38. Particle decays

The partial decay rate of a particle of mass \(M\) into \(n\) bodies in its rest frame is given in terms of the Lorentz-invariant matrix element \(\mathcal{M}\) by

\[
d\Gamma = \frac{(2\pi)^{n/2}}{(2M)^n} |\mathcal{M}|^2 d^3p_1 \cdots d^3p_n,
\]

where \(d\Phi_n\) is an element of \(n\)-body phase space given by

\[
d\Phi_n(P; p_1, \ldots, p_n) = \delta^n (P - \sum_{i=1}^n p_i) \prod_{i=1}^n \frac{d^3p_i}{(2\pi)^3 2E_i}.
\]

This phase space can be generated recursively, viz.,

\[
d\Phi_n(P; p_1, \ldots, p_n) = d\Phi_2(p_1, p_2) \times d\Phi_{n-2}(P; p_3, \ldots, p_n),
\]

where \(d\Phi_2(p_1, p_2) = d\Phi(p_1; p_2)\), and the probability that it travels a distance \(x\) or greater is

\[
P(x) = e^{-\lambda x},
\]

where \(\lambda = \frac{1}{\tau}\), and the probability that it travels a distance \(x\) or greater is

\[
P(x) = e^{-\lambda x}.
\]

38.2. Two-body decays:

![Figure 38.1: Definitions of variables for two-body decays.](image)

In the rest frame of a particle of mass \(M\), decaying into 2 particles labeled 1 and 2,

\[
E_1 = M^2 - m_1^2 + m_2^2, \quad E_2 = M^2 - m_2^2 + m_1^2.
\]

For example, if a 0.8 GeV/c kaon beam is incident on a proton target, the center of mass energy is 1.669 GeV and the center of mass momentum of either particle is 0.442 GeV/c. It is also useful to note that

\[
E_{cm} dE_{cm} = m_2 dE_{lab} = m_1 d\beta_1 d\Omega_{lab}.
\]

38.3. Lorentz-invariant amplitudes

The matrix elements for a scattering or decay process are written in terms of an invariant amplitude \(-i\mathcal{M}\). As an example, the \(S\)-matrix for \(2 \rightarrow 2\) scattering is related to \(\mathcal{M}\) by

\[
\langle p'| \phi | p \rangle = i \left( \frac{2\pi}{2M} \right)^{1/2} (2\pi)^{1/2} \delta^3(p - p') \mathcal{M} (p; p').
\]

The state normalization is such that

\[
\langle p'| \phi | p \rangle = \frac{1}{(2\pi)^{1/2}} \delta^3(p - p').
\]
38.4.3. Three-body decays:

\[ \mathbf{P}, M \rightarrow \mathbf{p}_1, m_1 \]
\[ \mathbf{p}_2, m_2 \]
\[ \mathbf{p}_3, m_3 \]

Figure 38.2: Definitions of variables for three-body decays.

Defining \( p_{ij} = p_i + p_j \) and \( m_{ij}^2 = p_{ij}^2 \), then
\[ m_1^2 + m_2^2 + m_3^2 = M^2 + m_2^2 + m_3^2 + m_1^2 \]
\[ = (p_{12} - p_1)^2 + m_2^2 + m_3^2 = 2m_{12}^2, \]
where \( E_3 \) is the energy of particle 3 in the rest frame of \( M \). In that frame, the momenta of the three decay particles lie in a plane. The relative orientation of these three momenta is fixed if their energies are known.

The momenta can therefore be specified in space by giving three Euler angles \((\alpha, \beta, \gamma)\) that specify the orientation of the final system relative to the initial particle \([1]\). Then
\[ d\Omega = \frac{1}{2(2\pi)^3} \int dE_1 dE_2 \sin \beta d\beta \cos \gamma d\gamma . \tag{38.18} \]

Alternatively
\[ d\Gamma = \frac{1}{(2\pi)^3} \int |\mathbf{p}_1|^2 \frac{d\mathbf{p}_1}{2m_{12}^2} dE_1 dF_2, \tag{38.19} \]
where \(|\mathbf{p}_1|^2 \frac{d\mathbf{p}_1}{2m_{12}^2}\) is the momentum of particle 1 in the rest frame of 1 and 2, and \( \Omega_3 \) is the angle of particle 3 in the rest frame of the decaying particle. \([\mathbf{p}_1] \) and \([\mathbf{p}_2] \) are given by
\[ [\mathbf{p}_1] = \left( m_1^2 - (m_1 + m_2)^2 \right) \left( m_1 - m_2 \right) \left( m_1 - m_2 \right) \right]^{1/2} \]
\[ 2m_{12}^2 \]
\[ \left( m_1^2 - (m_1 + m_2)^2 \right) \left( m_1 - m_2 \right) \left( m_1 - m_2 \right) \right]^{1/2} . \tag{38.20a} \]

If the decaying particle is a scalar or we average over its spin states, then integration over the angles in Eq. (38.18) gives
\[ d\Gamma = \frac{1}{2(2\pi)^3} \int |\mathbf{p}_1|^2 dE_1 dF_2 \]
\[ = \frac{1}{2(2\pi)^3} \int |\mathbf{p}_1|^2 dE_1 dF_2. \tag{38.21b} \]

This is the standard form for the Dalitz plot.

38.4.3.1. Dalitz plot: For a given value of \( m_{12}^2 \), the range of \( m_{23}^2 \) is determined by its values when \( p_{12} \) is parallel or antiparallel to \( p_1 \):
\[ (m_{23}^2)_{\text{max}} = \left( \frac{m_1^2 + m_2^2 - m_3^2}{2m_{12}} \right) \]
\[ = \left( \frac{m_1^2 + m_2^2 - m_3^2}{2m_{12}} \right) \]
\[ = \left( \frac{m_1^2 + m_2^2 - m_3^2}{2m_{12}} \right) \]
\[ = \left( \frac{m_1^2 + m_2^2 - m_3^2}{2m_{12}} \right). \]... (38.22a)

Here \( E_2 = (m_2^2 - m_1^2 + m_2^2)2m_{12} \) and \( E_2 = (m_2^2 - m_1^2 + m_2^2)2m_{12} \) are the energies of particles 2 and 3 in the rest frame of 1 and 3.

The scatter plot in \( m_{23}^2 \) and \( m_{23}^2 \) is called a Dalitz plot. If \( |\mathbf{p}_1|^2 \) is constant, the allowed region of the plot will be uniformly populated with events [see Eq. (38.21)]. A nonuniformity in the plot gives immediate information on \( |\mathbf{p}_1|^2 \). For example, in the case of \( D \rightarrow K \pi \), bands appear when \( m_{(K)} = m_{(K^{*(892)})} \), reflecting the appearance of the decay chain \( D \rightarrow K^{*(892)} \to K \pi \).

38.4.4. Kinematic limits: In a three-body decay the maximum of \( |\mathbf{p}_1| \), given by Eq. (38.20), is achieved when \( m_{12} = m_1 + m_2 \), i.e., particles 1 and 2 have the same vector velocity in the rest frame of the decaying particle. If, in addition, \( m_3 > m_1, m_2 \), then \( \left| \mathbf{p}_1 \right| \max > \left| \mathbf{p}_1 \right| \max, \left| \mathbf{p}_2 \right| \max \).

38.4.5. Multibody decays: The above results may be generalized to final states containing any number of particles by combining some of the particles into "effective particles" and treating the final states as 2 or 3 "effective particle" states. Thus, if \( p_{ij...} = p_1 + p_2 + p_3 + ... \), then
\[ m_{ij...} = \sqrt{p_{ij...}^2} \]
and \( m_{ij...} \), may be used in place of \( \mathbf{p}_1 \) in the relations in Sec. 38.4.5 or 38.4.3.1 above.

38.5. Cross sections

The differential cross section is given by
\[ \frac{d\sigma}{d\Omega} = \frac{(2\pi)^4 |\mathbf{p}_1|^2}{4\sqrt{p_{12}^2 - m_{12}^2}} \times d\Phi_{(p_1 + p_2)} \]

(38.23)

[See Eq. (38.11)]. In the rest frame of \( m_{12} \),
\[ \left| \mathbf{p}_{12} \right| = \sqrt{(p_1 + p_2)^2 - m_{12}^2} \]

(38.24)

while in the center-of-mass frame
\[ \left| \mathbf{p}_{12} \right| = \sqrt{(p_1 + p_2)^2 - m_{12}^2} - \gamma \]

(38.25b)
38.5.1. Two-body reactions:

\[ \mathbf{p}_1, m_1 \rightarrow \mathbf{p}_2, m_2 \]
\[ \mathbf{p}_3, m_3 \rightarrow \mathbf{p}_4, m_4 \]

Figure 38.5: Definitions of variables for a two-body final state.

Two particles of momenta \( p_1 \) and \( p_3 \) and masses \( m_1 \) and \( m_3 \) scatter to particles of momenta \( p_2 \) and \( p_4 \) and masses \( m_2 \) and \( m_4 \); the Lorentz-invariant Mandelstam variables are defined by

\[
\begin{align*}
\sigma &= (p_1 + p_3)^2 - (p_2 + p_4)^2, \\
\omega &= 2E_1E_3 - 2p_1p_3, \\
\tau &= (p_1 - p_3)^2 - (p_2 - p_4)^2, \\
\rho &= (p_1 - p_3)^2 + (p_2 - p_4)^2, \\
\end{align*}
\]

(38.26)

and they satisfy

\[
\sigma + \rho + \omega = (m_1^2 + m_2^2 + m_3^2 + m_4^2). 
\]

(38.29)

The two-body cross section may be written as

\[
d\sigma \propto \frac{1}{|\mathbf{p}_{\text{cm}}|^2} \left| \hat{d} \right|^2. 
\]

(38.30)

In the center-of-mass frame

\[
t = (E_{\text{cm}} - E_{\text{lab}})^2 - (p_{\text{cm}} - p_{\text{lab}})^2 = 4p_{\text{cm}}p_{\text{lab}} \sin^2(\theta_{\text{cm}}/2),
\]

(38.31)

where \( \theta_{\text{cm}} \) is the angle between particle 1 and 3. The limiting values \( \theta_{\text{cm}} = 0 \) and \( t_1 = \theta_{\text{cm}} = \pi \) for \( 2 \rightarrow 2 \) scattering are

\[
\theta_{\text{cm}}(t_1) = \frac{m_1^2 - m_2^2 - m_3^2 + m_4^2}{2m_2}, \\
\theta_{\text{cm}}(t_2) = \frac{m_1^2 + m_2^2 - m_3^2 + m_4^2}{2m_2}.
\]

(38.32)

In the literature the notation \( \theta_{\text{lab}} \) (\( \theta_{\text{cm}} \)) for \( \theta_{\text{cm}}(t_1) \) is sometimes used, which should be discouraged since \( \theta_{\text{lab}} > \theta_{\text{cm}} \). The center-of-mass energies and momenta of the incoming particles are

\[
E_{\text{cm}} = \frac{s + m_1^2 - m_2^2}{2m_2}, \\
E_{\text{lab}} = \frac{s + m_2^2 - m_1^2}{2m_2}.
\]

(38.33)

For \( E_{\text{cm}} \) and \( E_{\text{lab}} \), change \( m_1 \) to \( m_3 \) and \( m_2 \) to \( m_4 \). Then

\[
p_{\text{cm}} = \frac{E_{\text{cm}} - E_{\text{lab}}}{2\sqrt{s}} m_1^2, \\
p_{\text{lab}} = \frac{p_{\text{lab}}m_2^2}{2\sqrt{s}}.
\]

(38.34)

Here the subscript lab refers to the frame where particle 2 is at rest. For other relations see Eqs. (38.20)–(38.34).

38.5.2. Inelastic reactions: Choose some direction (usually the beam direction) for the z-axis; then the energy and momentum of a particle can be written as

\[
E = m_\gamma \cosh y, \quad p_z, \quad p_y, \quad p_\gamma = m_\gamma \sinh y,
\]

(38.35)

where \( m_\gamma \) is the transverse mass\n
\[
m_\gamma^2 = m^2 + p_{\text{lab}}^2 + p_\gamma^2,
\]

(38.36)

and the rapidity \( y \) is defined by

\[
y = \frac{1}{2} \ln \left( \frac{E + p_\gamma}{E - p_\gamma} \right).
\]

(38.37)

Under a boost in the z-direction to a frame with velocity \( \beta \), \( y = \tan^{-1} \beta \). Hence the shape of the rapidity distribution \( dN/dy \) is invariant. The invariant cross section may also be rewritten

\[
E^2 \frac{d^2 \sigma}{dp_\gamma dp_\gamma} = \frac{d^2 \sigma}{dp_\gamma dp_\gamma} \rightarrow \frac{d^2 \sigma}{dM d\phi}.
\]

The second form is obtained using the identity \( d\gamma/dp_\gamma = 1/E_1 \), and the third form represents the average over \( \phi \).

Feynman's \( z \) variable is given by

\[
z = \frac{p_\gamma}{p_{\text{max}}} \approx \frac{E_1 + p_\gamma}{E_1 + p_{\text{max}}} (\text{if } \gamma \ll 1).
\]

In the c.m. frame,

\[
z = \frac{p_{\gamma}}{\sqrt{s}} = \frac{2m_\gamma \sinh y}{\sqrt{s}}
\]

(38.39)

and

\[
\gamma_{\text{lab}} = \ln (\sqrt{s}/m). 
\]

(38.40)

For \( p \gg m \), the rapidity \( \gamma_{\text{lab}} \) may be expanded to obtain

\[
y = \frac{1}{2} \ln \frac{\cosh^2(\theta/2) + m^2/\beta^2 + \ldots}{\sinh^2(\theta/2) + m^2/\beta^2 + \ldots} \approx -\ln \tan(\theta/2) \approx \eta
\]

(38.41)

where \( \cos \theta = p_z/p_\gamma \). The pseudorapidity \( \eta \) defined by the second line is approximately equal to the rapidity \( y \) for \( p \gg m \) and \( \theta \gg 1/\beta \), and in any case can be measured when the mass and momentum of the particle is unknown. From the definition one can obtain the identities

\[
\sinh \eta = \alpha \theta, \quad \cosh \eta = 1/\sin \theta, \quad \tanh \eta = -\cos \theta.
\]

(38.42)

38.5.3. Partial waves: The amplitude in the center of mass for elastic scattering of spinless particles may be expanded in Legendre polynomials

\[
f(k, \theta) = \frac{1}{k} \sum_{\ell} (2\ell + 1) P_\ell(\cos \theta),
\]

(38.43)

where \( k \) is the c.m. momentum, \( \theta \) is the c.m. scattering angle, \( a_\ell \) is \( (\eta_k^2/2k - 1)/2 \), \( 0 \leq \eta_k \leq 1 \), and \( \delta_\ell \) is the phase shift of the \( \ell \)-th partial wave. For purely elastic scattering, \( \eta_k = 1 \). The differential cross section is

\[
\frac{d\sigma}{d\theta} = |f(k, \theta)|^2.
\]

(38.45)

The optical theorem states that

\[
\sigma_{\text{opt}} = \frac{4\pi}{k} \text{Im} f(k, 0),
\]

(38.46)

and the cross section in the \( \ell \)-th partial wave is therefore bounded:

\[
a_\ell = \frac{4\pi}{k} |(2\ell + 1)k|^2 \leq \frac{4\pi}{k^2} (2\ell + 1) k^2.
\]

(38.47)

The evolution with energy of a partial-wave amplitude \( a_\ell \) can be displayed in an Argand plot, as shown in Fig. 38.6.
The usual Lorentz-invariant matrix element \( \mathcal{M} \) (see Sec. 36.3 above) for the elastic process is related to \( f(k, 0) \) by

\[
\mathcal{M} = -8\pi \sqrt{f} f(k, 0),
\]

so

\[
s_{\text{tot}} = -\frac{1}{2} s_{\text{inel}} \text{Im} \mathcal{M}(t = 0),
\]

where \( s \) and \( t \) are the center-of-mass energy squared and momentum transfer squared, respectively (see Sec. 38.4.1).

### 38.5.3. Resonances

The Breit-Wigner (nonrelativistic) form for an elastic amplitude \( a_J \) with a resonance at c.m. energy \( E_R \), elastic width \( \Gamma_{el} \), and total width \( \Gamma_{\text{tot}} \) is

\[
a_J = \frac{\Gamma_{el}/2}{E_R - E - \Gamma_{\text{tot}}/2},
\]

where \( E \) is the c.m. energy. As shown in Fig. 38.7, in the absence of background the elastic amplitude traces a counterclockwise circle with center \( i z_{el}/2 \) and radius \( x_{el}/2 \), where the elasticity \( x_{el} = \Gamma_{el}/\Gamma_{\text{tot}} \). The amplitude has a pole at \( E = E_R - \Gamma_{\text{tot}}/2 \).

The spin-averaged Breit-Wigner cross section for a spin-1 resonance produced in the collision of particles of spin \( S_1 \) and \( S_2 \) is

\[
\sigma_{BW}(E) = \frac{(2J + 1)\pi}{(2S_1 + 1)(2S_2 + 1)} \frac{B_{2m}B_{2m}^{1/2}}{k^2(2E - E_R)^2 + \Gamma_{\text{tot}}^2/4},
\]

where \( k \) is the c.m. momentum, \( E \) is the c.m. energy, and \( B_{2m} \) and \( B_{2m} \) are the branching fractions of the resonance into the entrance and exit channels. The \( 2S + 1 \) factors are the multiplicities of the incident spin states, and are replaced by 2 for photons. This expression is valid only for an isolated state. If the width is not small, \( \Gamma_{\text{tot}} \) cannot be treated as a constant independent of \( E \). There are many other forms for \( \sigma_{BW} \), all of which are equivalent to the one given here in the narrow-width case. Some of these forms may be more appropriate if the resonance is broad.

### References

39. CROSS-SECTION FORMULAE FOR SPECIFIC PROCESSES

Revised October 2001 by R.N. Cahn (LBNL).

39.1. Leptoproduction

See section on Structure Functions (Sec. 16 of this Review).

39.2. e⁺e⁻ annihilation

For pointlike, spin-1/2 fermions, the differential cross section in the c.m. for e⁺e⁻ → f f via single photon annihilation is \( \Theta \) is the angle between the incident electron and the produced fermion; \( N_c = 1 \) if \( f \) is a lepton and \( N_c = 3 \) if \( f \) is a quark.

\[
\frac{d\sigma}{d\Omega} = N_c \frac{\alpha^2}{4\pi} |1 + \cos^2 \theta + (1 - |\beta|^2)| |Q_f^2|^{-3/2}.
\]

(39.1)

where \( \beta \) is the velocity of the final state fermion in the c.m. and \( Q_f^2 \) is the charge of the fermion in units of the proton charge. For \( \beta = 1 \),

\[
\sigma = N_c \frac{4\pi \alpha^2}{(2\pi)^3} \beta^2 = N_c \frac{8.8 |Q|^2 m_H}{3}.
\]

(39.2)

where \( s \) is in GeV² units.

At higher energies, \( z \) (mass \( M_Z \) and width \( \Gamma_Z \)) must be included. If the mass of a fermion is much less than the mass of the \( Z^0 \), then the differential cross section for e⁺e⁻ → f f is

\[
\frac{d\sigma}{d\Omega} = N_c \frac{\alpha^2}{4\pi} \left[ \frac{1 + \cos^2 \Theta}{|Q_f^2|^{-3/2}} - 2\chi \nu_f v_f Q_f + \chi_0 (u_f^2 + v_f^2) \right] + 2 \cos \Theta \left[ -2\chi_a \nu_f a_f + 4\chi_a \nu_f \nu_f \right]
\]

(39.3)

where

\[
\chi = \frac{1}{2} \ln \frac{s}{m_f^2 + m_f^2} - \frac{1}{2} \ln \frac{s}{m_f^2 + m_H^2},
\]

\[
\chi_0 = \frac{1}{2} \ln \frac{s}{m_f^2 + m_H^2},
\]

\[
\nu_f = -1 + \frac{1}{2} \frac{m_f^2}{m_H^2 + m_f^2},
\]

\[
\nu_f = -1 + \frac{1}{2} \frac{m_f^2}{m_H^2 + m_f^2},
\]

\[
\alpha_f = 4\pi \frac{m_f}{m_H},
\]

\[
\nu_f = 4\pi \frac{m_f}{m_H},
\]

\[
\nu_f = \frac{4\pi m_f}{m_H},
\]

(39.4)

where \( \Theta_f \) is the angle for \( u, \nu, e \), and neutrinos, while \( \nu_f \) is \(-1/2\) for \( d, s, b \), and negatively charged leptons.

At LEP II it may be possible to produce the orthogonal Higgs boson, \( H \), (see the mini-review on Higgs bosons) in the reaction \( e^+e^- \rightarrow HZ^0 \), which proceeds dominantly through a virtual \( Z^0 \). The Standard Model prediction for the cross section is

\[
\frac{d\sigma}{d\Omega} = \frac{m_f^4}{(2\pi)^3} \frac{2K}{\sqrt{s}} \frac{3M_Z^2}{s} \frac{1}{3} \ln \frac{m_f^2 + m_Z^2}{m_f^2 + m_H^2} \tan^2 \Theta + \sin^2 \Theta,
\]

(39.5)

where \( K \) is the c.m. momentum of the produced \( H \) or \( Z \). Near the production threshold, this formula needs to be corrected for the finite width of the \( Z^0 \).

39.3. Two-photon process at e⁺e⁻ colliders

When an e⁺ and an e⁻ collide with energies \( E_1 \) and \( E_2 \), they emit \( d_1 \) and \( d_2 \) virtual photons with energies \( \omega_1 \) and \( \omega_2 \) and 4-momenta \( q_1 \) and \( q_2 \). In the equivalent photon approximation, the cross section for e⁺e⁻ → e⁺e⁻ is related to the cross section for γγ → X by (Ref. 1)

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4\pi} \left[ 1 - \frac{\omega_1^2}{4E_1^2} - \frac{\omega_2^2}{4E_2^2} + \frac{m^2}{\omega_1^2} \right] \frac{m^2}{\omega_2^2} \frac{d\sigma}{d\Omega},
\]

where \( s = 4E_1E_2, W^2 = 4\omega_1\omega_2 \) and

\[
d_1 = \frac{\omega_1}{E_1} \left( 1 - \frac{\omega_1^2}{4E_1^2} - \frac{m^2}{\omega_1^2} \right)
\]

(39.6)

(39.7)

After integration (note that over \( q_1^2 \) in the region \( m^2/\omega_1^2 \leq q_1^2 \leq m^2/\omega_2^2 \), the cross section is

\[
\frac{\alpha^2}{4\pi} \int \frac{d\omega_1}{\omega_1^2} \frac{d\omega_2}{\omega_2^2} \left[ 1 - \frac{\omega_1^2}{4E_1^2} + \frac{m^2}{\omega_1^2} \right] \frac{d\sigma}{d\Omega},
\]

(39.8)

The quantity \( (m^2/\omega_2^2) \) depends on properties of the produced system \( X \), in particular, \( (m^2/\omega_2^2) \rightarrow m^2 \) for hadron production \( (X = h) \) and \( (m^2/\omega_2^2) \) for lepton pair production \( (X = e^+e^-) \), \( \ell = e, \mu, \tau \).

For production of a resonance of mass \( M_R \) and spin \( J \neq 1 \)

\[
\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}(m_f) \left[ \frac{m_f^2}{M_R^2} \ln \frac{m_f^2}{m_f^2 - 1} + \frac{1}{2} \left( \frac{m_f^2}{M_R^2} \right) \right],
\]

(39.9)

(39.10)

In the case of processes where \( p_\mu \) is large or the mass of the produced particle is large (there large means greater than 10 GeV), the parton model can be used to calculate the rate. Symbolically

\[
\sigma_{\text{hadronic}} = \sum_{ij} \int f_i(\mu, \nu_i) f_j(\mu, \nu_j) dx_i dx_j \left| \delta_{\text{hadronic}}(x_i, x_j) \right|,
\]

(39.11)

where \( f_i(\mu, \nu_i) \) is the parton distribution introduced above and \( Q^2 \) is a typical momentum transfer in the partonic process and \( \nu_i \) is the partonic cross section. Some examples will help to clarify. The production of a W⁺ in pp reactions at rapidity \( y \) in the center-of-mass frame is given by

\[
\frac{d\sigma}{d\nu} = \frac{G_F \nu \sqrt{s}}{3} \left[ 1 - \left( \frac{m^2}{\nu} \right) \frac{\nu}{2(2\pi)^3} \right] \left[ \frac{m^2}{\nu} \frac{d\sigma}{d\nu} \right],
\]

where \( x = \sqrt{s} \nu, \nu_2 = \sqrt{s} \nu, e = \nu, \) and \( \nu = M_W^2/s \). Similarly the production of a jet in pp or pγ collisions is given by

\[
\frac{d^2\sigma}{d\nu dy} = \sum_{ij} \int dx_i \frac{dx_j}{d\nu} f_i(\nu_1, x_1) f_j(\nu_2, x_2)
\]

(39.12)

(39.13)
where the summation is over quarks, gluons, and antiquarks. Here
\[ s = (p_1 + p_2)^2 , \quad (39.14) \]
\[ t = (p_1 - p_{34})^2 , \quad (39.15) \]
\[ u = (p_2 - p_{34})^2 . \quad (39.16) \]
\[ p_1 \text{ and } p_2 \text{ are the momenta of the incoming } p \text{ and } p \text{ for } \mathbf{F} \text{ and } \hat{s}, \hat{t}, \text{ and } \hat{u} \text{ with } p_1 \rightarrow z_1 p_1 \text{ and } p_2 \rightarrow z_2 p_2. \]

The partonic cross section \( \hat{s} \langle d\hat{s}/d\hat{t} \rangle \) can be found in Ref. 2. Example: for the process \( gg \rightarrow Q\overline{Q} \),
\[ \hat{s} \frac{d\hat{s}}{d\hat{t}} = 3\alpha_s^2 \left( \frac{\hat{s}^2 + \hat{t}^2}{8\hat{t}} \right) \left( \frac{4}{9\hat{u}} - \frac{1}{\hat{s}^2} \right). \quad (39.17) \]

The prediction of Eq. (39.13) is compared to data from the UA1 and UA2 collaborations in Figs. 40.1 in the Plots of Cross Sections and Related Quantities section of this Review.

The associated production of a Higgs boson and a gauge boson is analogous to the process \( e^+ e^- \rightarrow H\gamma \) in Sec. 39.2. The required parton-level cross sections \( \sigma \), averaged over initial quark colors, are
\[ \sigma(qq \rightarrow W^+ H) = \frac{m^2 q_j f_j}{36\pi \alpha^2 \alpha_s \alpha_W} \frac{2K}{\sqrt{s}} \frac{K^2 + 3M_W^2}{(s - M_W^2)^2} \]
\[ \sigma(q\overline{q} \rightarrow Z^0 H) = \frac{m^2 (q_j^2 + \overline{q}_j^2)}{144\pi^2 \alpha^2 \alpha_s \alpha_W} \frac{2K}{\sqrt{s}} \frac{K^2 + 3M_W^2}{(s - M_W^2)^2}. \]
Here \( V_{ij} \) is the appropriate element of the Kobayashi-Maskawa matrix and \( K \) is the cm. momentum of the produced \( H \). The axial and vector couplings are defined as in Sec. 39.2.

39.5. One-parton inclusive distributions

In order to describe one-parton inclusive production in \( e^+ e^- \) annihilation or deep inelastic scattering, it is convenient to introduce a fragmentation function \( D_k^h (z, Q^2) \) where \( D_k^h (z, Q^2) \) is the number of hadrons of type \( h \) and momentum between \( zp \) and \( (z + dz)p \) produced in the fragmentation of a parton of type \( i \). The \( Q^2 \) evolution is predicted by QCD and is similar to that of the parton distribution functions [see Section on Quantum Chromodynamics (Sec. 9 of this Review)]. The \( D_k^h (z, Q^2) \) are normalized so that
\[ \sum_h \int zD_k^h (z, Q^2) dz = 1. \quad (39.18) \]

If the contributions of the \( Z \) boson and three-jet events are neglected, the cross section for producing a hadron \( h \) in \( e^+ e^- \) annihilation is given by
\[ \frac{1}{\sigma_{had} dz} \int \frac{d\sigma}{dz} = \sum_i \epsilon_i \sigma_{i,j} (z, Q^2) \frac{1}{\sum_i \epsilon_i \sigma_{i,j} (z, Q^2)}, \quad (39.19) \]
where \( \epsilon_i \) is the charge of quark-type \( i \), \( \sigma_{had} \) is the total hadronic cross section, and the momentum of the hadron is \( zF_{em}/2 \).

In the case of deep inelastic muon scattering, the cross section for producing a hadron of energy \( E_h \) is given by
\[ \frac{1}{\sigma_{had}} \int \frac{d\sigma}{dz} \epsilon_i \sigma_{i,j} (z, Q^2) \frac{1}{\sum_i \epsilon_i \sigma_{i,j} (z, Q^2)}, \quad (39.20) \]
where \( E_h = \nu z \). (For the kinematics of deep inelastic scattering, see Sec. 38.4.2 of the Kinematics section of this Review.) The fragmentation functions for light and heavy quarks have a different \( z \) dependence; the former peak near \( z = 0 \). They are illustrated in Figs. 175a and 175b in the section on "Fragmentation Functions in \( e^+ e^- \) Annihilation" (Sec. 17 of this Review).

References:
1. V. M. Budnev, I. F. Ginsburg, G. V. Melедин, and V. G. Serbo, Phys. Reports 15C, 181 (1975);
2. G. F. Owens, F. Reya, and M. Gluck, Phys. Rev. D18, 1501 (1978);
40. PLOTS OF CROSS SECTIONS AND RELATED QUANTITIES

Jet Production in $pp$ and $\bar{p}p$ Interactions

Figure 40.1: Transverse energy dependence of the inclusive differential jet cross sections in the central pseudorapidity region. The error bars are either statistical (D0), statistical and $p_T$ dependent (UA2), statistical and energy dependent from unsmearing (UA1), uncorrelated (CDF), or total (R806) uncertainties. Comparison of the different experimental results is not straightforward, since the different experiments used different jet reconstruction algorithms. For instance, D0 and CDF used a fixed cone algorithm with a size $R=0.7$ for all their measurements, compared to a cone size of 1.2 for UA2. D0: Phys. Rev. D64, 072003 (2001); CDF: Phys. Rev. D64, 072001 (2001); UA1: Phys. Lett. B172, 461 (1986); UA2: Phys. Lett. B257, 232 (1991); R807: Phys. Lett. B123, 133 (1983). Next-to-Leading order QCD predictions, using CTEQ4HJ pdfs and $E_T = E/2$, are shown for $p_T$ at 630 GeV and 1.8 TeV. (Courtesy of V.D. Elvira, Fermilab, 2001)

Direct $\gamma$ Production in $pp$ Interactions

Differential Cross Section for W and Z Boson Production


Pseudorapidity Distributions in $p\bar{p}$ Interactions

Figure 40.4: Charged particle pseudorapidity distributions in $p\bar{p}$ collisions for 53 GeV ≤ $\sqrt{s}$ ≤ 1800 GeV. UA5 data from the Sps are taken from G.J. Able et al., Z. Phys. C33, 1 (1986), and from the ISR from K. Alpgard et al., Phys. Lett. 112B, 193 (1982). The UA5 data are shown for both the full inelastic cross-section and with singly diffractive events excluded. Additional non single-diffractive measurements are available from CDF at the Tevatron, F. Abe et al., Phys. Rev. D41, 2300 (1990) and Experiment P238 at the Sps, R. Harr et al., Phys. Lett. B401, 176 (1997). (Courtesy of D.R. Ward, Cambridge Univ., 1999.)
Average Hadron Multiplicities in Hadronic $e^+e^-$ Annihilation Events

Table 40.1: Average hadron multiplicities per hadronic $e^+e^-$ annihilation event at $\sqrt{s} = 10$, 29.470, 91, and 130-200 GeV. The rates given include decay products from resonances with $c < 10$ cm, and include the corresponding anti-particle state. Correlations of the systematic uncertainties were considered for the calculation of the averages. (Updated July 2003 by O. Bieberl, LMU, Munich.)

<table>
<thead>
<tr>
<th>Particle</th>
<th>$\sqrt{s} = 10$ GeV</th>
<th>$\sqrt{s} = 29-35$ GeV</th>
<th>$\sqrt{s} = 91$ GeV</th>
<th>$\sqrt{s} = 130-200$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudoscalar mesons:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^0$</td>
<td>66.6 ± 0.2</td>
<td>10.3 ± 0.4</td>
<td>65.9 ± 0.27</td>
<td>21.24 ± 0.39</td>
</tr>
<tr>
<td>$\eta$</td>
<td>32.0 ± 0.3</td>
<td>5.83 ± 0.28</td>
<td>9.42 ± 0.32</td>
<td></td>
</tr>
<tr>
<td>$K^0$</td>
<td>0.90 ± 0.04</td>
<td>1.48 ± 0.09</td>
<td>2.24 ± 0.06</td>
<td>2.81 ± 0.19</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.91 ± 0.05</td>
<td>1.48 ± 0.07</td>
<td>2.09 ± 0.06</td>
<td>2.10 ± 0.12</td>
</tr>
<tr>
<td>$\eta(958)$</td>
<td>0.20 ± 0.04</td>
<td>0.61 ± 0.07</td>
<td>1.09 ± 0.09</td>
<td></td>
</tr>
<tr>
<td>$D^+$</td>
<td>0.16 ± 0.03(b)</td>
<td>0.17 ± 0.03</td>
<td>0.15 ± 0.016</td>
<td></td>
</tr>
<tr>
<td>$D^0$</td>
<td>0.37 ± 0.06(b)</td>
<td>0.45 ± 0.07</td>
<td>0.454 ± 0.030</td>
<td></td>
</tr>
<tr>
<td>$D_{sJ}(980)^0$</td>
<td>0.13 ± 0.02(b)</td>
<td>0.45 ± 0.20(s)</td>
<td>0.311 ± 0.021</td>
<td></td>
</tr>
<tr>
<td>$B_{sJ}^0, B_{sJ}^{-}$</td>
<td>—</td>
<td>0.165 ± 0.026(b)</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Scalar mesons:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_0(980)$</td>
<td>0.024 ± 0.006</td>
<td>0.05 ± 0.02(c)</td>
<td>0.146 ± 0.012</td>
<td>0.27 ± 0.11(d)</td>
</tr>
<tr>
<td>$a_0(880)^+$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Vector mesons:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho(770)^0$</td>
<td>0.35 ± 0.04</td>
<td>0.81 ± 0.08</td>
<td>1.23 ± 0.098</td>
<td></td>
</tr>
<tr>
<td>$\omega(782)$</td>
<td>0.30 ± 0.08</td>
<td>—</td>
<td>1.016 ± 0.065</td>
<td></td>
</tr>
<tr>
<td>$K^0(892)^+</td>
<td>0.27 ± 0.03</td>
<td>0.64 ± 0.05</td>
<td>0.715 ± 0.059</td>
<td></td>
</tr>
<tr>
<td>$K^0(892)^0$</td>
<td>0.29 ± 0.03</td>
<td>0.56 ± 0.06</td>
<td>0.738 ± 0.024</td>
<td></td>
</tr>
<tr>
<td>$\phi(1020)$</td>
<td>0.044 ± 0.003</td>
<td>0.085 ± 0.011</td>
<td>0.0993 ± 0.0032</td>
<td></td>
</tr>
<tr>
<td>$D^+(2110)^+$</td>
<td>0.22 ± 0.04(b)</td>
<td>0.43 ± 0.07</td>
<td>0.387 ± 0.0057(b)</td>
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</tr>
<tr>
<td>$D^0(2010)^0$</td>
<td>0.23 ± 0.06(b)</td>
<td>0.27 ± 0.11</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>$B^+(b)$</td>
<td>—</td>
<td>—</td>
<td>0.101 ± 0.041(b)</td>
<td></td>
</tr>
<tr>
<td>$J/P(1S)$</td>
<td>0.00100 ± 0.00005(b)</td>
<td>—</td>
<td>0.0032 ± 0.0004(b)</td>
<td></td>
</tr>
<tr>
<td>$\psi(2S)$</td>
<td>—</td>
<td>—</td>
<td>0.0023 ± 0.0004(b)</td>
<td></td>
</tr>
<tr>
<td>$T(1S)$</td>
<td>—</td>
<td>—</td>
<td>0.00014 ± 0.00007(b)</td>
<td></td>
</tr>
<tr>
<td>Pseudovector mesons:</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$h_{12}(320)$</td>
<td>—</td>
<td>—</td>
<td>0.0041 ± 0.0011(b)</td>
<td></td>
</tr>
<tr>
<td>Tensor mesons:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_2(1270)^+$</td>
<td>0.09 ± 0.02</td>
<td>0.14 ± 0.04</td>
<td>0.166 ± 0.030</td>
<td></td>
</tr>
<tr>
<td>$f_2^+(1525)$</td>
<td>—</td>
<td>—</td>
<td>0.012 ± 0.006</td>
<td></td>
</tr>
<tr>
<td>$K_{1J}^+(1430)^+$</td>
<td>—</td>
<td>0.09 ± 0.03</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>$K_{1J}^0(1430)^0$</td>
<td>—</td>
<td>0.12 ± 0.06</td>
<td>0.084 ± 0.022(b)</td>
<td></td>
</tr>
<tr>
<td>$B^{++}(b)$</td>
<td>—</td>
<td>—</td>
<td>0.118 ± 0.023</td>
<td></td>
</tr>
<tr>
<td>$D_{sJ}^{(*)}(b)$</td>
<td>—</td>
<td>—</td>
<td>0.0052 ± 0.0011(b)</td>
<td></td>
</tr>
<tr>
<td>$D_{sJ}^{(*)}(b)$</td>
<td>—</td>
<td>—</td>
<td>0.0035 ± 0.0031(b)</td>
<td></td>
</tr>
<tr>
<td>Baryons:</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>0.233 ± 0.016</td>
<td>0.640 ± 0.050</td>
<td>1.048 ± 0.045</td>
<td>1.41 ± 0.18</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>0.080 ± 0.007</td>
<td>0.205 ± 0.010</td>
<td>0.391 ± 0.006</td>
<td>0.39 ± 0.03</td>
</tr>
<tr>
<td>$\Sigma^0$</td>
<td>0.023 ± 0.008</td>
<td>—</td>
<td>0.076 ± 0.011</td>
<td></td>
</tr>
<tr>
<td>$\Sigma^+$</td>
<td>—</td>
<td>—</td>
<td>0.094 ± 0.010</td>
<td></td>
</tr>
<tr>
<td>$\Sigma^-$</td>
<td>—</td>
<td>—</td>
<td>0.107 ± 0.011</td>
<td></td>
</tr>
<tr>
<td>$\Xi^-^*$</td>
<td>0.0029 ± 0.0007</td>
<td>0.0176 ± 0.0027</td>
<td>0.0228 ± 0.0010</td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1232)^+^*$</td>
<td>0.040 ± 0.010</td>
<td>—</td>
<td>0.086 ± 0.014</td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1355)^*$</td>
<td>0.000 ± 0.002</td>
<td>0.014 ± 0.004</td>
<td>0.0230 ± 0.0017</td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1385)^+$</td>
<td>0.005 ± 0.003</td>
<td>0.017 ± 0.004</td>
<td>0.0299 ± 0.0015</td>
<td></td>
</tr>
<tr>
<td>$\Xi(1530)^0$</td>
<td>0.0010 ± 0.0020</td>
<td>0.033 ± 0.0038</td>
<td>0.092 ± 0.0028</td>
<td></td>
</tr>
<tr>
<td>$\Omega(1550)^-$</td>
<td>0.0015 ± 0.0006</td>
<td>—</td>
<td>0.0055 ± 0.0005</td>
<td></td>
</tr>
<tr>
<td>$\Xi$</td>
<td>0.0007 ± 0.0004</td>
<td>0.014 ± 0.007</td>
<td>0.0016 ± 0.0003</td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1520)$</td>
<td>0.300 ± 0.030(b)</td>
<td>0.110 ± 0.050</td>
<td>0.078 ± 0.017</td>
<td></td>
</tr>
<tr>
<td>$\Sigma^0 + \Sigma^-$</td>
<td>0.014 ± 0.007</td>
<td>—</td>
<td>0.0031 ± 0.0016</td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1520)$</td>
<td>0.008 ± 0.002</td>
<td>—</td>
<td>0.0222 ± 0.0027</td>
<td></td>
</tr>
</tbody>
</table>
40. Plots of cross sections and related quantities

Notes for Table 40.1:

(a) $B(D_s \to \eta \pi^\pm)\gamma$ was used (RPP1994).
(b) The Standard Model $B(Z \to \bar{B}\bar{B}) = 0.217$ was used.
(c) $\sigma_p = p/e_{\text{beam}} > 0.1$ only.
(d) Both charge states.
(e) $B(D^\ast (3030)^\pm \to D^\pm \pi^\mp) \times B(D^\pm \to K^{\ast \pm})$ has been used (RPP2000).
(f) $B(D_s^+ \to D^+_s\pi^0), B(D_s^+ \to \phi \pi^+)\gamma$ have been used (RPP1998).
(g) Any charge state (i.e., $B_{q}^+, B_{q}^-, B_{q}^{*+}$).
(h) $B(Z \to \text{hadrons}) = 0.699$ was used (RPP1994).
(i) Any charge state (i.e., $B_{q}^+, B_{q}^-, B_{q}^{*+}$).

(j) The value was derived from the cross section of $A_j^\pm \to \eta K$, assuming the branching fraction to be $(32 \pm 0.7)\%$ (RPP1992).

(k) $\sigma_{\text{had}} = 3.33 \pm 0.05 \pm 0.21$ nb (CLEO: Phys. Rev. D49, 1254 (1994)) has been used in converting the measured cross sections to average hadron multiplicities.

(l) Assumes $B(D_s^{(*)+} \to D^0 K^0 + D^{(*)0} K^+) = 100\%$ and $B(D_s^{(*)+} \to D^{(*)0} K^+)$ = 45%.

References for Table 40.1:


Average $e^+e^-$, $pp$, and $p\bar{p}$ Multiplicity

Figure 40.5: Average multiplicity as a function of $\sqrt{s}$ for $e^+e^-$ and $p\bar{p}$ annihilations, and $pp$ and $ep$ collisions. The indicated errors are statistical and systematic errors added in quadrature, except when no systematic errors are given. Files of the data shown in this figure are given in http://home.cern.ch/b/biebel/www/4402

$e^+e^-$: Most $e^+e^-$ measurements include contributions from $K_S^0$ and $A$ decays. The $\gamma\gamma$ and MARK 1 measurements contain a systematic 5% error. Points at identical energies have been spread horizontally for clarity.


$e^+p$: Multiplicities have been measured in the current fragmentation region of the Breit frame.


$p\bar{p}$: The errors of the $p\bar{p}$ measurements are the quadratically added statistical and systematic errors, except for the bubble chamber measurements for which only statistical errors are given in the references. The values measured by UA5 exclude single diffractive dissociation.


(Courtesy of O. Biebel, MPI München, 2001.)
Figure 40.6: World data on the total cross section of $e^+e^- \rightarrow \text{hadrons}$ and the ratio $R(s) = \sigma(e^+e^- \rightarrow \text{hadrons},s) / \sigma(e^+e^- \rightarrow \mu^+\mu^-,s)$. $\sigma(e^+e^- \rightarrow \text{hadrons},s)$ is the experimental cross section corrected for initial state radiation and electron/positron vertex loops, $\sigma(e^+e^- \rightarrow \mu^+\mu^-,s) = 2m_e^2(s)\delta\lambda$. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are an educative guide: the broken one is a naive quark-parton model prediction and the solid one is 3-loop pQCD prediction (see “Quantum Chromodynamics” section of this Review, Eq. (9.12) or, for more details, A. G. Chetyrkin et al., hep-ph/0005309, p.3, Eqs. (1)-(3)). Breit-Wigner parameterizations of $J/\psi, \psi(2S)$, and $\Upsilon(nS), n = 1, 2$ are also shown. Note: The experimental shapes of these resonances are dominated by machine energy spread and are not shown. The full list of references to the original data and the details of the $R$ ratio extraction from them can be found in hep-ph/0312114.

Corresponding computer-readable data files are available at http://pdg.ihep.su/xsect/contents.html. (Courtesy of the COMPAS (Protvino) and HEPDATA (Durham) Groups, March 2004. Corrections by P. Janot (CERN) and M. Schmitt (Northwestern U.))
$R$ in Light-Flavor, Charm, and Beauty Threshold Regions

Figure 40.7: $R$ in the light-flavor, charm, and beauty threshold regions. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are the same as in Fig. 40.6. Note: CLEO data above $\Upsilon(4S)$ were not fully corrected for radiative effects, and we retain them on the plot only for illustrative purposes with a normalization factor of 0.8. The full list of references to the original data and the details of the $R$ ratio extraction from them can be found in hep-ph/0312114. The computer-readable data are available at http://pdg.lbl.gov/sect/contents.html. (Courtesy of the COMPAS (Protvino) and HEPDATA (Durham) Groups, March 2004.)
Annihilation Cross Section Near $M_Z$

Figure 40.8: Combined data from the ALEPH, DELPHI, L3, and OPAL Collaborations for the cross section in $e^+e^-$ annihilation into hadronic final states as a function of the center-of-mass energy near the Z pole. The curves show the predictions of the Standard Model with two, three, and four species of light neutrinos. The asymmetry of the curve is produced by initial-state radiation. Note that the error bars have been increased by a factor ten for display purposes. References:


**Combination**: The Four LEP Collaborations (ALEPH, DELPHI, L3, OPAL) and the Lineshape Subgroup of the LEP Electroweak Working Group, hep-ph/0301127.

(Courtesy of M. Grünwald and the LEP Electroweak Working Group, 2003)
Muon Neutrino and Anti-Neutrino Charged-Current Total Cross Section

Figure 40.9: $\sigma_T/E_\nu$ for the muon neutrino and anti-neutrino charged-current total cross section as a function of neutrino energy. The error bars include both statistical and systematic errors. The straight lines are the averaged values over all energies as measured by the experiments in Refs. [1-4]: $0.67 \pm 0.04 \times 10^{-38} \text{ cm}^2/\text{GeV}$. Note the change in the energy scale at 30 GeV. (Courtesy W. Seligman and M.H. Shaevitz, Columbia University, 2001.)

[9] V.B. Anikeev et al., Z. Phys. C70, 39 (1996);
Table 40.2: Total hadronic cross section. Analytic S-matrix and Regge theory suggest a variety of parameterizations of total cross sections at high energies with different applications and fit quality.

A ranking procedure, based on measures of different aspects of the quality of the fits to the current evaluated experimental database, allows one to single out the following parameterization of highest rank:

\[ \sigma^{ab} = Z^{ab} + B \log^2(s/\eta_1) + Y_1^{ab}(s_1/s)^\eta_1 - Y_2^{ab}(s_1/s)^\eta_2, \]

\[ \sigma^{ab} = Z^{ab} + B \log^2(s/\eta_1) + Y_1^{ab}(s_1/s)^\eta_1 + Y_2^{ab}(s_1/s)^\eta_2, \]

where \( Z^{ab}, B, Y^{ab}_{1,2} \) are in mb, \( s, s_1 \), and \( \eta_1, \eta_2 \) are in GeV\(^2\). The scales \( s_0, s_1 \), the rate of universal rise of the cross sections \( B \), and exponents \( \eta_1 \) and \( \eta_2 \) are independent of the colliding particles. The scale \( s_1 \) is fixed at 1 GeV\(^2\). Terms \( Z^{ab} + B \log^2(s/\eta_1) \) represent the pomeron. The exponents \( \eta_1 \) and \( \eta_2 \) represent low-lying C-even and C-odd exchanges, respectively. Requiring \( \eta_1 = \eta_2 \) results in somewhat poorer fits. In addition to total cross sections \( \sigma \), the measured ratios of the real-to-imaginary parts of the forward scattering amplitudes \( \rho = Re(T)/Im(T) \) were included in the fits by using \( s \) to a crossing symmetry and differential dispersion relations. Global fits were made to the 2003-updated data for \( \pi^+\pi^- \), \( K^\pm p \), \( \eta \), and \( \gamma \gamma \) collisions. Exact factorisation hypothesis was used for both \( Z^{ab} \) and \( \log^2(s/\eta_1) \) to extend the universal rise of the total hadronic cross sections to the \( \gamma \gamma \rightarrow \text{hadrons} \) and \( \gamma \gamma \rightarrow \text{hadrons} \) collisions. This resulted in reducing the number of adjusted parameters from 21 used for the 2002 edition to 19, and in the higher quality rank of the parameterization. The asymptotic parameters thus obtained were then fixed and used as inputs to a fit to a larger data sample that included cross sections on deuterons \( (d) \) and neutrons \( (n) \). All fits produced to data above \( \sqrt{s_{\text{min}}} = 5 \) GeV.

<table>
<thead>
<tr>
<th>Fits to ( \pi(p)p, \pi^0 p, K^\pm p, \gamma \gamma )</th>
<th>Beam/Target</th>
<th>Fits to groups</th>
<th>( \chi^2/\text{dof} ) by groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z )</td>
<td>( Y_1 )</td>
<td>( Y_2 )</td>
<td>( \rho )</td>
</tr>
<tr>
<td>35.45(48)</td>
<td>42.53(1.35)</td>
<td>33.34(1.04)</td>
<td>0.03(17)</td>
</tr>
<tr>
<td>35.20(1.46)</td>
<td>42.53(1.35)</td>
<td>33.34(1.04)</td>
<td>0.03(17)</td>
</tr>
<tr>
<td>20.86(40)</td>
<td>19.24(1.22)</td>
<td>6.03(19)</td>
<td>0.03(17)</td>
</tr>
<tr>
<td>17.91(36)</td>
<td>7.1(15)</td>
<td>13.45(40)</td>
<td>0.03(17)</td>
</tr>
</tbody>
</table>

The fitted functions are shown in the following figures, along with one-standard-deviation error bands. When the reduced \( \chi^2 \) is greater than one, a scale factor has been included to evaluate the parameter values, and to draw the error bands. Where appropriate, statistical and systematic errors were combined quadratically in constructing weights for all fits. On the plots, only statistical error bars are shown. Vertical arrows indicate lower limits on the \( \rho_{\text{min}} \) of \( \eta_{\text{min}} \) range used in the fits.

One can find the details of the global fits (all data on proton target and \( \gamma \gamma \) fitted simultaneously) and ranking procedure, as well as the exact parameterizations of the total cross sections, and corresponding ratios of the real to imaginary parts of the forward-scattering amplitudes in the recent paper of COMPETE Collab. [1]. Database used in the fits now includes the recent OPAL and L3 (LEP) \( \gamma \gamma \) data, highest energy data for \( \pi^0 \) and \( \pi^0 \) from SELEX (FNAL) experiment, \( \gamma \gamma \) from ZEUS (DESY), cosmic ray \( p \) data from the Fly's Eye and AKEGO (Agano), and \( \gamma \gamma \) data from Baksan experiments. The numerical experimental data were extracted from the PPDF accessible at http://wpppsdhep.m2.np1993/pdfs.html. Computer-readable data files are also available at http://pdg.lbl.gov. (Courtesy of the COMPAS group, IIHE, Protvino, August 2003.) On-line "Predictor" to calculate \( \sigma \) and \( \rho \) for any energy from five high rank models is also available at http://nucita02.phys.cas.cee/compete/predictor.html/.

References:
Figure 40.10: Summary of hadronic, $\gamma p$, and $\gamma \gamma$ total cross sections, and ratio of the real to imaginary parts of the forward hadronic amplitudes. Corresponding computer-readable data files may be found at http://pdg.lbl.gov/xsect/contents.html. (Courtesy of the COMPAS group, IHEP, Protvino, August 2003.)
Figure 40.11: Total and elastic cross sections for $pp$ and $\bar{p}p$ collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at http://pdg.lbl.gov/sect/contents.html. (Courtesy of the COMPAS group, IHEP, Protvino, August 2003.)
Figure 40.12: Total and elastic cross sections for pd (total only), np, pd (total only), and pn collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at http://pdg.lbl.gov/xsect/contents.html. (Courtesy of the COMPAS Group, IIHEP, Protvino, August 2003.)
Figure 40.13: Total and elastic cross sections for $\pi^+p$ and $\pi^+d$ (total only) collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at http://pdg.lbl.gov/xsect/contents.html. (Courtesy of the COMPAS Group, IHEP, Protvino, August 2003.)
Figure 40.14: Total and elastic cross sections for $K^{-}p$ and $K^{-}d$ (total only), and $K^{-}n$ collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at http://pdg.lbl.gov/xsect/contents.html. (Courtesy of the COMPAS Group, IHEP, Protvino, August 2003.)
Figure 4.0.15: Total and elastic cross sections for $K^+p$ and total cross sections for $K^+d$ and $K^+n$ collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at http://pdg.lbl.gov/bsect/contents.html. (Courtesy of the COMPAS Group, IHEP, Protvino, August 2003.)
Figure 4.16: Total and elastic cross sections for $\Lambda p$, total cross section for $\Sigma^-p$, and total hadronic cross sections for $\gamma p$, $\gamma d$, and $\gamma\gamma$ collisions as a function of laboratory beam momentum and the total center-of-mass energy. Corresponding computer-readable data files may be found at [http://pdg.lbl.gov/sect/contents.html](http://pdg.lbl.gov/sect/contents.html). (Courtesy of the COMPAS group, IHEP, Protvino, August 2003.)