure as before, we have the reduced inverse of propagator

$$\Pi(p)^{\nu\rho} = -p^2\eta^{\nu\rho} - \frac{2i\xi}{M_{Pl}}(\epsilon^{\nu\mu0\rho}p_0^2 p_{\mu} + \epsilon^{\nu\mu0\alpha}p_0 p_{\mu}p_{\alpha}\delta_0^{\rho})\tag{17}$$

when expressing explicitly the time-like four-vector n in a preferred frames as $n^{\rho} = (1, 0, 0, 0)$. Then by imposing

$$\det\Pi(p) = \det\begin{pmatrix} -p^2 & 0 & 0 & 0 \\ 0 & p^2 & -i\frac{2\xi}{M_{Pl}}(p^0)^2 p^3 & i\frac{2\xi}{M_{Pl}}(p^0)^2 p^2 \\ 0 & i\frac{2\xi}{M_{Pl}}(p^0)^2 p^3 & p^2 & -i\frac{2\xi}{M_{Pl}}(p^0)^2 p^1 \\ 0 & -i\frac{2\xi}{M_{Pl}}(p^0)^2 p^2 & i\frac{2\xi}{M_{Pl}}(p^0)^2 p^1 & p^2 \end{pmatrix} = p^4\left((\frac{2\xi}{M_{Pl}}\vec{p})^2(p^0)^4 - p^4\right) = 0,\tag{18}$$

we obtain the dispersion relation

$$(p^0)^2 = \vec{p}^2 \pm \frac{2\xi}{M_{Pl}}(p^0)^2|\vec{p}|,\tag{19}$$

which was obtained in [19] plus the conventional one $p^2 = 0$.

B.   Anisotropic scaling induced LIV

Now we turn to another framework of LIV proposed recently by Hořava [5]. His original proposal was to provide a UV completion of quantum theory of gravity. Lorentz symmetry appears naturally in this theory when the dynamical critical exponent flows to $z = 1$ at low energies. While at high energies, space and time present anisotropic scaling

$$t \to \lambda^z t, \quad \vec{r} \to \lambda \vec{r}, \tag{20}$$

thus Lorentz symmetry breaks down. However, this formalism does not break spatial isotropy, thus there is no need to assume a special background field configuration to realize rotational invariance, unlike the background tensor formalism discussed above. Aside from gravity, Hořava also constructed an anisotropic Yang-Mills theory with critical spatial dimension $D = 4$ [5]. As Chen and Huang recently gave a general construction of bosonic field theory demonstrating this anisotropic scaling behavior [6], we follow this new approach instead of [5]. In the new formalism, the photon action reads

$$S = \frac{1}{2} \int \, dt d^D x \frac{1}{g_E^2} \left(\vec{E}^2 - \sum_{J \geq 2} \frac{1}{g_E^{J-2}} \sum_{n=0}^{n_J} (-1)^n \frac{\lambda_{J,n}}{M^{2n+\frac{1}{2}(D+1)(J-2)}} \partial^{2n} \star F^J \right). \tag{21}$$

For simplicity, we consider the case $z = 2$ and $D = 3$. Then one immediately reads from the action that the scaling dimensions of the couplings are

$$[g_E]_s = \frac{1}{2}(z - D) + 1, \quad [\lambda_{J,n}]_s = z + D + \frac{1}{2}(z - D - 2)J - 2n. \tag{22}$$

Thus in this case the renormalizable condition ($[g_E]_s \geq 0 : z \geq D - 2$) for $\vec{E}$ is automatically satisfied. Actually, it is superrenormalizable. If the critical dimension (i.e., $[g_E]_s = 0$) is $D = 3$, then z must equal to 1, which just corresponds to the conventional Lorentz invariant gauge theory. Renormalizability also imposes the condition $[\lambda_{J,n}]_s \geq 0$, and for a free field theory, $J = 2$, $n \leq z - 1 = 1$. For simplicity, we set $\lambda_{2,0} = \frac{1}{2}$, then the free Lagrangian (with gauge fixing term) is

$$\mathcal{L}_{\text{free}} = \frac{1}{g_E^2} \left(\vec{E}^2 - \frac{1}{2} F_{ij} F^{ij} - \frac{\lambda_{2,1}}{M^2} (\partial_i F_{ik} \cdot \partial_j F_{jk} + \partial_i F_{jk} \cdot \partial_i F_{jk}) \right) - \frac{1}{\xi}(\partial \cdot A)^2$$

$$= \frac{1}{2g_E^2} A_\nu \left\{ \left(\Box \eta^{\nu\rho} - \partial^\nu \partial^\rho (1 - \frac{1}{\xi}) \right) - \frac{3\lambda_{2,1}}{M^2} \Delta (\Delta \delta_{kj} - \partial_k \partial_j) \delta_k^\nu \delta_j^\rho \right\} A_\rho. \tag{23}$$

By performing the same trick, we can obtain

$$\Gamma(p)^{\nu\rho} = -p^2 \eta^{\nu\rho} + \frac{3\lambda_{2,1}}{M^2} \delta_k^\nu \delta_j^\rho (p_k p_j - \vec{p}^2 \delta_{jk}), \tag{24}$$

and the corresponding dispersion relations read

$$p^2 = 0, \quad (p^0)^2 = \vec{p}^2 (1 + \frac{3\lambda_{2,1}}{M^2} \vec{p}^2). \tag{25}$$

III.   TIME OF FLIGHT ANALYSIS OF GRB IN LIV THEORY

In this section, we discuss the LIV effect on the observed GRBs, focusing especially on the time of flight of γ rays. Gamma-ray bursts (GRBs) are sudden, intense flashes originating from distant galaxies with cosmological distances, and they are the most luminous electromagnetic events we ever known. As already shown in the above formulas, LIV can modify conventional Maxwell equations and hence leads to modified dispersion relation in addition to the conventional one. However, due to the large mass scale suppression, this LIV effects must be very tiny to account for the conventional stringent terrestrial test. Fortunately, as first pointed out in [3], the cosmological origin plus high energy and the millisecond time structure of GRB make GRB an ideal object to observe the possible minuscule effects of LIV. Actually, many known stringent constraints to LIV parameters were drawn from astronomical observations, such as AGN [9], ultrahigh-energy cosmic rays (UHECR) [13], CMB [10, 11], etc.. The LIV induced modified dispersion relation can lead to many interesting phenomena. The most apparent consequence is the frequency dependence of photon group velocity, though this is not always the case. For example, if only $k_F \neq 0$ in the SME framework, photons

propagate independently with their energies. So if photons with different energies are emitted simultaneously, this frequency dispersion of group velocity then leads to the so called time-lag phenomenon. In addition to time lag, certain models, e.g. SME, indicate that photons with independent polarizations obey distinct dispersion relations. This was demonstrated in the above two models, see (11) and (19). All these models involve a conventional mode with an extraordinary helicity dependent one, thus can lead to the so called vacuum birefringence effects [16, 17]. The tiny changes in polarization grow linearly with propagation distance and hence can be accumulated to be observable for cosmological sources. This can provide a sensitive probe to LIV [10, 11, 12, 16, 17]. Aside from purely kinematic effects, the tiny LIV correction to dispersion relation can also dramatically change the thresholds of high-energy particle reactions, hence leads to distinct particle spectrum of UHECR with respect to that of Lorentz invariance cases. The observation of this spectrum can provide a unique signature of LIV [24]. On the contrary, the nonobservation of these effects can put very stringent constraints to LIV parameters [13]. Below we will primarily discuss the GRB time-lag caused by LIV.

First we make a brief review of the formula used in the description of GRB photon time-lag with respect to the source redshift. For an isotropic and homogeneous universe, one can derive a differential relation

$$dt = -\frac{dz}{H_0(1+z)\sqrt{\Omega_\Lambda + \Omega_K(1+z)^2 + \Omega_M(1+z)^3 + \Omega_R(1+z)^4}} \tag{26}$$

from the Friedman equation

$$(\frac{\dot{a}}{a})^2 + \frac{K}{a^2} = \frac{8\pi G_N \rho}{3}, \tag{27}$$

see [25] for details, where the present-day Hubble constant $H_0 \simeq 71$ km/s/Mpc and $\Omega_K = 0$ for a nearly flat universe. The matter density $\Omega_M \simeq 0.27$, radiation density $\Omega_R \simeq 0$ and vacuum energy density $\Omega_\Lambda \simeq 0.73$ are the cosmological parameters evaluated today. We note that (26) is the standard result derived from general relativity which is a locally Lorentz invariant (LI) theory. So in dealing with photon time-lag below, we implicitly assume that the gravity side is untouched. Though a unified treatment should also include the change of gravity due to possible LIV effects hence may also change (26) and the time-lag formula used below.

Then we make a general assumption of photon dispersion relation

$$E^2 = f(p; \mathrm{M}, \{\xi_i\}), \tag{28}$$

where $f(p; \mathrm{M}, \{\xi_i\})$ is a general function of p ($p = |\vec{p}|$), some unknown large scale M relevant to LIV and a set of parameters $\{\xi_i\}$. Inspired by the dispersion relations (19) and (25) derived from particular models discussed above and the fact that LIV corrections must be very tiny at low energies, we assume that the expansion of (28) around the conventional dispersion relation $E^2 = p^2$ is

$$E^2 = p^2(1 + \sum_{i=1}^{N} \xi_i (\frac{p}{\mathrm{M}})^i), \tag{29}$$

where N is a large number marking the precision of our expansion. Thus (19) and (25) can be regarded as just two special cases of (29), i.e., only $\xi_1 \neq 0$ and $\xi_2 \neq 0$ respectively (where (19) just adds helicity dependence assumption of LIV). In addition to its generality, the reason for beginning with (29) instead of those particular models is that various experiments have already ruled out (19) to a convincing level (see [31] and a recent review [36]). Indeed, dimension 5 LIV operators have suffered very stringent constraints both from frequency-dependent birefringence test with GRB [37, 39], Crab Nebula [38], and the UHECR spectrum analysis under special assumptions [13] (i.e. LIV corrections to electron dispersion relations are smaller than those of photon ones, e.g., in the Liouville string models of foamy structure of space-time [40], where only neutral gauge bosons receive quantum-gravity corrections).

When taking into account of the expansion of universe [26] and the assumption that gravity side remains intact,the time-lag led by modified dispersion relation (29) with leading order LIV correction of order n is

$$\delta t = \frac{1+n}{2} \xi_n \frac{\delta E_0^n}{M^n} \int_0^z \frac{(1+z')^n}{h(z')} dz', \tag{30}$$

where

$$h(z) = H_0 \sqrt{\Omega_\Lambda + \Omega_K(1+z)^2 + \Omega_M(1+z)^3 + \Omega_R(1+z)^4}, \tag{31}$$

E_0 is the redshifted photon energy observed on earth. $\delta E_0^n = E_l^n - E_h^n$, where E_l and E_h denote lower and higher energies of observed photons respectively in the time delay. The n-th order correction corresponds to dimension $n+4$ LIV operators. This can be seen from another formula

$$\delta t = \delta w^{d-4} \int_0^z \frac{(1+z')^{d-4}}{h(z')} dz' \sum_{jm} {}_0 Y_{jm}(\hat{n}) k_{(I)jm}^{(d)}, \tag{32}$$

which is suitable to the analysis of time-lag in the SME framework given recently by Kostelecký and Mewes [27].

Before we discuss the linear ($n=1$) and the quadratic ($n=2$) corrections to the photon dispersion relation, we first utilize the observed time delay in GRB090510 located at redshift $z = 0.903 \pm 0.003$ to give a rough estimate to photon mass. As is well known, photon mass is represented by dimension 2 operator (quadratic in photon fields) and may spoil gauge invariance (this is not the case in Chern-Simons theory [22] in space-time dimension 3). However, the presence of photon mass does not necessarily implies LIV. In a LI theory (Proca's theory), the existence of a unique speed c could be regarded as the limiting speed of light for arbitrary-high energy photons. So photon mass is absent in a LIV theory (e.g. SME [37]) if gauge invariance is still valid. But we can still give a rough estimate of its magnitude by using (32) and the fact that the bulk of the photons above 30 MeV arrived 258 ± 34 ms later than those below 1 MeV [18] in the observation of GRB090510 :

$$\sum_{jm} {}_0 Y_{jm}(\hat{n}) k_{(I)jm}^{(2)} \le 1.4801 * 10^{-24} \text{ GeV}^2. \tag{33}$$

This could be translated to the photon mass bound as $m_\gamma \le 1.217 * 10^{-3}$ eV, much larger than the mass upper bound given in [30], $m_\gamma \le 1 * 10^{-18}$ eV. This confirms the remarks given in [33]: "(departures of electrostatic and magnetostatic fields from the gauge invariant one) give more sensitive ways to detect a photon mass than the observation of velocity dispersion." Of course, one can obtain an effective mass bound comparable to this as $m_\gamma \le 1.217 * 10^{-19}$ eV [11], but the origin is different. From the bound derived we see that one would need a much larger photon mass to explain the time of flight data if one does not introduce the LIV effect (or other effect, e.g. source effect). More than that, as the presence of photon mass indicates that high energy photons propagate faster than low energy ones, this time advance of high energy photons may cancel possible time-lag induced by certain LIV models ($\xi_i < 0$ in (29), see also (35)). Thus mass effects may conspire with LIV effects to produce a nearly nonobservation of time-lag in certain time of flight analysis [14]. On the contrary, in some scenario with $\xi_i > 0$, the time-lag might be caused by the combined effects of mass and LIV, thus the situation is still complicated. Fortunately, due to the high precision laboratory experiment [32] (constrain m_γ to 10^{-17} eV level), those scenarios mentioned above do not happen and we can safely ignore the mass effects in our discussion about LIV constraints drawn from the time of flight analysis of GRB090510.

Without the trouble of possible mass effects, we can then securely discuss LIV effects in the time-lag phenomena below. As a byproduct of (32), we give a rough estimate to mass dimension 3 LIV operators

$$\sum_{jm} {}_0 Y_{jm}(\hat{n}) k_{(I)jm}^{(3)} \le 1.1558 * 10^{-21} \text{ GeV}. \tag{34}$$

We see that this bound is comparable to that obtained from the LIV effects on Schumann resonances in a natural earth-ionosphere cavity [28], though it is much weaker than the other astronomical constraints [10, 11] (which are constrained to less than 10^{-43} GeV).

Then we turn to nonrenormalizable LIV operators but using formula (30) of time-lag instead, as it is more suitable to our simple analysis based on general dispersion relation (29). As usual, we only discuss energy dependent corrections to photon group velocity to the quadratic level. Before looking into details, from (29) we derive modified group velocity

$$
\begin{aligned}
v_g \equiv \frac{\partial E}{\partial p} &= \frac{p}{E} \left(1 + \frac{1}{2}(i+2) \, \xi_i (\frac{p}{M})^i \right) \\
&= \frac{\left(1 + \frac{1}{2}(i+2) \, \xi_i (\frac{p}{M})^i \right)}{\sqrt{1 + \xi_i (\frac{p}{M})^i}} \\
&\simeq 1 + \frac{1}{2}(i+1) \, \xi_i (\frac{p}{M})^i,
\end{aligned} \tag{35}
$$

where Einstein sum over index i from 1 to N is indicated. Then by the same procedure in [26], we can give a time-lag formula which is accurate to 2nd order of $\frac{E}{M}$ and first order in Δz as

$$\delta t = \xi_1 \frac{E_l - E_h}{M} \int_0^z \frac{(1+z')}{h(z')} dz' + \frac{3}{8}(4\xi_2 - \xi_1^2) \frac{E_l^2 - E_h^2}{M^2} \int_0^z \frac{(1+z')^2}{h(z')} dz'. \tag{36}$$

It can be seen that (36) is consistent with (30) when $\xi_2 = 0$ and $\xi_1 = 0$ respectively for linear (though the linear correction is calculated to second order in the large mass suppression, it will be checked that this can not improve the linear constraints any more, thus in practical calculation, (30) is enough) and quadratic corrections to group velocity.

For linear energy dependent correction to 1st order, we derive from the most conservative claim that, the bulk of the photons above 30 MeV arrived 258±34 ms later than those below 1 MeV [18], the LIV scale $\frac{M}{\xi_1} \sim -5.02689 * 10^{16}$ GeV, which is 3-order less than the Planck scale if $|\xi_1|$ is of order 1. However, if utilizing the more stringent claim that, a single highest detected photon from GRB090510 with 31 GeV arrives 0.179 s later than the main LAT emission above 100 MeV, we can deduce a significant higher quantum gravity mass scale

$$\frac{M}{\xi_1} \sim -7.72017 * 10^{19} \text{ GeV}, \tag{37}$$

where the minus sign indicates the fact that photons with higher energies propagate slower than lower ones as already mentioned in the discussion of photon mass ($\xi_1 < 0$). By direct calculation of solving 2nd order equation of $\frac{M}{\xi_1}$ (i.e. setting $\xi_2 = 0$ in (36)), we find that this can not improve the result any more as mentioned. This illustrates that in a rough estimate of linear correction to group velocity, there is no need to take into account 2nd order correction of $(\frac{M}{\xi_1})^2$ as (36). The result (37) means that linear correction gives a LIV mass scale nearly 6.32 M_{Pl} if $\xi_1 \sim o(1)$, which is very close to that of [18]. Of course, if one chooses other data from the Table 2 in [18], one can obtain the same conclusion that quantum-gravity mass scale is significantly above the Planck mass (at most of order 102 M_{Pl} [18]) from this simple analysis. This conclusion is nothing more than a translation of the claim that the constraint on the linear energy dependent LIV parameter $|\xi_1|$ can be placed in the range $10^{-1} \sim 10^{-2}$, if we regard $M \sim M_{\text{Pl}}$. If this constraint is confirmed by other astrophysical observations, then it puts the constraints at least 2 order of magnitude stronger than [9] and [35], which gives $|\xi_1| < 17$ and $|\xi_1| < 58$ respectively. However, this constraints is not stronger enough as those obtained in [13] extracted from the UHECR spectrum and those in [37] from the frequency-dependent helicity observations of GRB930131 and GRB960924. However, we note that those most stringent constraints ($\xi_1 \leq 10^{-14}$) up to now rely either on particular assumptions (see [13]) or helicity dependent models, e.g. (19). Thus it is necessary to put the constraints obtained from (29) on the linear LIV parameter ξ_1 to the same level (still a hard task as a span of 12 orders to be conquered) from future observations. If so, we can finally make a more firm claim that dimension 5 LIV operators can be excluded firmly [36]. Then we may reach the conclusion in the near future that either Lorentz symmetry is exact, or at high energies there are some other symmetries such as SUSY plus CPT to protect our low energy theory from receiving CPT odd corrections [31].

For quadratic energy dependent correction, i.e. $\xi_1 = 0$, we obtain from the most conservative claim mentioned above the constraints on quantum-gravity mass scale $M \sim 5.84718 * 10^7$ GeV if $\xi_2 \sim o(1)$. While for the single 31 GeV event, we obtain the constraint as $\frac{M^2}{\xi_2} \sim -5.26767 * 10^{21}$ GeV^2, thus $M \sim 7.25787 * 10^{10}$ GeV. It is obvious that the constraints obtained from quadratic correction are much weaker than those from linear one as quadratic correction being suppressed more than one power of M. Thus in the future we should take more efforts to the search of more stringent constraints on quadratic LIV correction to photon group velocity. As mentioned above, the quadratic correction is produced by dimension 6 operators, which are the leading order nonrenormalizable CPT even LIV operators. As we known, various current constraints to dimension 6 operators are also much weaker than dimension 5 ones [37].

Before we close this section, we observe that our results are similar to that obtained recently in [9] and [34, 35]. We give these results in the table below:

Source	Mkn501 [35]	PKS 2155 - 304 [9]	GRB080916C [34]	GRB090510 [18]
redshift	0.034	0.116	4.35	0.900
δt(s)	240	27	16.54	0.179
E_h to E_l(GeV)	10^4 to 250	600 to 210	13.22 to 10^{-3}	31 to 0.1
M(GeV)	$6.06 * 10^{17}$	$7.51 * 10^{17}$	$1.55 * 10^{18}$	$7.72 * 10^{19}$
$\frac{t_{total}}{\delta t}$	$6.01 * 10^{13}$	$1.72 * 10^{15}$	$2.34 * 10^{16}$	$1.29 * 10^{18}$

TABLE I: Where the first three rows below the source row are the data given from [9, 18, 34, 35] respectively, the last 2 rows are the linear quantum-gravity masses and total time to time-lag ratios calculated from (30).

We find that the rough estimates about the linear mass scale are consistent with those given by the reference above and the order of magnitude of the linear mass scale ranges from 10^{17} to 10^{19}. It is easily seen from Table I that one can approach this large magnitude with the advantages both from the quotient of total time to time-lag (which

originates from the cosmological distance and short pulse nature of GRB) and the large absolute energy difference (range from GeV to TeV). To further constrain the linear order quantum-gravity mass hence the coefficient ξ_1 in the future, we need to amplify the large ratio of total time to time lag, as photons with energies much higher than already observed (TeV) can not reach us from cosmological distance due to the pair creation interaction with infrared background photons. One way to amplify the large time ratio is to exclude other non-LIV induced time-lag factors, like different response time of detectors [41] (which slightly increase linear quantum-gravity mass scale obtained in [18]), or one turns to other methods like spectrum analysis [13], otherwise the constraints can not be improved significantly. Further more, a statistic analysis by taking into account of statistic error [35] and a multi-source analysis to calculate the correlation between distance and time-lag [15] will make the results more concrete.

IV. CONCLUSION

In this paper, we reviewed several modified dispersion relations from standard model extension and Hořava theory in the photon sector. Dispersion relations are derived consistently from the inverse of photon free propagators, without taking quantum corrections into account. Inspired by these dispersion relations we give a more general one (29) to avoid some particular assumptions (e.g. helicity dependence). Then we apply this relation to the time of flight analysis of recently reported GRB090510. We obtain constraints on the linear LIV energy dependent coefficients to the level of $\xi_1 \sim o(.1)$, which is equivalent to the statement that the relevant linear quantum-gravity mass scale is $M \sim 7.72 * 10^{19}$ GeV. Using the same method we also get the quadratic mass scale $M_q \sim 7.26 * 10^{10}$ GeV, denoting a much loose constraints to dimension 6 operators. These results are consistent with those in [9, 18, 34, 35]. As a byproduct of the time-lag formula (32), we point out that one can safely ignore the photon mass effect in the discussion of LIV effects in the time-lag analysis of GRBs due to the stringent terrestrial constraints on photon mass, and we obtain a constraints $\sum_{jm} {}_0Y_{jm}(\hat{n})k^{(3)}_{(I)jm} \leq 1.1558 * 10^{-21}$ GeV to the dimension 3 operator.

From our analysis, we find that though the constraints obtained are far from reaching those from the spectrum analysis [13] and those from the helicity dependent analysis [37], our analysis relies little on extra assumptions except the expansion (29) and the formula (30). Thus to exclude the linear order quantum-gravity correction to photon group velocity is still too early as long as the constraints to the general linear order correction (29) have not approached the same level as in [36]. We find that the results have already reached the precision of probing Planck mass scale or even higher, slightly better than [9, 35]. If one can largely clarify other time lag uncertainties like [41], the constraints could be improved more.

Appendix

The previous referred parameter α, is just one of the parameters defined from various combinations of $(k_F)_{\kappa\lambda\mu\nu}$. These definitions arise for convenience from the consideration of the symmetry of this tensor. From the Lagrangian

$$\delta\mathrm{L} = -\frac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu}, \tag{38}$$

we find that $(k_F)_{\kappa\lambda\mu\nu}$ is antisymmetric to the two indices $\kappa\lambda$ and $\mu\nu$ respectively, and is symmetric to the interchange of these two pairs of indices. As we do not want to include a conceivable θ-type term proportional to $\frac{1}{2}\epsilon_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu}$, we require that $\epsilon_{\kappa\lambda\mu\nu}(k_F)^{\kappa\lambda\mu\nu} = 0$. By requiring that $(k_F)_{\kappa\lambda\mu\nu}$ is doubletraceless as any trace term would serve merely as a redefinition of kinematic terms and hence a field redefinition, $(k_F)_{\kappa\lambda\mu\nu}$ has the symmetry of Riemann tensor. Then we can define the decomposition of $(k_F)_{\kappa\lambda\mu\nu}$ in terms of its spatial and time indices, i.e.,

$$(k_{DE})^{jk} \equiv -2(k_F)^{0j0k}, \quad (k_{HB})^{il} \equiv \frac{1}{2}(k_F)^{jkmn}\epsilon^{ijk}\epsilon^{lmn},$$

$$(k_{DB})^{jk} \equiv -(k_{HE})^{kj} \equiv \frac{1}{2}(k_F)^{0jmn}\epsilon^{kmn}, \tag{39}$$

with Latin indices run from 1 to 3. Moreover, we define

$$\alpha_E = \frac{1}{3}\text{tr}(k_{DE}), \quad \alpha_B = \frac{1}{3}\text{tr}(k_{HB}), \tag{40}$$

and double tracelessness gives $\text{tr}(k_{HB} + k_{DE}) = 0$, i.e., $\alpha \equiv \alpha_E = -\alpha_B$. So we can extract the trace term to define

$$(\beta_E)^{jk} = (k_{DE})^{jk} - \alpha\delta^{jk}, \quad -(\beta_B)^{jk} = (k_{HB})^{jk} + \alpha\delta^{jk}. \tag{41}$$

By using Bianchi identity $(k_F)_{\kappa[\lambda\mu\nu]} = 0$, we have $\text{tr}(k_{DB}) = 0$. With these definitions, we can rewrite the Lagrangian (1) as

$$\begin{aligned}
\mathcal{L}_{\text{photon}} &= \frac{1}{2}(\vec{E}^2 - \vec{B}^2) + \frac{1}{2}\alpha(\vec{E}^2 + \vec{B}^2) + \frac{1}{2}\left((\beta_E)^{jk}E^jE^k + (\beta_B)^{jk}B^jB^k + (k_{DB})^{jk}E^jB^k\right) \\
&\quad + k_{AF}^0\vec{A}\cdot\vec{B} - \phi\vec{k}_{AF}\cdot\vec{B} + \vec{k}_{AF}\cdot(\vec{A}\times\vec{E}).
\end{aligned} \tag{42}$$

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