Gamma-ray and Cosmic-ray Tests of Lorentz Invariance Violation and Quantum Gravity Models and Their Implications

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Abstract.

The topic of Lorentz invariance violation (LIV) is a fundamental question in physics that has taken on particular interest in theoretical explorations of quantum gravity scenarios. I discuss various γ-ray observations that give limits on predicted potential effects of Lorentz invariance violation. Among these are spectral data from ground based observations of the multi-TeV γ-rays from nearby AGN, *INTEGRAL* detections of polarized soft γ-rays from the vicinity of the Crab pulsar, *Fermi Gamma Ray Space Telescope* studies of photon propagation timing from γ-ray bursts, and *Auger* data on the spectrum of ultrahigh energy cosmic rays. These results can be used to seriously constrain or rule out some models involving Planck scale physics. Possible implications of these limits for quantum gravity and Planck scale physics will be discussed.

INTRODUCTION

It has been the major goal of particle physics to discover a theoretical framework for unifying gravity with the other three known forces, *viz.*, electromagnetism, and the weak and strong nuclear forces. Such a theory must be compatible with quantum theory at very small scales corrsponding to very high energies. Even the possibly less ambitious goal of reconciling general relativity with quantum theory has been elusive and may require new concepts to accomplish.

There has been a particular interest in the possibility that a quantum gravity theories will lead to Lorentz invariance violation (LIV) at the Planck scale, $\lambda_{Pl} = \sqrt{G\hbar/c^3} \sim 1.6 \times 10^{-35}$ m. This scale corresponds to a mass (energy) scale of $M_{Pl} = \hbar/(\lambda_{Pl}c) \sim 1.2 \times 10^{19}$ GeV/c^2. It is at the Planck scale where quantum effects are expected to play a key role in determining the effective nature of space-time that emerges as general relativity in the classical continuum limit. The idea that Lorentz invariance (LI) may indeed be only approximate has been explored within the context of a wide variety of suggested Planck-scale physics scenarios. These include the concepts of deformed relativity, loop quantum gravity, non-commutative geometry, spin foam models, and some string theory (M theory) models. Such theoretical explorations and their possible consequences, such as observable modifications in the energy-momentum dispersion relations for free particles and photons, have been discussed under the general heading of "Planck scale phenomenology". There is an extensive literature on this subject. (See [1] for a review; some recent references are Refs. [2] – [4]. For a non-technical treatment

of the present basic approaches to a quantum gravity theory, see Ref. [5]). One should keep in mind that in a context that is separate from quantum gravity considerations, it is important to test LI for its own sake [6, 7]. *LIV gratia LIV*. The significance of such an approach is evident when one considers the unexpected discoveries of the violation of *P* and *CP* symmetries. In fact, it has been shown that a violation of *CPT* would imply LIV [8]

We will consider here some of the consequent searches for such effects using high energy astrophysics observations, particularly observations of high energy cosmic γ-rays and ultrahigh energy cosmic rays.

LIV PERTURBATIONS

We know that Lorentz invariance has been well validated in particle physics; indeed, it plays an essential role in designing machines such as the new LHC (Large Hadron Collider). Thus, any LIV extant at accelerator energies ("low energies") must be extremely small. This consideration is reflected by adding small Lorentz-violating terms in the free particle Lagrangian. Such terms can be postulated to be independent of quantum gravity theory, *e.g.*, Refs. [6, 7]. Alternatively, it can be assumed that the terms are small because they are suppressed by one or more powers of p/M_{Pl} (with the usual convention that $c = 1$.) In the latter case, in the context of effective field theory (EFT), such terms are assumed to approximate the effects of quantum gravity at "low energies" when $p \ll M_{Pl}$.

One result of such assumptions is a modification of the dispersion relation that relates the energy and momentum of a free particle or photon. This, in turn, can lead to a maximmum attainable velocity (MAV) of a particle different from *c* or a variation of the velocity of a photon *in vacuo* with photon energy. Both effects are clear violations of relativity theory. Such modifications of kinematics can result in changes in threshold energies for particle interactions, suppression of particle interactions and decays, or allowance of particle interactions and decays that are kinematically forbidden by Lorentz invariance [7].

A simple formulation for breaking LI by a small first order perturbation in the electromagnetic Lagrangian which leads to a renormalizable treatment has been given by Coleman and Glashow [7]. The small perturbative noninvariant terms are both rotationally and translationally invariant in a preferred reference frame which one can assume to be the frame in which the cosmic background radiation is isotropic. These terms are also taken to be invariant under $SU(3) \otimes SU(2) \otimes U(1)$ gauge transformations in the standard model.

Using the formalism of Ref. [7], we denote the MAV of a particle of type *i* by c_i, a quantity which is not necessarily equal to $c \equiv 1$, the low energy *in vacua* velocity of light. We further define the difference $c_i - c_j \equiv \delta_{ij}$. These definitions can be generalized and can be used to discuss the physics implications of cosmic-ray and cosmic γ-ray observations [9, 10, 11].

ELECTROWEAK INTERACTIONS

In general then, $c_e \neq c_\gamma$. The physical consequences of such a violation of LI depend on the sign of the difference between these two MAVs. Defining

$$c_e \equiv c_\gamma(1+\delta), \quad 0 < |\delta| \ll 1, \tag{1}$$

one can consider the two cases of positive and negative values of δ separately [7, 9].

Case I: If $c_e < c_\gamma$ ($\delta < 0$), the decay of a photon into an electron-positron pair is kinematically allowed for photons with energies exceeding

$$E_{\text{max}} = m_e \sqrt{2/|\delta|}. \tag{2}$$

The decay would take place rapidly, so that photons with energies exceeding E_{max} could not be observed either in the laboratory or as cosmic rays. From the fact that photons have been observed with energies $E_\gamma \geq 50$ TeV from the Crab nebula, one deduces for this case that $E_{\text{max}} \geq 50$ TeV, or that $-\delta < 2 \times 10^{-16}$.

Case II: For this possibility, where $c_e > c_\gamma$ ($\delta > 0$), electrons become superluminal if their energies exceed $E_{\text{max}}/2$. Electrons traveling faster than light will emit light at all frequencies by a process of 'vacuum Čerenkov radiation.' This process occurs rapidly, so that superluminal electron energies quickly approach $E_{\text{max}}/2$. However, because electrons have been seen in the cosmic radiation with energies up to ~ 2 TeV, it follows that $E_{\text{max}} \geq 2$ TeV, which leads to an upper limit on δ for this case of 3×10^{-14}. Note that this limit is two orders of magnitude weaker than the limit obtained for Case I. However, this limit can be considerably improved by considering constraints obtained from studying the γ-ray spectra of active galaxies [9].

Constraints on LIV from AGN Spectra

A constraint on δ for $\delta > 0$ follows from a change in the threshold energy for the pair production process $\gamma + \gamma \rightarrow e^+ + e^-$. This follows from the fact that the square of the four-momentum is changed to give the threshold condition

$$2\varepsilon E_\gamma(1 - cos\theta) - 2E_\gamma^2\delta \geq 4m_e^2, \tag{3}$$

where ε is the energy of the low energy photon and θ is the angle between the two photons. The second term on the left-hand-side comes from the fact that $c_\gamma = \partial E_\gamma/\partial p_\gamma$. It follows that the condition for a significant increase in the energy threshold for pair production is $E_\gamma\delta/2 \geq m_e^2/E_\gamma$, or equivalently, $\delta \geq 2m_e^2/E_\gamma^2$. The observed γ-ray spectrum of the active galaxies Mkn 501 and Mkn 421 while flaring [12] exhibited the high energy absorption expected from γ-ray annihilation by extragalactic pair-production interactions with extragalactic infrared photons [13, 14]. This led Stecker and Glashow [9] to point out that the Mkn 501 spectrum presents evidence for pair-production with no indication of LIV up to a photon energy of ~ 20 TeV and to thereby place a quantitative constraint on LIV given by $\delta < 2m_e^2/E_\gamma^2 \simeq 10^{-15}$.

GAMMA-RAY CONSTRAINTS ON QUANTUM GRAVITY AND EXTRA DIMENSION MODELS

As previously mentioned, LIV has been proposed to be a consequence of quantum gravity physics at the Planck scale [15, 16]. In models involving large extra dimensions, the energy scale at which gravity becomes strong can occur at a quantum gravity scale, $M_{QG} << M_{Pl}$, even approaching a TeV [17]. In the most commonly considered case, the usual relativistic dispersion relations between energy and momentum of the photon and the electron are modified [16, 18] by a term of order p^3/M_{QG}.

Generalizing the LIV parameter δ from equation (1) to an energy dependent form, we find

$$\delta \equiv \frac{\partial E_e}{\partial p_e} - \frac{\partial E_\gamma}{\partial p_\gamma} \simeq \frac{E_\gamma}{M_{QG}} - \frac{m_e^2}{2E_e^2} - \frac{E_e}{M_{QG}}. \qquad (4)$$

It follows that the threshold condition for pair production given by equation (3) implies that $M_{QG} \geq E_\gamma^3/8m_e^2$. Since pair production occurs for energies of at least 20 TeV, we find a constraint on the quantum gravity scale [10] $M_{QG} \geq 0.3M_{Pl}$. This constraint contradicts the predictions of some proposed quantum gravity models involving large extra dimensions and smaller effective Planck masses. In a variant model of Ref. [19], the photon dispersion relation is changed, but not that of the electrons. In this case, we find the even stronger constraint $M_{QG} \geq 0.6M_{Pl}$.

ENERGY DEPENDENT PHOTON DELAYS FROM GRBS AND TESTS OF LORENTZ INVARIANCE VIOLATION

One possible manifestation of Lorentz invariance violation, from Planck scale physics produced by quantum gravity effects, is a change in the energy-momentum dispersion relation of a free particle or a photon. If this results from the linear Planck-supressed term as in equation (4) above, this results in a photon velocity retardation that is of first order in E_γ/M_{QG} [17, 18]. In a ΛCDM cosmology, where present observational data indicate that $\Omega_\Lambda \simeq 0.7$ and $\Omega_m \simeq 0.3$, the resulting difference in the propagation times of two photons having an energy difference ΔE_γ from a γ-ray burst (GRB) at a redshift z will be

$$\Delta t_{LIV} = H_0^{-1} \frac{\Delta E_\gamma}{M_{QG}} \int_0^z \frac{dz'(1+z')}{\sqrt{\Omega_\Lambda + \Omega_m(1+z')^3}} \qquad (5)$$

for a photon dispersion of the form $c_\gamma = c(1 - E_\gamma/M_{QG})$, with c being the usual low energy velocity of light [20]. In other words, δ, as defined earlier, is given by $-E_\gamma/M_{QG}$.

The *Fermi* Gamma-ray Space Telescope, (see Figure 1), with its γ-ray *Burst Monitors (GBM)* covers an energy range from 8 keV to 40 MeV and its *Large Area Telescope (LAT)* covers an energy range from 20 MeV to > 300 GeV. [1] It can observe and study

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[1] See paper the of Silvia Rainò, these proceedings.

both GRBs and flares from active galactic nuclei over a large range of both energy and distance. This was the case with the GRB 090510, a short burst at a cosmological distance corresponding to a redshift of 0.9 that produced photons with energies extending from the X-ray range to a γ-ray of energy ~ 31 GeV. This burst was therefore a perfect subject for the application of equation (5). *Fermi* observations of GRB090510 have yielded the best constraint on any first order retardation of photon velocity with energy $\Delta t \propto (E/M_{QG})$. This result would require a value of $M_{QG} \gtrsim 1.2 M_{Pl}$ [21][2] In large extra dimension scenarios, one can have effective Planck masses smaller than 1.22×10^{19} GeV, whereas in most QG scenarios, one expects that the minimum size of space-time quanta to be λ_{Pl}. This implies a value for $M_{QG} \lesssim M_{Pl}$ in all cases.

In particular, we note the string theory inspired model of Ref. [2]. This model invisions space-time as a gas of D-particles in a higher dimensional bulk where the observable universe is a D3 brane. The photon is represented as an open string that interacts with the D-particles, resulting a retardation $\propto E_\gamma/M_{QG}$. The new *Fermi* data appear to rule out this model as well as other models that predict such a retardation.

The dispersion effect will be smaller if the dispersion relation has a quadratic dependence on E_γ/M_{QG} as suggested by effective field theory considerations [22, 23]. This will obviate the limits on M_{QG} given above. These considerations also lead to the prediction of vacuum birefringence (see next section).
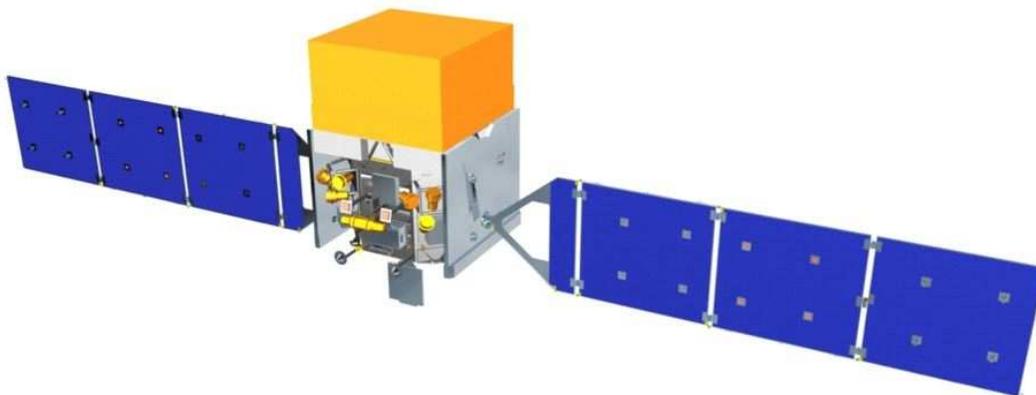

FIGURE 1. Schematic of the *Fermi* satellite, launched in June of 2008. The *LAT* is located at the top (yellow area) and the *GBM* array is located directly below.

LOOKING FOR BIREFRINGENCE EFFECTS FROM QUANTUM GRAVITY

A possible model for quantizing space-time which has been actively investigated is *loop quantum gravity* (see the review given in Ref. [24] and references therein.) A signature of this model is that the quantum nature of space-time can produce a vacuum birefringence effect. (See also the EFT treatment in Ref. [22].) This is because electromagnetic

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[2] See also the paper of Francesco de Palma, these proceedings.

waves of opposite circular polarizations will propagate with different velocities, which leads to a rotation of linear polarization direction through the angle

$$\theta(t) = [\omega_+(k) - \omega_-(k)]t/2 = \xi k^2 t/2M_{Pl} \qquad (6)$$

for a plane wave with wave-vector k [25]. Again, for simple Planck-suppressed LIV, we would expect that $\xi \simeq 1$.

Some astrophysical sources emit highly polarized radiation. It can be seen from equation (6) that the rotation angle is reduced by the large value of the Planck mass. However, the small rotations given by equation (6) can add up over astronomical or cosmological distances to erase the polarization of the source emission. Therefore, if polarization is seen in a distant source, it puts constraints on the parameter ξ. Observations of polarized radiation from distant sources can therefore be used to place an upper bound on ξ.

Equation (6) indicates that the higher the wave number $|k|$, the stronger the rotation effect will be. Thus, the depolarizing effect of space-time induced birefringence will be most pronounced in the γ-ray energy range. It can also be seen that the this effect grows linearly with propoagation time.

The difference in rotation angles for wave-vectors k_1 and k_2 is

$$\Delta\theta = \xi(k_2^2 - k_1^2)d/2M_{Pl}, \qquad (7)$$

replacing the time t by the distance from the source to the detector, denoted by d.

The best secure bound on this effect, $|\xi| \lesssim 10^{-9}$, was obtained using the observed 10% polarized soft γ-ray emission from the region of the Crab Nebula [26].

Clearly, the best tests of birefringence would be to measure the polarization of γ-rays from GRBs. We note that linear polarization in X-ray flares from GRBs has been predicted [27]. Most γ-ray bursts have redshifts in the range 1-2 corresponding to distances of greater than a Gpc. Should polarzation be detected from a burst at distance d, this would place a limit on $|\xi|$ of

$$|\xi| \lesssim 5 \times 10^{-15}/d_{0.5} \qquad (8)$$

where $d_{0.5}$ is the distance to the burst in units of 0.5 Gpc [23]. Detectors that are dedicated to polarization measurements in the X-ray and γ-ray energy range and which can be flown in space to study the polarization from distant astronomical sources are now being designed [28, 29].

LIV AND THE ULTRAHIGH ENERGY COSMIC RAY SPECTRUM

The "GZK Effect"

Shortly after the discovery of the 3K cosmogenic background radiation (CBR), Greisen [30] and Zatsepin and Kuz'min [31] predicted that pion-producing interactions of such cosmic ray protons with the CBR should produce a spectral cutoff at $E \sim$ 50 EeV. The flux of ultrahigh energy cosmic rays (UHECR) is expected to be attenuated by such photomeson producing interactions. This effect is generally known as the "GZK

effect". Owing to this effect, protons with energies above ~ 100 EeV should be attenuated from distances beyond ~ 100 Mpc because they interact with the CBR photons with a resonant photoproduction of pions [32].

Modification of the GZK Effect Owing to LIV

Let us consider the photomeson production process leading to the GZK effect. Near threshold, where single pion production dominates,

$$p + \gamma \rightarrow p + \pi. \tag{9}$$

Using the normal Lorentz invariant kinematics, the energy threshold for photomeson interactions of UHECR protons of initial laboratory energy E with low energy photons of the CBR with laboratory energy ω, is determined by the relativistic invariance of the square of the total four-momentum of the proton-photon system. This relation, together with the threshold inelasticity relation $E_\pi = m/(M+m)E$ for single pion production, yields the threshold conditions for head on collisions in the laboratory frame

$$4\omega E = m(2M + m) \tag{10}$$

for the proton, and

$$4\omega E_\pi = \frac{m^2(2M+m)}{M+m} \tag{11}$$

in terms of the pion energy, where M is the rest mass of the proton and m is the rest mass of the pion [32].

If LI is broken so that $c_\pi > c_p$, the threshold energy for photomeson is altered.[3]

Because of the small LIV perturbation term, the square of the four-momentum is shifted from its LI form so that the threshold condition in terms of the pion energy becomes

$$4\omega E_\pi = \frac{m^2(2M+m)}{M+m} + 2\delta_{\pi p}E_\pi^2 \tag{12}$$

where $\delta_{\pi p} \equiv c_\pi - c_p$, again in units where the low energy velocity of light is unity.

Equation (12) is a quadratic equation with real roots only under the condition

$$\delta_{\pi p} \leq \frac{2\omega^2(M+m)}{m^2(2M+m)} \simeq \omega^2/m^2. \tag{13}$$

Defining $\omega_0 \equiv kT_{CBR} = 2.35 \times 10^{-4}$ eV with $T_{CBR} = 2.725 \pm 0.02$ K, equation (13) can be rewritten

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[3] This requirement precludes the 'quasi-vacuum Čerenkov radiation' of pions, *via* the rapid, strong interaction, pion emission process, $p \rightarrow N + \pi$. This process would be allowed by LIV in the case where $\delta_{\pi p}$ is negative, producing a sharp cutoff in the UHECR proton spectrum. (For more details, see Refs. [7, 11, 33].

$$\delta_{\pi p} \leq 3.23 \times 10^{-24} (\omega/\omega_0)^2. \tag{14}$$

Kinematics

If LIV occurs and $\delta_{\pi p} > 0$, photomeson production can only take place for interactions of CBR photons with energies large enough to satisfy equation (14). This condition, together with equation (12), implies that while photomeson interactions leading to GZK suppression can occur for "lower energy" UHE protons interacting with higher energy CBR photons on the Wien tail of the spectrum, other interactions involving higher energy protons and photons with smaller values of ω will be forbidden. Thus, the observed UHECR spectrum may exhibit the characteristics of GZK suppression near the normal GZK threshold, but the UHECR spectrum can "recover" at higher energies owing to the possibility that photomeson interactions at higher proton energies may be forbidden. We now consider a more detailed quantitative treatment of this possibility, *viz.*, GZK coexisting with LIV.

The kinematical relations governing photomeson interactions are changed in the presence of even a small violation of Lorentz invariance. The modified kinematical relations containing LIV have a strong effect on the amount of energy transfered from a incoming proton to the pion produced in the subsequent interaction, *i.e.*, the inelasticity [11, 34, 35].

The primary effect of LIV on photopion production is a reduction of phase space allowed for the interaction. This results from the limits on the allowed range of interaction angles integrated over in order to obtain the total inelasticity. For real-root solutions for interactions involving higher energy protons, the range of kinematically allowed angles becomes severely restricted. The modified inelasticity that results is the key in determining the effects of LIV on photopion production. The inelasticity rapidly drops for higher incident proton energies.

Figure 2 shows the calculated proton inelasticity modified by LIV for a value of $\delta_{\pi p} = 3 \times 10^{-23}$ as a function of both CBR photon energy and proton energy [35]. Other choices for $\delta_{\pi p}$ yield similar plots. The principal result of changing the value of $\delta_{\pi p}$ is to change the energy at which LIV effects become significant. For a choice of $\delta_{\pi p} = 3 \times 10^{-23}$, there is no observable effect from LIV for E_p less than ~ 200 EeV. Above this energy, the inelasticity precipitously drops as the LIV term in the pion rest energy approaches m_π.

With this modified inelasticity, the proton energy loss rate by photomeson production is given by

$$\frac{1}{E}\frac{dE}{dt} = -\frac{\omega_0 c}{2\pi^2 \gamma^2 \hbar^3 c^3} \int\limits_{\eta}^{\infty} d\varepsilon \ \varepsilon \ \sigma(\varepsilon) K(\varepsilon) \ln[1 - e^{-\varepsilon/2\gamma\omega_0}] \tag{15}$$

where we now use ε to designate the energy of the photon in the cms, η is the photon threshold energy for the interaction in the cms, $K(\varepsilon)$ denotes the inelasticity, and $\sigma(\varepsilon)$
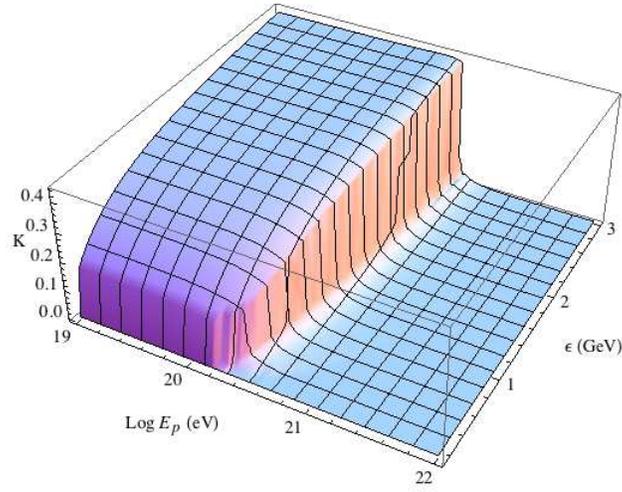
FIGURE 2. The calculated proton inelasticity modified by LIV for $\delta_{\pi p} = 3 \times 10^{-23}$ as a function of CBR photon energy and proton energy [35].

is the total γ-p cross section with contributions from direct pion production, multipion production, and the Δ resonance.

The corresponding proton attenuation length is given by $\ell = cE/r(E)$, where the energy loss rate $r(E) \equiv (dE/dt)$. This attenuation length is plotted in Figure 3 for various values of $\delta_{\pi p}$ along with the unmodified pair production attenuation length from pair production interactions, $p + \gamma_{CBR} \rightarrow e^+ + e^-$.
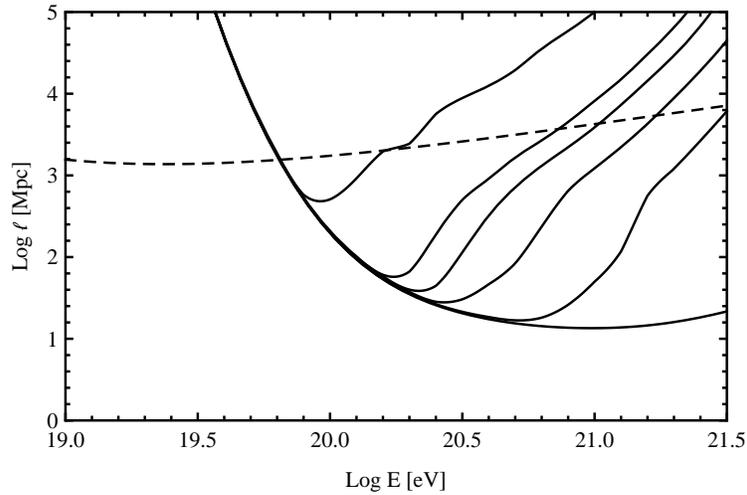

FIGURE 3. The calculated proton attenuation lengths as a function proton energy modified by LIV for various values of $\delta_{\pi p}$ (solid lines), shown with the attenuation length for pair production unmodified by LIV (dashed lines). From top to bottom, the curves are for $\delta_{\pi p} = 1 \times 10^{-22}, 3 \times 10^{-23}, 2 \times 10^{-23}, 1 \times 10^{-23}, 3 \times 10^{-24}, 0$ (no Lorentz violation) [35].

UHECR SPECTRA WITH LIV AND COMPARISON WITH PRESENT OBSERVATIONS

The effect of by a very small amount of LIV on the UHECR spectrum was analytically calculated in Ref. [35] in order to determine the resulting spectral modifications. It can be demonstrated that there is little difference between the results of using an analytic calculation *vs.* a Monte Carlo calculation (*e.g.*, see Ref. [36]). In order to take account of the probable redshift evolution of UHECR production in astronomical sources, they took account of the following considerations:

(*i*) The CBR photon number density increases as $(1+z)^3$ and the CBR photon energies increase linearly with $(1+z)$. The corresponding energy loss for protons at any redshift z is thus given by

$$r_{\gamma p}(E,z) = (1+z)^3 r[(1+z)E]. \tag{16}$$

(*ii*) They assumed that the average UHECR volume emissivity is of the energy and redshift dependent form given by $q(E_i,z) = K(z)E_i^{-\Gamma}$ where E_i is the initial energy of the proton at the source and $\Gamma = 2.55$. For the source evolution, we assume $K(z) \propto (1+z)^{3.6}$ with $z \leq 2.5$ so that $K(z)$ is roughly proportional to the empirically determined z-dependence of the star formation rate. $K(z=0)$ and Γ are normalized fit the data below the GZK energy.

Using these assumptions, one can calculate the effect of LIV on the UHECR spectrum. The results are actually insensitive to the assumed redshift dependence because evolution does not affect the shape of the UHECR spectrum near the GZK cutoff energy [37, 38]. At higher energies where the attenuation length may again become large owing to an LIV effect, the effect of evolution turns out to be less than 10%. The curves calculated in Ref. [11] assuming various values of $\delta_{\pi p}$, are shown in Figure 4 along with the latest *Auger* data from Ref. [39]. They show that *even a very small amount of LIV that is consistent with both a GZK effect and with the present UHECR data can lead to a "recovery" of the UHECR spectrum at higher energies.*

Allowed Range for the LIV Parameter $\delta_{\pi p}$

Stecker and Scully [11] have updated compared the theoretically predicted UHECR spectra with various amounts of LIV to the latest *Auger* data from the procedings of the 2009 International Cosmic Ray Conference [39], [40]. This update is shown in Figure 4. The amount of presently observed GZK suppression in the UHECR data is consistent with the possible existence of a small amount of LIV. The value of $\delta_{\pi p}$ that results in the smallest χ^2 for the modeled UHECR spectral fit using the observational data from *Auger* [39] above the GZK energy. The best fit LIV parameter found was in the range given by $\delta_{\pi p} = 3.0^{+1.5}_{-3.0} \times 10^{-23}$, corresponding to an upper limit on $\delta_{\pi p}$ of 4.5×10^{-23}.
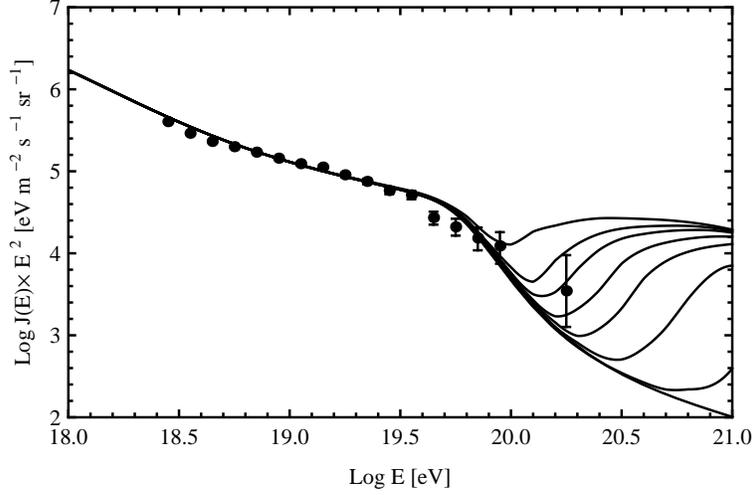
FIGURE 4. Comparison of the latest Auger data with calculated spectra for various values of $\delta_{\pi p}$, taking $\delta_p = 0$ (see text). From top to bottom, the curves give the predicted spectra for $\delta_{\pi p} = 1 \times 10^{-22}, 6 \times 10^{-23}, 4.5 \times 10^{-23}, 3 \times 10^{-23}, 2 \times 10^{-23}, 1 \times 10^{-23}, 3 \times 10^{-24}, 0$ (no Lorentz violation) [11].

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Implications for Quantum Gravity Models

An effective field theory approximation for possible LIV effects induced by Planck-scale suppressed quantum gravity for $E \ll M_{Pl}$ was considered in Ref. [41]. These authors explored the case where a perturbation to the energy-momentum dispersion relation for free particles would be produced by a CPT-even dimension six operator suppressed by a term proportional to M_{Pl}^{-2}. The resulting dispersion relation for a particle of type a is

$$E_a^2 = p_a^2 + m_a^2 + \eta_a \left(\frac{p^4}{M_{Pl}^2} \right) \tag{17}$$

In order to explore the implications of our constraints for quantum gravity, one can take the perturbative terms in the dispersion relations for both protons and pions, to be given by the dimension six dispersion terms in equation (17) above. Making this identification, the LIV constraint of $\delta_{\pi p} < 4.5 \times 10^{-23}$ in the fiducial energy range

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[4] The *HiRes* data [42] do not reach a high enough energy to further restrict LIV.

[5] We note that the overall fit of the data to the theoretically expected spectrum is somewhat imperfect, even below the GZK energy and even for the case of no LIV. It appears that the *Auger* spectrum seems to steepen even below the GZK energy. As a conjecture, one can assume that the derived energy may be too low by about 25%, within the uncertainty of both systematic-plus statistical error given for the energy determination. This gives better agreement between the theoretical curves and the shifted data [11]. The constraint on LIV would be only slightly reduced if this shift is assumed.

around $E_f = 100$ EeV indirectly implies a powerful limit on the representation of quantum gravity effects in an effective field theory formalism with Planck suppressed dimension six operators. Equating the perturbative terms in both the proton and pion dispersion relations, one obtains the relation [11]

$$2\delta_{\pi p} \simeq (\eta_\pi - 25\eta_p)\left(\frac{0.2E_f}{M_{Pl}}\right)^2,\qquad(18)$$

where the pion fiducial energy is taken to to be $\sim 0.2E_f$, as at the Δ resonance that dominates photopion production and the GZK effect [32]. Equation (18), together with the constraint $\delta_{\pi p} < 4.5 \times 10^{-23}$, indicates that any LIV from dimension six operators is suppressed by a factor of at least $\mathcal{O}(10^{-6}M_{Pl}^{-2})$, except in the unlikely case that $\eta_\pi - 25\eta_p \simeq 0$. These results are in agreement with those obtained independently by Maccione et al. from the Monte Carlo runs [41]. It can thus be concluded that an effective field theory representation of quantum gravity with dimension six operators that suppresses LIV by only a factor of M_{Pl}^2 *i.e.* $\eta_p, \eta_\pi \sim 1$, is effectively ruled out by the UHECR observations.

BEYOND CONSTRAINTS: SEEKING LIV

As we have seen (see Figure 4), even a very small amount of LIV that is consistent with both a GZK effect and with the present UHECR data can lead to a "recovery" of the primary UHECR spectrum at higher energies. This is the clearest and the most sensitive evidence of an LIV signature. The "recovery" effect has also been deduced in Refs. [41] and [43] [6]. In order to find it (if it exists) three conditions must exist: (*i*) sensitive enough detectors need to be built, (*ii*) a primary UHECR spectrum that extends to high enough energies (~ 1000 EeV) must exist, and (*iii*) one much be able to distinguish the LIV signature from other possible effects.

Obtaining UHECR Data at Higher Energies

We now turn to examining the various techniques that can be used in the future in order to look for a signal of LIV using UHECR observations. As can be seen from the preceding discussion, observations of higher energy UHECRs with much better statistics than presently obtained are needed in order to search for the effects of miniscule Lorentz invariance violation on the UHECR spectrum.

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[6] In Ref. [43], a recovery effect is also claimed for high proton energies in the case when $\delta_{\pi p} < 0$. However, we have noted that the 'quasi-vacuum Čerenkov radiation' of pions by protons in this case will cut off the proton spectrum and no "recovery" effect will occur.

Auger North

Such an increased number of events may be obtained using much larger ground-based detector arrays. The *Auger* collaboration has proposed to build an *"Auger North"* array that would be seven times larger than the present southern hemisphere Auger array (`http://www.augernorth.org`).

Space Based Detectors

Further into the future, space-based telescopes designed to look downward at large areas of the Earth's atmosphere as a sensitive detector system for giant air-showers caused by trans-GZK cosmic rays. We look forward to these developments that may have important implications for fundamental high energy physics.

Two potential spaced-based missions have been proposed to extend our knowledge of UHECRs to higher energies. One is *JEM-EUSO* (the Extreme Universe Space Observatory) [44], a one-satellite telescope mission proposed to be placed on the Japanese Experiment Module (JEM) on the International Space Station. The other is *OWL* (Orbiting Wide-angle Light Collectors) [45], a two satellite mission for stereo viewing, proposed for a future free-flyer mission. Such orbiting space-based telescopes with UV sensitive cameras will have wide fields-of-view (FOVs) in order to observe and use large volumes of the Earth's atmosphere as a detecting medium. They will thus trace the atmospheric fluorescence trails of numbers of giant air showers produced by ultrahigh energy cosmic rays and neutrinos. Their large FOVs will allow the detection of the rare giant air showers with energies higher than those presently observed by ground-based detectors such as *Auger*. Such missions will thus potentially open up a new window on physics at the highest possible observed energies.

CONCLUSIONS

The *Fermi* timing results for GRB090510 rule out and string-inspired D-brane model predictions as well as other quantum gravity predictions of a retardation of photon velocity that is simply proportional to E/M_{QG} because they would require $M_{QG} > M_{Pl}$. More indirect results from γ-ray birefringence limits, the non-decay of 50 TeV γ-rays from the Crab Nebula, and the TeV spectra of nearby AGNs also place severe limits on violations of special relativity (LIV). Limits on Lorentz invariance violation from observations of ultrahigh energy cosmic-rays provide severe constraints for other quantum gravity models, appearing to rule out retardation that is simply proportional to $(E/M_{QG})^2$. Various effective field theory frameworks lead to such energy dependences.

New theoretical models of Planck scale physics and quantum gravity need to meet all of the present observational constraints. One scenario that may be considered is that gravity, *i.e. G*, becomes weaker at high energies. We know that the strong, weak and electromagnetic interactions all have energy dependences, given by the running of the coupling constants. If G decreases, then the effective $\lambda_{Pl} = \sqrt{G\hbar/c^3}$ would decrease and the effective $M_{Pl} = \hbar/(\lambda_{Pl}c)$ would increase. In that case, the space-time quantum scale

would be less than the usual definition of λ_{Pl}. Such speculation is presently *cogitare ex arcis*, but might be plausible if a transition to a phase where the various forces are unified occurs at very high energies [46].

At the time of the present writing, high energy astrophysics observations have led to strong constraints on LIV. Currently, we have no positive evidence for LIV. This fact, in itself, should help guide theoretical research on quantum gravity, already ruling out some models. Will this lead to a new null result comparable to Michelson-Morley? Will a totally new concept be needed to describe physics at the Planck scale? If all of the known forces are unified at the Planck scale, this would not be surprising. One thing is clear: a consideration of all empirical data will be necessary in order to finally arrive at a true theory of physics at the Planck scale.

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