

Origin and growth of primordial black holes

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In a previous paper we have argued that primordial black holes can arise from the formation and subsequent gravitational collapse of bound states of stable supermassive elementary particles (gravitinos) during the early radiation era. Here we offer a comprehensive picture, describing the evolution and growth of the resulting mini-black holes through *both* the radiation and matter dominated phases until the onset of inhomogeneities, by means of an exact metric solving Einstein's equations. We show that, thanks to a special enhancement effect producing an effective horizon above the actual event horizon, this process can explain the observed mass values of the earliest giant black holes.

In a previous paper [1] we have proposed a new mechanism to explain the origin of supermassive black holes in the early Universe by means of the condensation of superheavy elementary particles during the early radiation phase. Accordingly, the existence of primordial black holes would be due to the gravitational collapse of such bound states, shortly after their formation, to small black holes, whose masses must lie above a critical value to avoid Hawking evaporation. The subsequent growth of these black holes during the radiation era is then modeled by the exact solution of Einstein's equation derived in [1], such that towards the end of the radiation era the emerging macroscopic black holes can grow to nearly solar mass objects.

As we have explained in [1], superheavy gravitinos can serve as microscopic seeds for generating mini-black holes if their mass is sufficiently large so that their gravitational attraction exceeds the repulsive or attractive electric forces between them. Furthermore, these seed particles must be stable against decay into Standard Model matter. Although other kinds of particles with similar properties might serve the same purpose, we have argued in [1] that gravitinos are distinguished in view of a possible unification of the fundamental interactions. This follows from the structure of the fermionic sector of the $N = 8$ supermultiplet [2] (however, as we have explained there, the underlying theory must transcend maximal $N = 8$ supergravity). Namely, identifying the 48 non-Goldstino spin- $\frac{1}{2}$ fermions of the $N = 8$ supermultiplet with three generations of quarks and leptons (including right-chiral neutrinos) of the Standard Model, one is left with eight massive gravitinos with the properties described in [1]. These properties are radically different from those of the more familiar sterile gravitinos of low energy $N = 1$ supergravity models; in particular, unlike the latter, superheavy gravitinos *do* participate in Standard Model interactions.

In this paper we discuss the further evolution of these black holes during the matter dominated phase, and show that the proposed mechanism can indeed explain the observed mass values of supermassive black holes, as reported in [3]. Namely, in [1] we did not follow the evo-

lution of the emergent macroscopic black holes beyond equilibrium time t_{eq} into the matter dominated era, nor did we provide mass estimates for the large black holes that emerge at the time of the formation of inhomogeneities. In this paper we close this gap by offering a much more comprehensive picture, modeling the growth of mini-black holes into giant black holes 'from beginning to end'. The fact that this can be done by means of a closed form metric solving the Einstein equations that encompasses *both* the radiation and the matter dominated phase is a main new result of the present paper.

According to [1] the gravitino mass M_g is hypothesized to lie between M_{BPS} and M_{PI} , where the 'BPS-mass' M_{BPS} is the mass for which the electrostatic repulsion between two (anti-)gravitinos of the same charge equals their gravitational attraction. Here M_{PI} is the reduced Planck mass ($\sim 4.34 \cdot 10^{-9}$ kg, corresponding to the Planck time $t_{PI} = 2.70 \cdot 10^{-43}$ s). For numerical estimates we will take $M_{BPS} \sim 0.01 \cdot M_{PI}$, so that

$$0.01 \cdot M_{PI} < M_g < M_{PI} \quad (1)$$

This ensures that the force remains attractive also between gravitinos of the same electric charge. The minimal seed mass for a primordial black hole in the early radiation phase is determined by asking the total energy of a bound system of N (anti-)gravitinos to be negative. For the minimum number of (anti-)gravitinos in a bound state this leads to the estimate [1]

$$N \gtrsim 10^{12} \quad (2)$$

Importantly, the cosmic time t drops out in the derivation of this inequality, hence the value of N remains the same throughout the radiation phase. To be sure, if the bound state is meta-stable, the collapse can be delayed in such a way that an even larger number N of (anti-)gravitinos can accrue before gravitational collapse occurs, in which case the seed mass value $M_{seed} \sim NM_g$ could be even larger. The minimum mass of a black hole resulting from gravitational collapse of such a bound state is therefore (we set $c = 1$ throughout)

$$M_{seed} \sim 10^{12} M_g \sim 10^3 \text{ kg} \Rightarrow GM_{seed} \sim 10^{-24} \text{ m} \quad (3)$$

where we assumed $M_g \sim 10^{-9}$ kg. Now, a black hole of such a small mass would be expected to decay immediately by Hawking radiation: from the well known formula for the lifetime of a black hole (see *e.g.* [4]) we have

$$\tau_{evap}(m) = t_{Pl} \left(\frac{m}{M_{Pl}} \right)^3 \quad (4)$$

This is the result which would hold in empty space. However, during the early radiation phase this is not the only process that must be taken into account, because of the presence of extremely hot and dense radiation, which can ‘feed’ black hole growth. The absorption of radiation thus provides a competing process which can stabilize the black hole against Hawking decay, such that with the initially extremely high temperatures of the radiation era mass accretion can overwhelm Hawking evaporation *even for very small black holes*. More precisely, for a black hole of given mass m the criterion for accretion to overcome the rate for Hawking radiation reads

$$T_{rad}(t) > T_{Hawking}(m) = \frac{\hbar}{8\pi G m} \quad (5)$$

The break-even point is reached when the radiation temperature equals the Hawking temperature, at time $t_0 = t_0(m)$ when $T_{rad}(t_0) = T_{Hawking}(m)$. For larger times $t > t_0$ (and lower radiation temperatures) a black hole of mass m will decay. Imposing this equality, or alternatively using eqn.(26) of [1] we deduce the relevant mass at time t , which gives

$$m^4(t) \simeq \frac{M_{Pl}^3}{t_{Pl}} \cdot \frac{1}{G^2 \rho_{rad}(t)} = \frac{32\pi M_{Pl}^3}{3G t_{Pl}} \cdot t^2 \quad (6)$$

When read from right to left this equation tells us which is the latest time for a mini-black hole of given mass m to remain stable against Hawking decay during the radiation phase. This is the case for $t < t_0 \equiv t(m) \propto m^2$, after which time the black hole will decay. Conversely, for a given time t any mini-black hole of initial mass greater than $m(t)$ will be able to survive and can start growing, whereas those of smaller mass decay. With (3) as the reference value we thus take the initial mass to be $\sim M_{seed}$, and assume that the time range available for the formation of such a mini-black hole is

$$t_{min} = 10^8 \cdot t_{Pl} \simeq 10^{-34} \text{ s} < t < t_{max} \simeq 10^{-18} \text{ s} \quad (7)$$

During this time interval a black hole of initial mass (3) can survive and start growing by accreting radiation. While the upper bound is thus determined by setting $t_{max} \equiv t(M_{seed})$, the lower bound has been chosen mainly to stay clear of the quantum gravity regime and a possible inflationary phase.

Once we have a stable mini-black hole we can study its further evolution through the radiation phase by means of the exact solution derived in [1], until matter starts to dominate over radiation at time $t \sim t_{eq} \sim 47000$ yr, when

these objects have grown into macroscopic black holes. With (7) we get the following range of masses

$$10^{-12} M_{\odot} \lesssim m(t_{eq}) \lesssim 10^{-3} M_{\odot} \quad (8)$$

However, the solution in [1] does *not* apply to the matter dominated phase. To investigate the further evolution one would conventionally switch to a different description by invoking the Eddington formula [3, 5]

$$m(t) = M_0 \exp \left(\frac{4\pi G m_p t}{\epsilon c \sigma_T} \right) \simeq M_0 \exp \left(\frac{t}{45 \text{ Myr}} \right) \quad (9)$$

where m_p is the proton mass, σ_T is the Thompson cross section, and ϵ is the fraction of the mass loss that is radiated away, which is usually taken as $\epsilon = 0.1$. Taking $M_0 = 10^{-3} M_{\odot}$ as the initial value at time $t = t_{eq}$, the mass at $t = 690 \cdot 10^6$ yr comes out to be

$$m(690 \text{ Myr}) \simeq 5 \cdot 10^3 M_{\odot}, \quad (10)$$

less than the actually observed value ($\sim 8 \cdot 10^8 M_{\odot}$) [3].

Since the ‘blind’ application of the Eddington formula does not produce the desired order of magnitude, we here want to proceed differently. This is because this formula was originally developed to describe the evolution of luminous stars [5], and it is therefore rather doubtful whether one can use it in the present context. In particular, its derivation relies on the Newtonian approximation and is based on a simple equilibrium condition, balancing the rate of mass absorption against the luminosity of infalling matter, where the luminosity is assumed to grow linearly with the mass of the black hole. Therefore, a fully relativistic treatment by means of an exact solution of Einstein’s equations seems preferable, even if it does not (yet) take into account matter self-interactions.

To present this new solution we employ conformal coordinates, with conformal time η , instead of the cosmic time coordinate t used above. One main advantage of this coordinate choice is that the causal structure of the space-time is often easier to analyze (for the solution to be presented below it is the same as that of the Schwarzschild solution). Secondly, we wish to exploit the remarkable fact that the use of conformal time allows us to exhibit a simple closed form solution that encompasses *both* the radiative and the matter dominated phase. With conformal time η , the Friedmann equations read (for a spatially flat universe and vanishing Λ)

$$\dot{a}^2 = \frac{8\pi G}{3} \rho a^4, \quad a\ddot{a} - \dot{a}^2 = -\frac{4\pi G}{3} (\rho + 3p) a^4 \quad (11)$$

where

$$\dot{a} \equiv da/d\eta, \quad dt = a(\eta) d\eta. \quad (12)$$

The requisite solution is

$$a(\eta) = A\eta + B^2\eta^2 \Rightarrow t = \frac{1}{2}A\eta^2 + \frac{1}{3}B^2\eta^3 \quad (13)$$

The density and pressure following from (11) are

$$8\pi G\rho(\eta) = \frac{3A^2}{a^4(\eta)} + \frac{12B^2}{a^3(\eta)}, \quad 8\pi Gp(\eta) = \frac{A^2}{a^4(\eta)} \quad (14)$$

where, for our Universe (starting from nucleosynthesis)

$$A = 2.1 \cdot 10^{-20} \text{ s}^{-1}, \quad B = 6.2 \cdot 10^{-19} \text{ s}^{-1}. \quad (15)$$

These numbers can be calculated from known data up to rescaling $\eta \rightarrow \lambda\eta$, $A \rightarrow \lambda^{-2}A$, $B \rightarrow \lambda^{-3/2}B$, $a \rightarrow \lambda^{-1}a$. The latter scale is conventionally fixed by setting $a(t_0) = 1$, where $t_0 \simeq 13.8 \cdot 10^9 \text{ yr}$ is the present time. At the equilibrium between radiation and matter we have [6]

$$a(\eta_{eq}) \simeq \frac{1}{3400}, \quad t_{eq} \simeq 1.5 \cdot 10^{12} \text{ s} \quad (16)$$

At the last scattering we have [6]

$$a(\eta_{LS}) \simeq \frac{1}{1090}, \quad t_{LS} \simeq 1.2 \cdot 10^{13} \text{ s} \quad (17)$$

and these numbers give (15).

We now generalize the solution of [1] by substituting (13) into the metric ansatz

$$ds^2 = a(\eta)^2 \left[-\tilde{C}(r)d\eta^2 + \frac{dr^2}{\tilde{C}(r)} + r^2 d\Omega^2 \right] \quad (18)$$

Here the *a priori* unknown function $\tilde{C}(r)$ is uniquely fixed by imposing two requirements. For the limiting case of pure radiation ($B = 0$) the trace of the Einstein tensor resulting from (18) must vanish: $T^\mu_\mu = 0 \Rightarrow (r^2\tilde{C})'' \stackrel{!}{=} 2$ [1]. In the other limiting case of pure matter ($A = 0$), the pressure must vanish: $p = 0 \Rightarrow (r\tilde{C})' \stackrel{!}{=} 1$. Imposing both requirements leads to the unique solution

$$\tilde{C}(r) \equiv C(r) := 1 - \frac{2G\mathbf{m}}{r} \quad (19)$$

and this choice will be used in the following. The new and essential feature here is that the metric (18) allows us to evolve the black hole through *both* the radiative and matter dominated periods, with a smooth transition between the two.

In (19) we use a different font for the fixed mass parameter because $\mathbf{m}$ is *not* the physical mass, unlike $m(t)$ above. This is most easily seen by replacing

$$\frac{G\mathbf{m}}{r} \rightarrow \frac{G\mathbf{m}a(\eta)}{ra(\eta)} \equiv \frac{G\mathbf{m}a(\eta)}{r_{phys}} \Rightarrow m(\eta) = \mathbf{m}a(\eta) \quad (20)$$

Using (3), (7) and the above relation with $\eta_{min} = 10^{-7} \text{ s}$ and $\eta_{max} = 10 \text{ s}$, as well as $G\mathbf{m}_{min} = GM_{seed}/a_{max}$ and $G\mathbf{m}_{max} = GM_{seed}/a_{min}$ we get

$$G\mathbf{m}_{min} \sim 5 \cdot 10^{-6} \text{ m}, \quad G\mathbf{m}_{max} \sim 5 \cdot 10^2 \text{ m} \quad (21)$$

For the metric ansatz (18) with $C(r)$ from (19) the non-vanishing components of the Einstein tensor, hence the associated energy momentum tensor, are given by:

$$\begin{aligned} 8\pi G T_{\eta\eta} &= \frac{3\dot{a}^2}{a^2} = \frac{3(A + 2B^2\eta)^2}{(A\eta + B^2\eta^2)^2} \\ 8\pi G T_{r\eta} &= \frac{2G\mathbf{m}}{r^2 C(r)} \cdot \frac{\dot{a}}{a} = \frac{2G\mathbf{m}}{r^2 C(r)} \cdot \frac{A + 2B^2\eta}{A\eta + B^2\eta^2} \\ 8\pi G T_{rr} &= \frac{\dot{a}^2 - 2a\ddot{a}}{a^2 C(r)^2} = \frac{1}{C(r)^2} \cdot \frac{A^2}{(A\eta + B^2\eta^2)^2} \end{aligned} \quad (22)$$

together with [10]

$$T_{\theta\theta} = C(r) r^2 T_{rr}, \quad T_{\varphi\varphi} = \sin^2 \theta T_{\theta\theta} \quad (23)$$

To endow (22) with physical meaning, we must interpret the r.h.s. in terms of physical sources of energy and momentum. To this aim we rewrite the energy momentum tensor in the form [7]

$$T_{\mu\nu} = pg_{\mu\nu} + (p + \rho) u_\mu u_\nu - u_\mu q_\nu - u_\nu q_\mu \quad (24)$$

neglecting higher derivatives in u_μ and matter self-interactions (viscosity, *etc.*). For the density and pressure to match between (24) and (22) we must include an extra inverse factor $C(r)$ in comparison with (14) to account for the curvature

$$\begin{aligned} 8\pi G\rho(\eta, r) &= \frac{1}{C(r)} \left(\frac{3A^2}{a^4(\eta)} + \frac{12B^2}{a^3(\eta)} \right) \\ 8\pi Gp(\eta, r) &= \frac{1}{C(r)} \frac{A^2}{a^4(\eta)} \end{aligned} \quad (25)$$

again with $a(\eta)$ from (13). The 4-velocity is [11]

$$u_\mu = -\frac{a(\eta)}{C(r)^{1/2}} (C(r) \cosh \xi, \sinh \xi, 0, 0) \quad (26)$$

while the heat flow vector is given by

$$8\pi Gq_\mu = -\frac{2G\mathbf{m}\dot{a}(\eta)}{r^2 C(r)^{3/2} a(\eta)^2} (C(r) \sinh \xi, \cosh \xi, 0, 0) \quad (27)$$

These vectors obey $u^\mu u_\mu = -1$ and $u^\mu q_\mu = 0$. The parameter $\xi = \xi(\eta, r) > 0$ is determined from

$$\tanh \xi = \frac{G\mathbf{m}\eta}{r^2} \cdot \left(1 - \frac{B^4\eta^2}{A^2 + 3AB^2\eta + 3B^4\eta^2} \right) \quad (28)$$

The signs in (26) and (27) are chosen such that for the contravariant components of the 4-velocity we have $u^\eta > 0$ and $u^r < 0$, hence *inward* flow of matter. (Choosing the opposite sign for the components of u_μ would correspond to a shrinking white hole.)

To keep ξ real and finite we must demand $\tanh \xi < 1$. It is easily seen that

$$\begin{aligned} \tanh \xi &\sim \frac{G\mathbf{m}\eta}{r^2} \quad \text{for } B^2\eta \ll A \quad (\text{radiation}) \\ &\sim \frac{2}{3} \frac{G\mathbf{m}\eta}{r^2} \quad \text{for } B^2\eta \gg A \quad (\text{matter}) \end{aligned} \quad (29)$$

The representation (24) is valid as long as all quantities remain real and finite. This requires $r^2 > \mathcal{O}(1)G\mathfrak{m}\eta$, with a strictly positive $\mathcal{O}(1)$ prefactor. When r reaches the value for which $\tanh\xi = 1$ the components of u_μ and q_μ diverge, and the expansion (24) breaks down. For the external observer the average velocity of the infalling matter then reaches the speed of light, so for all practical purposes everything happening inside this shell is shielded from the outside (even though light rays can still escape from this region, as long as $r > 2G\mathfrak{m}$). As we are not concerned with $\mathcal{O}(1)$ factors here we define

$$r_H(\eta) := a(\eta)\sqrt{G\mathfrak{m}\eta} \quad (30)$$

and interpret the associated outward moving shell as an *effective* horizon (or ‘pseudo-horizon’) that lies above the actual event horizon; note that $r_H(\eta)$ is invariant under the coordinate rescalings mentioned after (15). Physically, we expect the matter inside the shell $r_{phys} \lesssim r_H(\eta)$ to be rapidly sucked up into the black hole, once the outside region $r_{phys} > r_H(\eta)$ gets depleted of ‘fuel’ due to the formation of inhomogeneities. The extra matter inside the shell $r_{phys} \lesssim r_H(\eta)$ thus enhances the growth substantially, beyond the linear growth with the scale factor implied by (20).

At the onset of inhomogeneities, we must stop using the metric (18) because the growth of the black hole gets decoupled from the growth of the scale factor $a(\eta)$, after which the black hole evolves in a more standard fashion by much slower accretion (for this reason there is also no point in extending the metric ansatz (18) into the present epoch, which is dominated by Dark Energy). To estimate its mass we take the value of r_H at that particular time to define an effective Schwarzschild radius, thus equating the mass with the maximum energy that can possibly fit

inside a shell of radius r_H . This approximation is justified not only because of the apparent divergent kinetic energy of the infalling matter near r_H , but also because of the strong increase of the density and pressure inside this shell, which is due to the extra factor $C^{-1}(r)$ in (25).

The relevant time t at which to evaluate $r_H(t) \equiv r_H(\eta(t))$ lies well after decoupling, since the inhomogeneities in the CMB are still tiny, of order $\mathcal{O}(10^{-5})$. Rather, we take $t_{inhom} \simeq 10^8 \text{ yr} \simeq 3.2 \cdot 10^{15} \text{ s}$, which is the time when the first stars are born [8]. This corresponds to $\eta_{inhom} \simeq 2.7 \cdot 10^{17} \text{ s} \Rightarrow a(\eta_{inhom}) \simeq 0.034$. Substituting (21) into (30) and using $r_S(M_\odot) = 3 \text{ km}$ we can calculate the range of possible black hole masses at $t \sim 100 \text{ Myr}$ as

$$10^5 M_\odot \lesssim m_{\text{BH}} \lesssim 2 \cdot 10^9 M_\odot \quad (31)$$

consistent with observations [3]. To reach such large mass values the replacement of $G\mathfrak{m}$ by $\sqrt{G\mathfrak{m}\eta}$ in (30), as advocated in this paper, is evidently of crucial importance.

Acknowledgments: K.A.M. was partially supported by the Polish National Science Center grant. The work of H.N. has received funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme (grant agreement No 740209). The title of this paper was inspired by a philosophical treatise by one of the first author’s ancestors [9].

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- [10] We take this opportunity to correct two misprints in [1]: the extra factor of C in (23) below is missing in (46) there. Furthermore, in eqn.(50) of [1] it should read

$$8\pi Gp(\eta, r) = \frac{r}{A^2\eta^4(r - 2G\mathfrak{m})}$$

- [11] There is a second solution with the same ρ and p , but $u_r = q_\eta = 0$, which we discard as unphysical because it would imply the absence of motion of matter other than the co-motion with the cosmic frame.