

Notes Concerning "On the Origin of Gravity and the Laws of Newton" by E. Verlinde

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We point out that certain equations which, in a very recent paper written by E. Verlinde, are postulated as a starting point for a thermodynamical derivation of classical gravity, are actually consequences of a specific microscopic model of spacetime, which has been published earlier.

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I. INTRODUCTION

A remarkable paper was published very recently by Verlinde. [1] In that paper it was argued, among other things, that Newton's second law of dynamics and Newton's universal law of gravitation arise naturally and unavoidably in a theory in which space is emergent through a holographic scenario. [2] In these notes we shall focus our attention to Verlinde's derivation of Newton's law of gravitation.

The starting point of Verlinde's derivation was an assumption that whenever we have a sphere with area A , the sphere acts as a storage device for information such that the total number of bits of information stored on the surface of the sphere is

$$N = \frac{Ac^3}{G\hbar}. \quad (1)$$

Verlinde then identified as the energy E of a system the rest energy Mc^2 of the mass inside the sphere, and he assumed that the energy is divided evenly over the bits N such that

$$E = \frac{1}{2}Nk_B T. \quad (2)$$

If we put $E = Mc^2$ in Eq. (2), identify T as the Unruh temperature

$$T_U := \frac{1}{2\pi} \frac{\hbar a}{k_B c} \quad (3)$$

of an observer with a proper acceleration a , and use the fact that the area of a two-sphere with radius r is

$$A = 4\pi r^2, \quad (4)$$

we find, by means of Newton's second law $F = ma$, that the force exerted by a point-like mass M on a mass m at a distance r is:

$$F = G \frac{mM}{r^2}, \quad (5)$$

which is Newton's universal law of gravitation. Even prior to Verlinde's work a very similar derivation of Eq. (5) was discovered by Padmanabhan. [3]

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The validity of Verlinde's derivation hinges on Eqs. (1) and (2), (Eqs. (3.10) and (3.11) in Verlinde's paper) together with an identification of the absolute temperature T as the Unruh temperature T_U . Are Eqs. (1) and (2) valid? In which sense can we say that the temperature T may be identified as the Unruh temperature T_U ?

The questions raised above could be addressed in a precise fashion, if we had a concrete microscopic model of spacetime in our disposal. Using that model we should be able to calculate the partition function of spacetime, which, in turn, would yield an expression to the entropy - and therefore information - stored on its two-dimensional surfaces. We should also be able to calculate the dependence of the average energy associated with those surfaces on the given temperature T . Finally, we should be able to gain better understanding on the role played by the Unruh temperature T_U in the thermodynamics of spacetime. Even if our model were far from correct, it could possess features, which are generic to every plausible microscopic model of spacetime, thus providing a useful starting point for more advanced models.

Such a model indeed exists, and it was developed in details in Refs. [4–6]. In what follows, we shall outline that model briefly, and comment on its bearing on Verlinde's approach.

In Refs. [4–6] spacetime was modelled by a specific graph with particular objects on its vertices, which were called, for certain reasons, as "microscopic quantum black holes". The idea was to reduce all properties of spacetime to the quantum states of those holes. At macroscopic scales the quantum black holes are assumed to generate areas for spacelike two-surfaces of spacetime. More precisely, the possible eigenvalues of the areas of spacelike two-surfaces are assumed to be of the form:

$$A = \alpha(n_1 + n_2 + \dots + n_N)\ell_{Pl}^2, \quad (6)$$

where α is a numerical coefficient to be determined later, N is the number of quantum black holes on the surface, and

$$\ell_{Pl} := \sqrt{\frac{\hbar G}{c^3}} \approx 1.6 \times 10^{-35} m \quad (7)$$

is the Planck length. The numbers n_1, n_2, \dots, n_N are non-negative integers, which label the quantum states of the N quantum black holes lying on the surface. In other words, each quantum black hole on the surface is assumed to contribute to the surface an area, which is of the form:

$$A_n = \alpha n \ell_{Pl}^2, \quad (8)$$

where $n = 0, 1, 2, \dots$. The microstates of the surface are assumed to be uniquely determined by the combinations of the non-vacuum ($n \neq 0$) area eigenstates of the holes, whereas the microstates corresponding to the same total area of the surface belong to the same macroscopic state of the surface.

A specific attention in the model was paid to the so called *acceleration surfaces*. In very broad terms, acceleration surface may be described as a spacelike two-surface propagating in spacetime such that each point of the surface accelerates with the same constant proper acceleration a in a direction orthogonal to the surface (For a mathematically precise definition, see Refs. [5] and [6]). An example of an acceleration surface is provided by a $t = \text{constant}$ slice of a timelike hypersurface $r = \text{constant}$ of the Schwarzschild spacetime equipped with the Schwarzschild coordinates r and t . Indeed, each point of such a slice accelerates with the same proper acceleration

$$a = \left(1 - \frac{2M}{r}\right)^{-1/2} \frac{M}{r^2}. \quad (9)$$

In the Newtonian limit and SI units this gives:

$$a = G \frac{M}{r^2}, \quad (10)$$

which is the acceleration of particles in a free fall in the gravitational field created by a point-like mass M .

One of the reasons for defining the concept of acceleration surface is that with acceleration surfaces it is possible to associate the concept of energy in a natural manner. In the model of Refs. [4–6] the macroscopic energy of an acceleration surface with proper acceleration a and area A was defined to be, from the point of view of an observer at rest with respect to the surface,

$$E := \frac{c^2 a A}{4\pi G}. \quad (11)$$

If one takes the acceleration surface to be a two-sphere with radius r surrounding a point-like mass M , and identifies E as Mc^2 , one immediately finds, in the Newtonian limit, that the gravitational force exerted by the mass M on a mass m is

$$F = G \frac{Mm}{r^2}. \quad (12)$$

In other words, Newton's universal law of gravitation is, in our model, a consequence of our macroscopic definition of energy, rather an outcome of the microscopic properties of the constituents of spacetime.

Using Eqs.(6) and (11) we find that the energies of an acceleration surface are of the form:

$$E_n = n \frac{\alpha \hbar a}{4\pi c}, \quad (13)$$

where

$$n := n_1 + n_2 + \dots + n_N. \quad (14)$$

Hence we are able to calculate the *partition function*

$$Z(\beta) := \sum_{n=1}^{\infty} g(E_n) e^{-\beta E_n} \quad (15)$$

of the acceleration surface. It follows from our definitions of the microscopic and the macroscopic states of spacelike two-surfaces of spacetime that the degeneracy $g(E_n)$ of a state of an acceleration surface with energy E_n is simply the number of the possible ways of writing the positive integer n as a sum of at most N positive integers. More precisely, $g(E_n)$ is the number of the ordered strings (n_1, n_2, \dots, n_m) , where $n_1, n_2, \dots, n_m \in \{1, 2, 3, \dots\}$ and $1 \leq m \leq N$ such that

$$n_1 + n_2 + \dots + n_m = n. \quad (16)$$

A rather involved calculation yields an explicit -and surprisingly simple- analytical expression for the partition function $Z(\beta)$. [6] Using the partition function $Z(\beta)$ one may obtain explicit expressions both for the average energy

$$E(\beta) = -\frac{\partial}{\partial \beta} \ln Z(\beta) \quad (17)$$

and the entropy

$$S(\beta) = k_B \left[\ln Z(\beta) - \beta \frac{\partial}{\partial \beta} \ln Z(\beta) \right] \quad (18)$$

of an acceleration surface in a given temperature T . The key role in the results obtainable from these expressions is played by the *characteristic temperature*

$$T_C := \frac{\alpha}{4\pi \ln 2} \frac{\hbar a}{k_B c} \quad (19)$$

of the acceleration surface. It also turns out useful to define a quantity

$$\bar{E}(\beta) := \frac{E(\beta)}{N}, \quad (20)$$

which tells the average energy per a constituent of the acceleration surface.

The results of our model relevant to our comments on Verlinde's paper are the following: [6]

$$\lim_{N \rightarrow \infty} \bar{E}(T) = 0, \quad \forall T < T_C, \quad (21a)$$

$$\bar{E}(T_C) = k_B T_C \ln 2, \quad (21b)$$

$$\left. \frac{d\bar{E}(T)}{dT} \right|_{T=T_C} = \frac{1}{6} (\ln 2)^2 N + O(1), \quad (21c)$$

$$S(E) = \frac{2\pi k_B c}{\hbar a} E \quad \forall E \leq 2Nk_B T_C. \quad (21d)$$

Eq. (21a) implies that in the large N limit the microscopic quantum black holes on the acceleration surface are, in average, in vacuum, when $T < T_C$, whereas Eq. (21c) implies that when $T \approx T_C$, the curve $\bar{E} = \bar{E}(T)$ is practically vertical (See Fig. 1). The verticality of the curve $\bar{E} = \bar{E}(T)$, when $T \approx T_C$ indicates a *phase transition* at the temperature $T = T_C$. A detailed analysis shows that during this phase transition the quantum black holes on the acceleration surface jump, in average, from the vacuum, where $n = 0$, to the second excited states, where $n = 2$. When $T \gg T_C$, one finds that $E \approx Nk_B T$ as a very good approximation. [6]

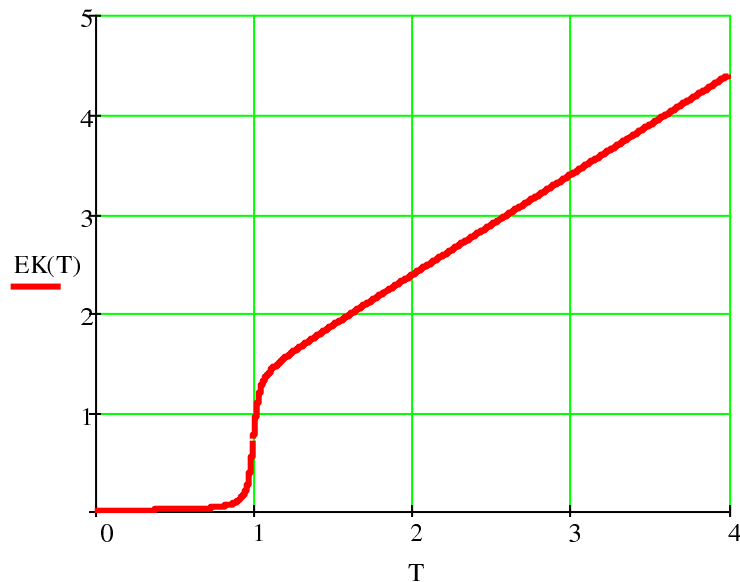


FIG. 1: The average energy $\bar{E} (= EK(T))$ of an acceleration surface per a hole as a function of the absolute temperature T , when the number of the holes on the surface is $N = 100$. The absolute temperature T has been expressed in the units of T_C , and the average energy \bar{E} in the units of $k_B T_C$. We have chosen $\alpha = 2 \ln 2$. If $T < T_C$, \bar{E} is effectively zero, which means that the black holes on the acceleration surface are in vacuum. When $T = T_C$, the curve $\bar{E} = \bar{E}(T)$ is practically vertical, which indicates a phase transition at the temperature $T = T_C$. During this phase transition the black holes on the acceleration surface are excited from the vacuum to the second excited states. The latent heat per a hole corresponding to this phase transition is $\bar{L} = 2(\ln 2)k_B T_C \approx 1.4k_B T_C$. When $T > T_C$, the curve $\bar{E} = \bar{E}(T)$ is approximately linear.

The fact that the black holes constituting the acceleration surface are in vacuum, whenever $T < T_C$ implies that the characteristic temperature T_C is the lowest possible temperature which an observer at rest with respect to the acceleration surface may measure, and therefore T_C may be identified as the Unruh temperature T_U of Eq. (3)

measured by an accelerated observer. Indeed, the characteristic temperature T_C coincides with the Unruh temperature T_U , if we fix the undetermined constant α in Eq. (6) as:

$$\alpha = 2 \ln 2. \quad (22)$$

So we see that our model predicts the Unruh effect: When the matter fields are in thermal equilibrium with spacetime, an accelerated observer will detect thermal radiation with a characteristic temperature, which is proportional to the proper acceleration a of the observer.

Eq. (21d) holds with the choice of Eq. (22) for the constant α . Using Eqs. (21b) and (21d) one finds that at the Unruh temperature $T_U = T_C$ the entropy S of an acceleration surface may be written as:

$$S(T_U) = N k_B \ln 2. \quad (23)$$

Because the relationship between H , the number of bits of information stored on a system, and its entropy S is, in general:

$$H = \frac{S}{k_B \ln 2}, \quad (24)$$

we find that the amount of information carried by the acceleration surface at the Unruh temperature T_U is:

$$H = N. \quad (25)$$

In other words, each quantum black hole on the acceleration surface carries, in average, exactly one bit of information. This is a very nice result, and using Eq. (11) we observe that at the Unruh temperature T_U the relationship between the area A and the integer N is:

$$N = \frac{1}{2 \ln 2} \frac{A c^3}{G \hbar}. \quad (26)$$

Moreover, Eq. (21b) implies that at the Unruh temperature T_U the energy of the acceleration surface is:

$$E = N k_B T_U \ln 2. \quad (27)$$

As one may observe, up to a numerical coefficient $2 \ln 2$, Eqs. (26) and (27), respectively, are identical to Eqs. (1) and (2).

In these notes we have shown that Eqs. (1) and (2) which, in Verlinde's paper, [1] constitute a starting point for his derivation of Newton's universal law of gravitation from the thermodynamical properties of spacetime follow, up to an unimportant numerical factor, from a specific microscopic model of spacetime. [4–6] In that model spacetime is assumed to be a graph with objects called as "microscopic quantum black holes" on its vertices. The attention is focused at the so called acceleration surfaces, and each quantum black hole on an acceleration surface is assumed to contribute to the surface an area which, up to a certain numerical coefficient, is an integer times the square of the Planck length. According to our model the Unruh temperature T_U of an accelerated observer is the lowest possible temperature which an accelerated observer may measure, provided that the matter fields are, from the point of view of the observer, in thermal equilibrium with spacetime. At the Unruh temperature the acceleration surface performs a phase transition, where the quantum black holes on the surface jump, in average, from the vacuum to the second excited states. Eqs. (1) and (2) hold in the Unruh temperature, as it was conjectured by Verlinde.

The results obtainable from our model provide support for Verlinde's results. Indeed, it is possible to construct a concrete microscopic model of spacetime, where equations acting as a starting point of Verlinde's approach follow as special cases. However, it should be noted that the logic in Verlinde's paper and in our model is somewhat different: In Verlinde's paper one assumes that the information and the energy carried by the constituents of a two-sphere

surrounding a point-like mass M obey, in the Unruh temperature, Eqs. (1) and (2). An identification of Mc^2 as the total energy associated with that two-sphere implied, through Eqs. (1) and (2), Newton's universal law of gravitation. In our model, in turn, one defines energy as an entirely macroscopic concept. In that definition a certain relationship between the total energy, proper acceleration and the area of an acceleration surface is postulated and the postulate in question implies, in the Newtonian limit, Newton's law of gravitation. In the microscopic level the postulate implies, at the Unruh temperature and up to a certain numerical coefficient, Eqs. (1) and (2) for the constituents of the two-sphere.

Taken as a whole, our model and Verlinde's approach may be seen as complementary: In Verlinde's approach Newton's universal law of gravitation as a consequence of certain thermal and entropic properties of the constituents of spacetime, whereas in our model those properties are, in a sense described above, necessary consequences of Newton's law of gravitation. If the idea of gravity as an emergent effect, rather than as a fundamental force turns out to be correct, we may currently be in a somewhat similar position as were the founders of quantum mechanics and atomic physics about 100 years ago. Instead of attempting to understand the microstructure of matter, however, we should, in this time, attempt to understand the microstructure of spacetime itself. It remains to be seen, whether any features of our still very preliminary microscopic model of spacetime will survive in more advanced models. Be that as it may, exciting times will lie ahead.

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