High birth velocities of radio pulsars

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NEUTRON stars are usually born during the supernova explosion of a massive star. Any small asymmetry during the explosion can result in a substantial 'kick' velocity[1] to the neutron star. Pulsars (rapidly rotating, magnetized neutron stars) have long been known to have high space velocities[2,3], but new measurements of proper motion[4–6], adoption of a new distance scale for the pulsars[7] and the realization that some previous velocities were systematically low by a factor of 2 (ref. 8) have prompted us to reassess these velocities. Here, taking into account a strong selection effect that makes the observed velocities unrepresentative of those acquired at birth[9], we arrive at a mean pulsar birth velocity of 450 ± 90 km s^{-1}. This exceeds the escape velocity from binary systems, globular clusters and the Galaxy, and so will affect our understanding of the retention of neutron stars in these systems. Those neutron stars that are retained by the Milky Way will be distributed more isotropically than has been thought[10–12], which may result in a distribution like that of the γ-ray burst sources.

Estimates of pulsar transverse velocities V_t have been based primarily on the measurement of proper motion μ (mas yr^{-1}) and the distance D (kpc): $V_t = 4.74\,\mu D$ km s^{-1}. There are 87 reported determinations of proper motion made by interferometry[3–6]. We use all these data, apart from pulsar B0736−40 (ref. 6) which lies behind the Gum nebula and has an unknown distance in excess of 0.5 kpc. The distance D is obtained from the new electron density model of Taylor and Cordes[7] which is based on a number of new independent distance measurements and clearly shows that previous models had underestimated the distance to nearby pulsars. The velocities of pulsars can also be estimated from observations of the speed of the interstellar scintillation patterns[9,13]. However, Harrison and Lyne[8] have recently shown that these are systematically low by an average factor of 2 because of a localization of the scattering medium close to the galactic plane. For this reason, we have relied, where possible, on the more direct proper motion measurements. For 14 pulsars which only have upper limits to their proper motion, and a further 13 which do not have proper motion data, we have used scintillation data[9,14] to estimate V_t. These scintillation speeds have been calculated using the new distance model[7] and the appropriate correction for the medium localization[8]. Thus our sample contains 99 pulsars, of which only 8 have upper limits to V_t.

This sample has a mean V_t of 300 ± 30 km s^{-1}, compared with 134 km s^{-1} obtained 12 years ago from 26 measurements[3]. The increase in value reflects both the change in the adopted distance scale[7] and the greater number of young and higher velocity pulsars observed in the recent astrometric surveys[4–6]. The much improved statistics also show that this velocity is not representative of the velocities at birth because we know that young pulsars have higher velocities on average than older ones[4]. This can be seen in Fig. 1, where the transverse speed of pulsars is plotted against characteristic age, and surely does not arise because pulsars slow down significantly as they age. Rather, it is due primarily to a selection effect, first recognised by Cordes[9]: young pulsars are born from Population I stars, close to the galactic plane, and those pulsars with high velocity (1,000 km s^{-1} ≈ 1 kpc Myr^{-1}) mostly move rapidly away from the plane. After ~10 Myr, the mean distance of the fastest pulsars from an observer on the galactic plane is significantly greater than their mean distance at birth, reducing their likelihood of detection and hence lowering the apparent mean velocity. By this time, the pulsars remaining within detectable range are those with small velocity and a few high velocity ones moving roughly parallel to or towards the plane of the Galaxy[4].

During the first 3 Myr, the strength of the above selection effect is minimal, and the observed transverse speed distribution of the 29 pulsars younger than this should be a good representation of the true V_t distribution at birth. This distribution has a mean of 345 ± 70 km s^{-1} and an r.m.s. of 499 km s^{-1}. The mean velocity of only 105 ± 25 km s^{-1} for the 10 oldest pulsars illustrates the strength of the effect.

The three-dimensional (3D) space velocities are of course somewhat greater than the observed two-dimensional (2D) transverse velocities, and we can estimate these statistically if we make the reasonable assumption that the velocities at birth are isotropic. We have used a simple, iterative Monte Carlo technique to determine the one-dimensional (1D) velocity distribution required to give the observed 2D transverse distribution, and hence to generate the 3D space velocity distribution. To approximate the observed 2D distribution, we found that the data are well described by a function of the form $x^m/(1+x^n)$, where $x = V_t/V_0$. The scaling factor V_0 and the constants m and n were found to be 330 km s^{-1}, 0.13 and 3.3 respectively. A cumulative plot of this function is compared with the observed

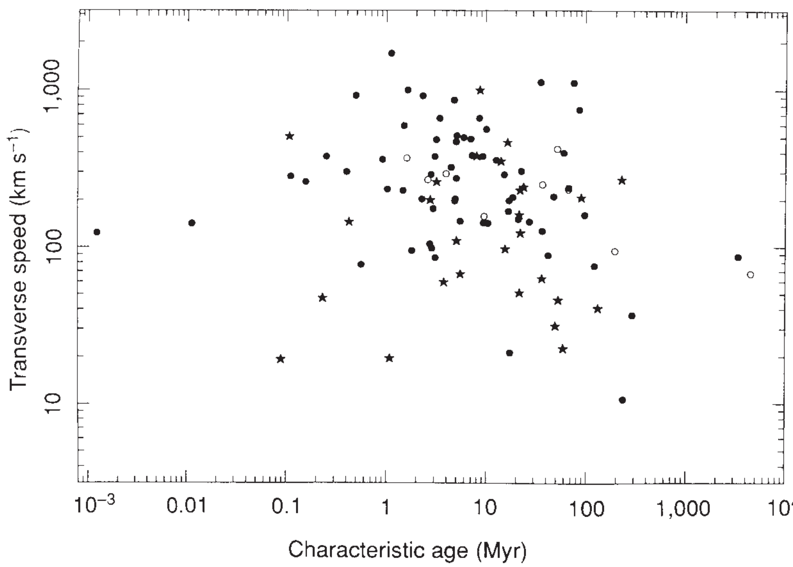

FIG. 1 The transverse speed V_t of 99 pulsars plotted against characteristic age $\tau = P/2\dot{P}$, where P and $\dot{P}$ are the pulsar rotation period and time derivative, respectively. Pulsars with only upper limits to their transverse speed are represented by open circles at half the upper limits. Starred symbols represent transverse speeds obtained from scintillation data. The mean transverse speed for the 29 pulsars younger than 3 Myr is 345 km s^{-1}. In the unlikely circumstance that the two young pulsars in this group with upper limits have zero speed, this value would only be reduced by 6%.

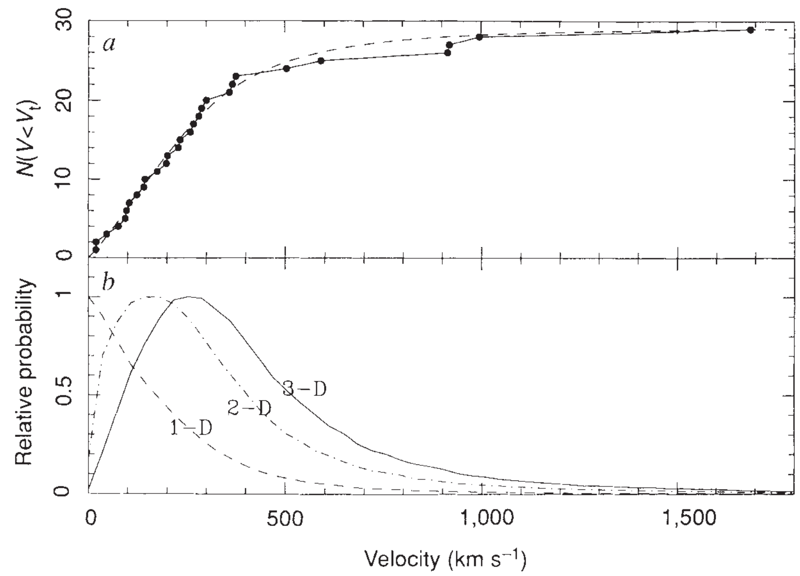

FIG. 2 *a*, The cumulative distribution of transverse speeds V_t for the 29 pulsars younger than 3 Myr. For these pulsars, the selection effect (see text) against detecting high-velocity pulsars is negligible and this is a good representation of the 2D speed distribution at birth. The dashed curve is an analytic approximation to the data having the same mean and r.m.s. (see text). *b*, The one-, two- and three-dimensional velocity distributions corresponding to the analytic function in *a*, obtained using an iterative Monte Carlo simulation. The 3D distribution shows that pulsars have a mean space velocity of ~450 km s^{-1}.

data in Fig. 2*a*, and the derived distributions are shown in Fig. 2*b*. From the 3D distribution, we deduce that the space velocity of pulsars at birth has a mean of 450 ± 90 km s^{-1} and an r.m.s. value of 535 km s^{-1}. This mean space velocity is a factor of three higher than the value of ~150 km s^{-1} deduced from the earlier sample[3]. The increase arises from the three main factors described above; namely a younger, faster sample (×1.4), the new distance model (×1.6) and the selection effect (×1.2).

There are two other indications that the birth velocities are large. First, such high birth velocities will give rise to a rapid migration from the galactic plane with a mean velocity V_z equal to 210 km s^{-1}, the mean of the 1D distribution in Fig. 2*b*. This is clearly seen for the whole observed pulsar population in Fig. 3 as an increase in the mean distance Z of pulsars from the galactic plane with age. Assuming that pulsars are born in a progenitor Population I distribution with a width $Z_0 = 80$ pc, then pulsars of age τ will have a mean height given approximately by $Z = (Z_0^2 + V_Z^2\tau^2)^{1/2}$. This function is also shown in Fig. 3 and describes the data well for ages below about 2 Myr, confirming the large velocity dispersion. For older pulsars, the selection effect described earlier reduces the detected number of large-Z pulsars and hence the mean Z of the observed population.

Second, the high transverse velocities found here are close to estimates of the velocities of young pulsars required by associations of pulsars with supernova remnants[15]. These are usually made on the basis of similarity of age, distance and position on the sky. Often the pulsar has moved significantly from the centre of the remnant since the supernova explosion, so that the pulsar velocity can be determined from the ratio of the separation and the age. Taking 13 reasonably convincing associations, we find that the mean transverse pulsar velocity is $\sim 530 \pm 180$ km s^{-1}. Using the same velocity distribution as above, this implies a mean space velocity of 690 ± 230 km s^{-1}.

We have shown that most pulsars are born with velocities of ~450 km s^{-1}. These large values indicate that their origin must lie in the kinetics of asymmetric supernova collapse rather than the disruption of binary systems[16,17]. The kinetic energy of the neutron stars so formed are an order of magnitude greater than previously thought. Although the mechanisms of such collapse are not well understood, the high degree of asymmetry now required might also be expected to show corresponding asymmetry in the gaseous supernova remnant in the opposite direction to the pulsar velocity.

Although most recent statistical studies[18-20] of the pulsar population and its evolution have taken account of the spatial separation of fast and slow pulsars, our results show that this

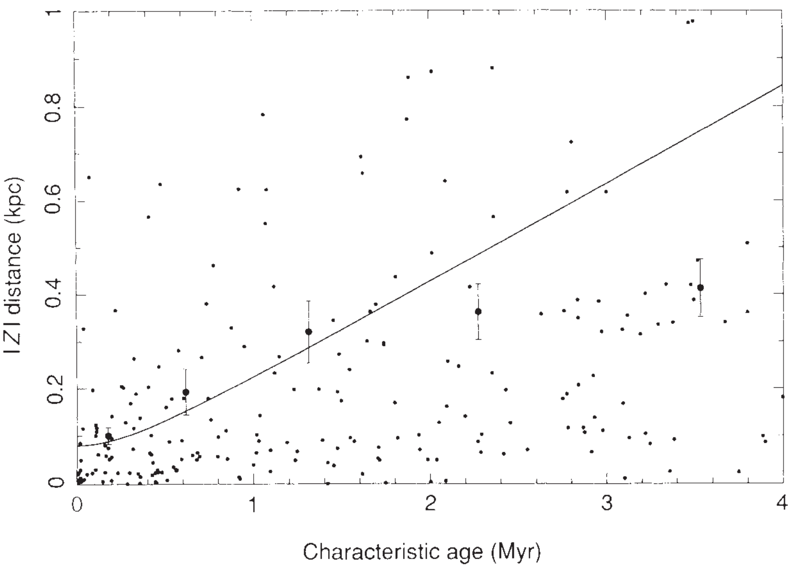

FIG. 3 The distance of pulsars from the galactic plane $|Z|$ shown as a function of characteristic age τ for all the pulsars detected in the major pulsar surveys. The large symbols represent the means of groups of 40 pulsars. The line shows the variation expected from a simple dynamical model with the mean velocity derived from Fig. 2.

has been underestimated by a factor of 3 and they will need to be repeated. Similarly, the increase in birth velocity will clearly have a major effect upon our understanding of the number of pulsars that remain bound in binary systems, globular clusters and the Galaxy. Although such high velocities make it easier to understand the small number of pulsars in binary systems[21], the small proportion (~0.7%) with velocities below ~50 km s^{-1} make the large population of neutron stars in globular clusters surprising unless they are formed in a less violent manner, such as accretion-induced collapse of a white dwarf[22].

With mean birth velocities of 450 km s^{-1}, more than half of all pulsars will escape the galactic gravitational potential. Those that remain bound will form a much larger spherical halo of old neutron stars than previous models suggested[10-12]. Given that the Burst and Transient Source Experiment[23] is continuing to observe an isotropic distribution of γ-ray bursts, our results clearly have a bearing on the possibility that old, high velocity pulsars are the source of the bursts[24,25]. □

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Relation between fractal dimension and spatial correlation length for extensive chaos

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SUSTAINED nonequilibrium systems can be characterized by a fractal dimension $D \geqslant 0$, which can be considered to be a measure of the number of independent degrees of freedom[1]. The dimension D is usually estimated from time series[2] but the available algorithms are unreliable and difficult to apply when D is larger than about 5 (refs 3, 4). Recent advances in experimental technique[5-8] and in parallel computing have now made possible the study of big systems with large fractal dimensions, raising new questions about what physical properties determine D and whether these physical properties can be used in place of time-series to estimate large fractal dimensions. Numerical simulations[9-11] suggest that sufficiently large homogeneous systems will generally be extensively chaotic[12], which means that D increases linearly with the system volume V. Here we test an hypothesis that follows from this observation: that the fractal dimension of extensive chaos is determined by the average spatial disorder as measured by the spatial correlation length ξ associated with the equal-time two-point correlation function—a measure of the correlations between different regions of the system. We find that the hypothesis fails for a representative spatiotemporal chaotic system. Thus, if there is a length scale that characterizes homogeneous extensive chaos, it is not the characteristic length scale of spatial disorder.

If a large homogeneous nonequilibrium system is extensively chaotic, we can relate its fractal dimension D (roughly the minimum number of degrees of freedom needed to describe a system[1]) to its characteristic size L by the equation:

$$D = (L/\xi_c)^d \quad \text{for } L \gg \xi_c \qquad (1)$$

where ξ_c is defined to be the chaos correlation length[12] and d is

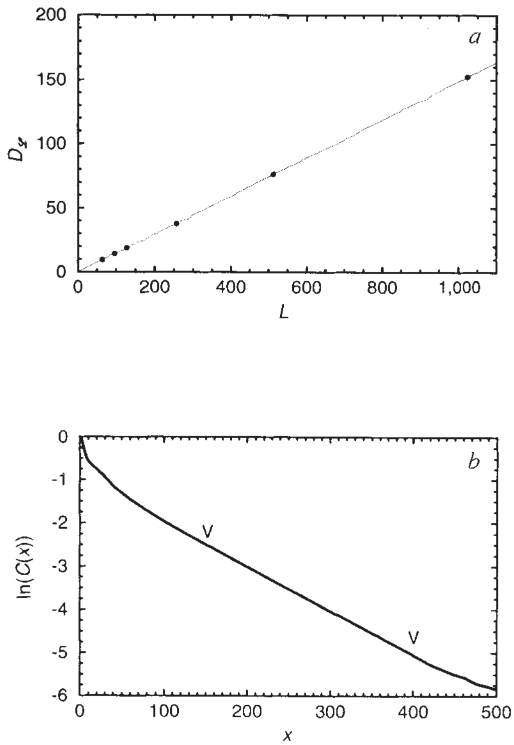

FIG. 1 *a*, Plot of the Lyapunov fractal dimension $D_{\mathcal{L}}$ against system size L for equation (4) with periodic boundary conditions. We used the parameter values $c_1 = 3.5$ and $c_3 = 0.8$, a constant time step of $\Delta t = 0.05$, a spatial resolution of two Fourier modes per unit length and a total integration time of $T = 50{,}000$ time units. The error in each fractal dimension is smaller than the size of the plotted points. *b*, Log–linear plot of the magnitude of the spatial correlation function $C(x) = \langle u^*(x+x', t)u(x', t)\rangle$ for spatiotemporal chaotic solutions of equation (4) for the parameter values $c_1 = 3.5$, $c_3 = 0.81$ and $L = 8{,}192$. After integrating for a time $T = 30{,}000$ to allow transients to decay, the correlations were calculated over an interval of 70,000 time units. The correlation function was further averaged over 64 different initial conditions. The two Vs indicate the region over which an exponential decay was identified. There is no change in the correlation function for larger system sizes.