The OPERA neutrino velocity result and the synchronisation of clocks

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The CERN-OPERA experiment [1] claims to have measured a one-way speed of neutrinos that is apparently faster than the speed of light c. One-way speed measurements such as these inevitably require a convention for the synchronisation of clocks in non-inertial frames since the Earth is rotating. We argue that the effect of the synchronisation convention is not properly taken into account in the analysis of [1] and may well invalidate their interpretation of superluminal neutrino velocity.

The CERN-OPERA experiment attempts to measure the velocity of neutrinos generated at CERN and received at the Laboratorio Nazionale del Gran Sasso (LNGS). The method requires accurate knowledge of the distance baseline traveled by the neutrino along with detailed budgeting of any delays affecting the electronic transmission of timing signals at both ends of the baseline. The overall time of flight is established by time stamping local reference clocks at both sites using a single, *common view* GPS clock signal received separately at both sites. The time difference between time stamped clock signals at either end is calibrated using a second atomic clock transported between the two GPS receivers.

The stated result, after allowing for all budgeted time delays, is that the Time Of Flight (TOF) of the neutrinos is shorter by 60 ± 6.9(stat.) ± 7.4(sys.) ns than that expected if they traveled at the speed of light c. If true, such a claim would have profound consequences for our understanding of fundamental physics.

Various authors have already commented on more or less exotic Lorentz violating theories for the neutrino that may account for the result [2–10]. Here we address the question of whether the stated measurement of the neutrino velocity is self-consistent.

One-way speed of light measurements in non-inertial frames of reference have always suffered from an interpretation problem due to a requirement for synchronising two or more clocks [11]. The problem does not exist in two-way speed of light measurements in which the (light) signal is reflected back to the origin of the apparatus where its arrival is measured with the same clock that measured its departure.

In this *letter* we will focus on the problem of synchronisation of the time signals used to calculate the TOF of the neutrino beam by the OPERA experiment. We will argue that there is sufficient ambiguity in the synchronisation and as such, the result may not be interpreted as done in [1].

The synchronisation of clocks is a fundamental problem in accelerating frames as only inertial observers are equivalent in general relativity. The OPERA experiment *attempts* to get around this problem by time-stamping their time chains using the clock signal of a single GPS satellite. GPS satellite signals have relativistic corrections applied to their tick rate and transmission frequencies. The aim of the corrections is to make the time signal as close as possible to a Universal Time Coordinate (UTC) equivalent to the time coordinate of an inertial frame of reference encompassing the Earth and all GPS satellites. The corrections applied counter the Sagnac effect due to the Earth's rotation, the time dilation due to the position of the satellites in the Earth's gravitational potential and some second order corrections (see [12] for an extended review). The largest of these corrections are of $\mathcal{O}(10^2)$ ns.

Due to both receiver and atmospheric transmission effects it is still not possible to use the GPS signal received at the two ends of the baseline as synchronous UTC markers with the required ns precision. To counter this the OPERA experiment employed a travelling *time-transfer device* (TTD) to calibrate the difference in time signals at each receiver. We assume this device to be a transportable atomic clock of sufficient accuracy [13]. The TTD constitutes a classic moving clock synchronisation conundrum in relativity.

The problem lies in the fact that the TTD was moved between baseline ends in an accelerating frame due to the Earth's rotation. At first order in $1/c^2$ this introduces *three* relativistic time distortions which will induce a discrepancy between the clock at the CERN end of the baseline where the TTD was initially synchronised and the the time shown by the TTD when it arrives at LNGS. The first effect is the time dilation due to moving the TTD through a non-uniform gravitational potential. This effect is exacerbated by the non-inertial frame as it makes it path dependent as opposed to dependent only on the potential difference between the starting and end points. The second is a Doppler type effect due to the velocity v of the TTD with respect to the Earth's rotating frame of reference. The third is the Sagnac effect due to the rotation of the Earth as the TTD travels to its destination. All three effects must be integrated along the path $\mathcal{C}$ taken by the TTD en-route to LNGS. The time discrepancy accumulated by the TTD over the path $\mathcal{C}$ can be stated as [12]

$$\Delta t \;\sim\; -\int_{\mathcal{C}} \frac{(V - V_{\rm CERN})}{c^2}\, d\tau + \frac{1}{2}\int_{\mathcal{C}} \frac{v^2}{c^2}\, d\tau + 2\frac{\omega_E}{c^2}\int_{\mathcal{C}} dA_z \,, \qquad (1)$$

where τ is the proper time along the path, dA_z is the infinitesimal area swept out by the vector connecting the (polar) z-axis of the reference frame to the TTD position along the path and projected onto the equatorial plane, and $\omega_E = 7.2921151467 \times 10^{-5}$ s^{-1} is the rotation rate of the Earth. Of the remaining terms, V is the gravitational potential at a given radius r and latitude θ and $V_{\rm CERN}$ is a reference value of the potential at the starting point. The rotating Earth is not a perfect sphere and a quadrupolar model for the effective gravitational potential of the Earth including a centripetal contribution is given by

$$V = -\frac{GM_E}{r}\left[1 - J_2\left(\frac{r_E}{r}\right)^2 P_2(\cos\theta)\right] + \frac{1}{2}(\omega_E\, r)^2 \quad (2)$$

where G is Newton's constant, M_E is the mass of the Earth with $GM_E = 3.986004418 \times 10^{14}$ m^3 s^{-2}, $r_E = 6.378137 \times 10^6$ m, P_2 is Legendre's polynomial of second order, and $J_2 = 1.0826300 \times 10^{-3}$ is the Earth's quadrupole moment. The radius r can be obtained from the geoid model for the equipotential surface and is described by a polynomial expansion in $x = \sin\theta$ as

$$r \sim 6356742.025 + 21353.642\, x^2 + 39.832\, x^4 \quad \text{m}\,, \quad (3)$$

to fourth order in x. At CERN's approximate latitude we have $V_{\rm CERN}/c^2 = -6.951546823 \times 10^{-10}$ and the difference between potentials at CERN and LNGS is $\Delta V/c^2 = 7.82 \times 10^{-14}$.

Let us now make some simple assumptions about the path $\mathcal{C}$ taken by the TTD on its way from CERN to LNGS; that is that the TTD moved at a constant speed between the two ends of the baseline and the journey took 12 hours. We will assume the journey followed a radius from the origin given by the quadrupolar model for the Earth's shape between the two latitudes.

Of the three effects included in (1) only the time dilation effect acts in such a way that the TTD runs slower than the reference clock at CERN and could therefore explain the anomalously low TOF observed by OPERA if the time stamp at OPERA were calibrated using the TTD. The time dilation estimated along the simple trajectory turns out to be $\Delta t \sim 2$ ns. The Sagnac effect acts in the opposite direction with a value of $\Delta t \sim 4$ ns whilst the Doppler effect is negligible at these speeds. At first glance therefore it appears that the overall Δt by which the TTD is shifted from the CERN clock is of $\mathcal{O}(1)$ ns in the direction *opposite* to what is required to explain the anomaly. However, whilst the Sagnac effect only accumulates as the TTD is moving, the time dilation effect accumulates even when the TTD is stationary and at a value of the potential which differs from that at CERN. As an example of how this can further affect the synchronisation procedure, let us assume that the TTD was stationary at the LNGS site for 4 days while the apparatus for clock comparison was set up. Using the value of $\Delta V/c^2$ quoted above this would result in a total shift of $\Delta t \sim 30$ ns.

We therefore argue that the true time-link discrepancy for the two baseline ends should be a (negative) time dilation effect that can easily be of $\mathcal{O}(10)$ ns plus a (positive) Sagnac effect of $\mathcal{O}(1)$ ns [14]. The quoted time-link discrepancy of 2.3 ns in [1] may well be irrelevant since it is obtained by comparing the asynchronous LNGS reference clock with the TTD which, as we have argued, is no longer synchronised with the CERN clock and is shifted by amounts comparable to those estimated here by the time a comparison is made at LNGS. Such a comparison has no quantitative meaning if the details and timings of the journey that the TTD undertook from CERN up to the clock comparison event at LNGS are not taken into account.

It is worth noting that the time dilation effect is sensitive to many aspects of the journey taken by the TTD. Even disregarding the acceleration phases of such a journey which would move the TTD between distinct non-inertial frames, the effect depends strongly on the velocity of the TTD during the trip and its altitude above the Earth's surface - was it transported by car or aeroplane? Did it sit for hours in a baggage terminal? Did the TTD stop for lunch at a scenic spot in the Alps? etc. It may seem counter-intuitive that such details of a non-relativistic journey could lead to appreciable effects but the time differences the result hinges on are extremely small.

Additional effects that exacerbate the path dependence of the time dilation effect is the deformation of the gravitational potential beyond the simple quadrupole model along the route taken by the TTD. These have not been taken into account here but may be relevant particularly around the mountainous regions along the TTD route.

More importantly, we have only considered the path taken by the TTD along a surface trajectory. The path taken by the neutrinos is some 3 kms below the surface at its midpoint along the trajectory connecting CERN and LNGS. At this level of accuracy the surface time measured by all clocks involved will differ from the proper time along the true trajectory and this further complicates the interpretation of the OPERA results.

In conclusion, a number of effects have not been taken into account when considering the synchronisation convention adopted by the OPERA experiment. The simple estimates given in this work show that the time difference expected after transporting the TTD between baseline ends invalidate its use as a synchronising master clock at the required level of precision. Although [1] does not state explicitly that these effects have not been taken into account, it is hard to imagine how a calculation as elaborate as that required to compute Δt accurately would not have been referred to in their discussion. At the very least, some discussion of the time line between synchronisation at CERN and comparison at LNGS should be included in their analysis. It appears that OPERA has fallen foul of the same stumbling block of past one-way speed of light measurements in non-inertial frames. The resulting measurement that the neutrino velocity differs from c is not only unsurprising but should be *expected* in their setup.

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[12] N. Ashby, Living Reviews in Relativity **6** (2003), URL `http://www.livingreviews.org/lrr-2003-1`.

[13] At the time of writing some of the references in [1] concerning these measurements refer to publications that are not public or lead to recursive http links.

[14] We have adopted the same sign convention as [1] such that a negative correction will alleviate the measured discrepancy.