IL NUOVO CIMENTO VOL. LXIII B, N, 1 11 Settembre 1969

# Gravitational Coupling of Negative Matter (*).

A. INOMATA and D. PEAK

*Department of Physics, State University of New York - Albany, N. Y.*

(ricevuto il 30 Dicembre 1968)

**Summary.** — The relevance of the sign of mass in general relativity is examined by analysing a simple model universe in which Dirac matter distributes uniformly. Mass reversal, converting a source of positive matter into one of negative matter, gives rise to a concomitant change in sign of the gravitational coupling. The principle of equivalence is invoked in order to generalize the result to all negative-matter sources. The admissibility of a Dirac source in general relativity implies that the sign of mass is irrelevant in gravitational interactions.

## 1. – Introduction.

Matter of negative mass—negative matter—may exist in a way inaccessible to our immediate experience. The prime reason for ordinarily ignoring negative mass is its empirical absence, but there is no *a priori* reason for its exclusion from physical theories. The existence of negative mass remains open to question. For the sign of mass to be physically relevant, however, some interaction must exist that discriminates negative matter from normal matter. Such an interaction would indeed disclose asymmetry under mass reversal.

Newtonian mechanics, as BONDI pointed out (1), distinguishes three kinds of mass: inertial mass, active gravitational mass, and passive gravitational mass. The law of inertia (*i.e.* Newton's second law) determines the inertial mass through its reaction to a mass-independent force. The law of gravitation describes the force acting between two gravitational masses, one active,

(*) Supported in part by the United States National Science Fundation. A preliminary account of this work was given in *Bull. Am. Phys. Soc.*, **13**, 662 (1968).

(1) H. BONDI: *Rev. Mod. Phys.*, **29**, 423 (1957).

the source inducing gravitation, and the other passive, the object susceptible to gravitation. These laws are both empirical, abstracted from the phenomena of ordinary massive bodies. Whether or not they would be equally applicable to phenomena involving negative matter is simply untested. It is merely a conjecture that Newton's laws are all extendible by reversing the sign of some of the masses involved.

From this conjectural point of view, a number of ideas have been put forward concerning various extraordinary phenomena ([1-4]). If its inertial mass is reversed, the motion of a body will be in the direction opposite to its momentum. If the gravitational mass alone can assume either sign, then Newton's laws assert that unlike masses repel each other while like masses attract; as FÖPPL first argued ([2]), this possible gravitational repulsion could account naturally for the absence of negative matter in our surroundings. Assuming likewise, SCHÜSTER, in his « holiday dream », attributed to antimatter, which is to him the sink of gravitation, the cause of some surprises in the universe ([3]). Alternatively, if the identity of inertial and gravitational mass, well established for normal matter, is also valid in the case of negative mass, then negative matter causes all matter to gravitate away from it, while positive mass attracts any matter. Under these conditions we may envision the chase of a negative-mass particle after one of positive mass; such a process is of fundamental importance to Hoffmann's proposition for explaining the prodigious energy output of quasars ([4]).

Nothing inherent in the relativistic formulation prejudices the choice of positive over negative inertial mass, for the mass at rest is defined as the magnitude of the energy-momentum four-vector. The most familiar relativistic particles are those in high-energy physics. Notice that experiment, however, determines the rest mass $m_0$ of a high-energy particle only through its rest

---

([2]) A. FÖPPL: *Sitzber. Math. Phys. Kl. Kongl. Bayrisch. Akad. Wiss. München*, **27**, 97 (1890). While FÖPPL developed a logically consistent theory of positive and negative masses, the possibility of stars composed of negative mass was considered by K. PEARSON: *Am. Journ. Math.*, **13**, 309 (1891). For a historical account on negative mass, see M. JAMMER: *Concepts of Mass in Classical and Modern Physics*, Ch. 10 (New York, 1964).

([3]) A. SCHÜSTER: *Nature*, **58**, 367, 618 (1898). Although our attention is focused on the coupling of negative matter, it is important to clarify the relationship between negative matter and antimatter. An interesting discussion on the gravitational properties of antimatter has been given by L. I. SCHIFF: *Proc. Natl. Acad. Sci. U.S.*, **45**, 69 (1959); see also S. WEINBERG: *Phys. Rev.*, **135**, B 1049 (1964); K. HIIDA and Y. YAMAGUCHI: *Progr. Theor. Phys. Suppl.*, Extra Number, 261 (1965).

([4]) B. HOFFMANN: article in *Perspective in Geometry and Relativity* (Indiana, 1966), p. 176. See also Y. P. TERLETSKY: article in *Quasi-Steller Sources and Gravitational Collapse* (Chicago, 1965), p. 466; and *Paradoksy Teorii Otnositel'nosti*, Chap. 6 (Moscow, 1966).

energy, *i.e.* through $\lim_{p\to 0}(p^2+m_0^2)^{\frac{1}{2}}$, where $p$ is the momentum; there seems to be no means to measure the mass itself. This situation might indicate that the sign of mass is experimentally indeterminate; an indeterminacy of this nature would imply the physical irrelevance of the sign of mass, which in turn reveals itself as mass-reversal invariance of the theory ([5]). In fact, it is known that the strong, electromagnetic and weak interactions are symmetrical under mass reversal ([6]).

Confronted, then, with the question of relevance of the sign of inertial mass, one may ask if it is possible to discern the difference between positive and negative gravitational mass. Einstein's theory of general relativity is indeed the relativistic theory of gravitation, in which the equivalence of inertial and gravitational mass is required. Therefore, it would perhaps be more correct to put the question raised above in the following way: Is the sign of mass relevant in general relativity?

In the present paper, we examine the gravitational coupling in general relativity when negative mass serves as a source of gravitation. For this purpose, we adopt a simple model of a universe filled with Dirac matter ([7]). Mass reversal is then performed in order to replace the source of positive matter by a negative source. The model analysis shows that the transformation of positive mass into negative mass is accompanied by a change in sign of the coupling. In other words, the gravitational coupling of a special class of negative Dirac matter must necessarily be opposite in sign to that of normal matter. Appealing to the principle of equivalence may allow one to extend the above result to any source of gravitation. Since negative matter coupled to the gravitational field by a negative coupling is equivalent in effect to normal matter by a positive coupling, it follows that the sign of mass is irrelevant in general relativity ([8]). Throughout this paper, we employ the metric of signature $(+++-)$ and natural units, $c=\hbar=1$.

---

([5]) This experimental indeterminacy may imply more generally the irrelevance of the phase of mass, and the corresponding automorphism is the chiral gauge transformation. See A. INOMATA: *Progr. Theor. Phys.*, **28**, 569 (1962).

([6]) J. TIOMNO: *Nuovo Cimento*, **1**, 226 (1955); S. HORI and A. WAKASA: *Nuovo Cimento*, **6**, 304 (1957); J. J. SAKURAI: *Nuovo Cimento*, **7**, 649 (1958).

([7]) A. INOMATA: *Nuovo Cimento*, **46** B, 132 (1966).

([8]) The theoretical assumptions determining the sign of the gravitational force have been extensively studied by S. DESER and F. A. E. PIRANI: *Ann. of Phys.*, **43**, 436 (1967). It is reported that in general relativity the sign of the coupling is arbitrary unless the gravitational field energy is ensured to be positive-definite. An exceptional case is the interaction between gravitational geons, in that the sign is completely determined. The present model may be considered as belonging to the latter case since it can be looked upon as a geometrization of the massive Dirac field in Rainich-Misner-Wheeler's sense. For geometrization of the massless Dirac field, see A. INOMATA and W. A. MCKINLEY: *Phys. Rev.*, **140**, B 1467 (1965).

## 2. – The premise.

The Einstein equation

$$G_{\mu\nu} = \varkappa T_{\mu\nu} \tag{1}$$

links the geometry of space-time to the stress-energy tensor $T_{\mu\nu}$ of the matter source, with $\varkappa$ the gravitational coupling [9]. Through eq. (1), any type of matter can, in principle, serve as the source of geometrized gravitation. Inasmuch as the quantum version of general relativity is unclear, the source will be limited to classical entities. The standard matter sources are those of perfect fluids and radiation fields. A source not given much attention, but well suited for our consideration, is that provided by the Dirac field [10].

The physical reality of the Dirac field in the *status quo ante*-quantization is somewhat obscure. This obscurity may be ascribed to the classically unfamiliar feature of spin [11]. Nevertheless, it seems reasonable to assume that the Dirac field has its own office in the $c$-number theory, at least, as an approximate description of some physical fabric. We premise, then, that Dirac matter, described by a $c$-number field, is an admissible source to geometry in the Einstein equation (1).

## 3. – The principle of equivalence.

The principle of equivalence, basic to general relativity, works to keep local physics in order. The principle is usually understood as requiring the universality of gravitational coupling. The null result of the Eötvös experiment shows that the passive gravitational mass is equivalent to the inertial mass and strongly supports the universal coupling of normal matter. This requirement, however, is not always compatible with the premise just made. As we shall see, the Dirac matter of negative mass couples to gravitation only with a negative sign. The literal interpretation of the universal coupling leads

---

[9] We ignore the cosmological term from the Machian aspect; see A. INOMATA: *Progr. Theor. Phys.*, **39**, 1071 (1968).

[10] In the view that the Einstein equation (1) is derivable from the Lorentz-invariant field theory, the Dirac source is quite natural; S. GUPTA: *Rev. Mod. Phys.*, **29**, 334 (1957); W. E. THIRRING: *Ann. of Phys.*, **16**, 96 (1961); V. I. OGIEVETSKY and I. V. POLUBARINOV: *Ann. of Phys.*, **25**, 358 (1963); S. WEINBERG: *Phys. Rev.*, **138**, B 990 (1965).

[11] See J. A. WHEELER: *Geometrodynamics* (New York, 1962), in which the problems associated with geometrization of the Dirac field are extensively discussed.

one to reject all source-dependent couplings, including the negative coupling specific to a negative source.

The empirically tenous nature of negative mass serves as a warning to be cautious in accepting the conventional interpretation. To understand the significance of the principle of equivalence for relativity, DICKE makes a distinction between the weak principle of equivalence supported directly by the Eötvös experiment and the strong principle of equivalence that Einstein's theory rests upon (12). The principle in the weak form states that the local gravitational acceleration is substantially independent of the composition and structure of the matter being accelerated. The strong principle requires more severely that all laws of physics be locally reducible to the standard Lorentz invariant forms. He points out that the strong principle of equivalence is also basically supported by the Eötvös experiment except for the question of the invariance of small constants in local physics; the question is left open because of a finite accuracy of the experiment.

How then does each of these principles apply to untested negative matter? The Eötvös experiment does not guarantee the validity of the weak principle for negative mass. Insofar as negative matter is treated within the framework of general relativity, however, the principle of equivalence must be accepted in the strong form. This is after all our fundamental assumption. Therefore, all particles, of normal matter, of negative matter, and presumably of antimatter, are considered to fall down with the same acceleration in the gravitational field induced by a positive source. A question remains whether they fall up or down under the influence of negative matter. Neither the weak nor the strong principle specifies the sign of coupling. The universality of gravitational coupling with a fixed sign is not at all a necessary consequence of the principle. The weak principle, which claims the universal coupling of normal matter, may be regarded as suggesting the universal gravitational attraction.

In view of the dearth of empirical evidences, a more careful interpretation of the principle of equivalence is warranted; namely, that the gravitational coupling is universal to all matter of the same sign. The coupling of negative matter may be of the same sign as, or opposite sign to that of normal matter, though our model analysis yields a result in favor of the universal attraction.

## 4. – The model.

The principle of equivalence enables us to discuss the general character of the gravitational coupling through a particular model. Once the detail

(12) R. H. DICKE: *Experimental Relativity* (New York, 1964), p. 4.

of coupling is known for a particular source, the same should be true, in accordance with the principle, to all other sources.

To form a simple model for this purpose, we consider the Dirac field of mass $m$, which is constrained by

$$\nabla_\mu \psi = -\tfrac{1}{4} m \gamma_\mu \psi \,, \tag{2}$$

where $\nabla_\mu$ is the covariant differential operator ([13]) and the $\gamma$'s are the Dirac matrices satisfying $\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2g_{\mu\nu}$ and $\nabla_\mu \gamma_\nu = 0$. The adjoint field of $\psi$ is defined by $\bar{\psi} = \psi^\dagger \eta$ with the help of a Hermitian matrix $\eta$ such that $\gamma_\mu^\dagger = -\eta\gamma_\mu\eta^{-1}$ and $\nabla_\mu \eta = 0$. The Dirac field of this type has the stress-energy tensor of the form ([7])

$$T_{\mu\nu} = -\varrho g_{\mu\nu} \,, \tag{3}$$

where

$$\varrho = \tfrac{1}{4} m \bar{\psi}\psi \,. \tag{4}$$

From eq. (2) and its adjoint equation it follows that the scalar bilinear $\bar{\psi}\psi$ appearing in eq. (3) is constant in space-time. Thus, eqs. (3) and (4) imply the physical situation that the mass, or energy, of this constrained field distributes uniformly throughout the entire space-time.

When the Dirac matter of this type acts as a source of gravitation, the Einstein equation tells us that the geometry of space-time is characterized by the Einstein tensor

$$G_{\mu\nu} = -3K g_{\mu\nu} \tag{5}$$

with

$$K = \tfrac{1}{3} \varkappa \varrho \,. \tag{6}$$

Note that $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$, $R_{\mu\nu} = R_{\mu\lambda\nu}{}^\lambda$, and $R = R_\mu{}^\mu$, where $R_{\mu\nu\varrho\sigma}$ is the Riemann curvature tensor. Then we find a simple solution of eq. (6),

$$R_{\mu\nu\varrho\sigma} = K(g_{\mu\varrho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\varrho}) \,, \tag{7}$$

which describes a space-time of constant curvature; this we adopt as our model.

---

([13]) The covariant differential operator $\nabla_\mu$ varies in form depending on its operand. Obviously operating eq. (2) with $\gamma^\mu$ yields the Dirac equation (10); the generalized form of the Klein-Gordon operator working on $\psi$ is given by $\Box = (\gamma^\mu \nabla_\mu)^2$.

For the model to be complete, however, the solution of eq. (2) must be ensured to exist everywhere in the space-time (7); that is, the constraint (2) must be completely integrable in the constantly curved geometry (7). The integrability condition to be satisfied is

$$(\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu)\psi = \tfrac{1}{4} R_{\mu\nu\varrho\sigma}\gamma^\varrho \gamma^\sigma \psi \,. \tag{8}$$

Substitution of eqs. (2) and (7) into (8) yields

$$\varkappa\varrho = \tfrac{3}{4} m^2 \,, \tag{9}$$

the condition necessary for the model to be self-consistent.

In brief, our model is a universe of constant curvature filled with the Dirac matter specified by eqs. (2) and (9). This is indeed a de Sitter universe responsible for Mach's principle.

## 5. – The mass reversal.

The Dirac equation is a linearized form of the Klein-Gordon equation. Although the mass symmetry of the latter is apparent, it is not in the case of the former. This does not necessarily imply that Dirac's theory is asymmetric with respect to positive and negative mass. The rise of this awkward situation may be traced back to the fact that the linearization is not unique. There are, in fact, a number of linear equations equivalent (up to invariance under proper Lorentz transformations) to the Dirac equation, all of which are related by a class of similarity transformations [14].

To see the mass symmetry of the Dirac equation

$$(\gamma^\mu \nabla_\mu + m)\psi = 0 \,, \tag{10}$$

defined in curved space-time, let us introduce a matrix $\gamma_5$ by

$$\gamma_5 = \frac{1}{4!}\, e_{\mu\nu\varrho\sigma}\gamma^\mu \gamma^\nu \gamma^\varrho \gamma^\sigma \,, \tag{11}$$

---

[14] V. G. SOLOVIEV: *Nucl. Phys.*, **6**, 618 (1958); S. OZAKI: *Progr. Theor. Phys.*, **23**, 221 (1960).

which anticommutes with $\eta$ and all $\gamma$'s. It also has the properties

$$\gamma_5^{\,2} = 1 , \qquad \nabla_\mu \gamma_5 = 0 . \tag{12}$$

The chirality transformation

$$\psi \to \gamma_5 \psi \tag{13}$$

is a similarity transformation which converts the Dirac equation (10) into

$$(\gamma^\mu \nabla_\mu - m)\psi = 0 . \tag{14}$$

This transformation has the effect equivalent to the mass inversion

$$m \to -m . \tag{15}$$

Naturally the combined transformations (13) and (15), which defines the mass reversal of TIOMNO at each point in space-time [6], leaves the theory invariant. The idea of mass reversal presumes the gaugelike character of the sign of mass. However, if the physical reality of negative mass is to be taken seriously, a question arises as to which equation, (10) or (14), the negative-mass field is to obey.

Suppose the Dirac equation with a negative mass parameter (14) is the basic equation for the negative-mass field while the positive-mass field satisfies the Dirac equation (10) as usual. Then the signs of mass and energy are independent. A field of negative mass, obeying eq. (14), may have both positive and negative energy solutions. Let $\psi(x)$ be a solution of eq. (10). Then $\gamma_5 \psi(x)$ is certainly a solution of negative mass. Since $\gamma_5$ anticommutes with $\eta$, the scalar bilinear $\bar{\psi}\psi$ charges its sign under the chirality transformation. As a result, the mass density $\varrho$, defined by eq. (7), reverses its sign.

Suppose, alternatively, that the positive- and negative-mass fields are simultaneous solutions of the normal Dirac equation (10). Since the local Dirac solutions consist of only those corresponding to four independent spin and energy states, the signs of mass and energy are no longer independent [15]. To find the negative-mass solution, let us associate with the chirality transformation [13] the space-time inversion

$$x \to -x , \tag{16}$$

---

[15] The center of a wave packet formed of negative-energy solutions describes a uniform motion of velocity in the opposite direction to its momentum and behaves like a negative-mass particle; see A. MESSIAH: *Quantum Mechanics*, vol. **2** (Amsterdam, 1962), p. 952.

defined at every point of space-time. As far as the local operational effect is concerned, this space-time inversion (16) and the mass inversion (15) are equivalent. The covariant differential operator $\nabla_\mu$, working on $\psi$, takes the form $\partial_\mu - \Gamma_\mu$, where $\Gamma_\mu$ is the Fock-Ivanenko connection ([16]) having the property $\Gamma_\mu(x) + \Gamma_\mu(-x) = 0$ . The space-time inversion (16) therefore reverses the sign of the differential operator and transform eq. (14) back into eq. (10). It is then apparent that $\gamma_5\psi(-x)$ is a solution of eq. (10) as well. Although the scalar bilinear changes its sign after the chirality transformation, it is by the constraint (2) a constant in space-time and hence an invariant under the space-time inversion (16). Again the mass density $\varrho$ simply reverses its sign.

Clearly, either of the transformations

$$\psi(x) \to \gamma\psi_5(\pm x) \tag{17}$$

causes the conversion of the positive mass source to the negative source

$$\varrho \to -\varrho \tag{18}$$

and nothing else. This shows that if $\psi(x)$ represents a solution corresponding to positive matter then either $\gamma_5\psi(x)$ or $\gamma_5\psi(-x)$ corresponds to negative matter. It is important to note that the presence of such choices for the negative-mass solution is specific to the present model in which the scalar bilinear is a space-time constant. In general, the simultaneous solutions $\psi(x)$ and $\gamma_5\psi(-x)$ can be identified with those locally describing the two states of energy, positive and negative ([17]).

According to the necessary requirement (9) for the self-consistency of the model, the product $\varkappa\varrho$ must be positive definite regardless of the sign of mass. The alternation of the sign of $\varrho$ therefore carries with it the change in sign of $\varkappa$,

$$\varkappa \to -\varkappa\,. \tag{19}$$

In other words, in order for the model to accommodate the negative source, it is necessary to admit the negative gravitational coupling peculiar to the new source.

---

([16]) See, *e.g.*, J. L. ANDERSON: *Principles of Relativity Physics* (New York, 1967), p. 360.

([17]) Under the transformation $\psi(x) \to \gamma_5\psi(-x)$, the stress-energy tensor of the Dirac field free from the constraint (2) transforms as $T_{\mu\nu}(x) \to -T_{\mu\nu}(-x)$. The energy is defined in the vicinity of a local space-time point by $E = \int T_{00}(x)\,\mathrm{d}^3x$. Since this is constant in time, one can carry out the space integral at $t = 0$ without loss of generality. Apparently, the integral remains unchanged under the inversion of all space variables. Thus, in general, the energy evaluated for the transformed field has the opposite sign to that of the original field.

## 6. – Conclusions.

An immediate conclusion that can be drawn from the model analysis presented in the preceding Sections is that the negative source of the Dirac matter constrained by eq. (2) entails the gravitational coupling equal in magnitude but opposite in sign to that of the positive source. The principle of equivalence, as we reinterpret it, requires the universality of the gravitational coupling to all sources of the same sign, thus generalizing the above conclusion as follows: If negative matter exists, then the gravitational coupling of any source of such matter is equal in magnitude but opposite in sign to that of a positive source.

In the model we have employed, mass and energy are synonymous in the presence of the self-induced gravitational field. Consider the case where the negative-mass solution is equivalent to the negative-energy solution. As is well known, the negative-energy solution is indispensable in forming a complete set of Dirac solutions. Thus, excluding the negative-mass solution amounts to abandoning the Dirac source altogether, contrary to our premise. It is important in this context that the principle of equivalence does not rule out the negative coupling of a negative source. Once the mass symmetry is established in general relativity, however, the sign of mass would have to be considered, like the electromagnetic gauge, physically irrelevant, and the reinterpretation of the principle of equivalence will be redundant.

What is the the effect of the negative coupling of negative matter in the Newtonian limit? To see this, let is consider Schwarzschild's solution

$$\mathrm{d}s^2=\left(1-\frac{\varkappa m}{4\pi r}\right)\mathrm{d}t^2-\left(1-\frac{\varkappa m}{4\pi r}\right)^{-1}\mathrm{d}r^2-r^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\varphi^2)\,, \tag{20}$$

where $m$, the active gravitational mass, appears as a product with $\varkappa$, the coupling constant. For $\varkappa>0$ and $m>0$, test particles will, in the first approximation, describe the Newtonian orbits corresponding to an attractive case. The orbits corresponding to a repulsive force occur when $\varkappa>0$ and $m<0$. What our result tells us is that the signs of $\varkappa$ and $m$ are in phase; namely, that the product $\varkappa m$ can never be negative. Consequently, no repulsive solution is possible. This seems to indicate that the phenomenon of a negative mass chasing a positive mass is implausible.

* * *

Helpful conversations with Prof. R. H. DICKE are gratefully acknowledged.

## RIASSUNTO (*)

Si esamina l'importanza del segno della massa in relatività generale analizzando un semplice modello di universo in cui la materia di Dirac si distribuisce uniformemente. L'inversione di massa, convertendo una sorgente di materia positiva in una di materia negativa, dà origine ad una variazione concomitante del segno dell'accoppiamento gravitazionale. Si invoca il principio di equivalenza per generalizzare il risultato a tutte le sorgenti di materia negativa. L'ammissibilità di una sorgente di Dirac nella relatività generale implica che il segno della massa è irrilevante nelle interazioni gravitazionali.

(*) *Traduzione a cura della Redazione.*

## Гравитационная константа связи для отрицательного вещества.

**Резюме** (*). — Исследуется уместность знака массы в общей теории относительности, посредством анализа простой модельной Вселенной, в которой вещество Дирака распределено неоднородно. Изменение знака массы, путем преобразования источника положительного вещества в источник отрицательного вещества, приводит к сопутствующему изменению знака гравитационной константы связи. Используется принцип эквивалентности для того, чтобы обобщить результат для всех отрицательных источников вещества. Приемлемость дираковского источника в общей теории относительности означает, что знак массы является неуместным в гравитационных взаимодействиях.

(*) *Переведено редакцией.*