Testing Quantum Gravity with a Single Quantum System

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Until recently, table-top tests of quantum gravity (QG) were thought to be practically impossible. However, due to a radical new approach to testing QG that uses principles of quantum information theory (QIT) and quantum technology, such tests now seem, remarkably, within sight. In particular, a promising test has been proposed where the generation of entanglement between two massive quantum systems, both in a superposition of two locations, would provide evidence of QG. In QIT, quantum information can be encoded in discrete variables, such as qubits, or continuous variables. The latter approach, called continuous-variable QIT (CVQIT), is extremely powerful as it has been very effective in applying QIT to quantum field theory. Here we apply CVQIT to QG, and show that another signature of QG would be the creation of non-Gaussianity, a continuous-variable resource that is necessary for universal quantum computation. In contrast to entanglement, non-Gaussianity can be applied to a single rather than multi-partite quantum system, and does not rely on local interactions. We use these attributes to describe a table-top test of QG that is based on just a single quantum system in a single location.

Shortly after Einstein formulated general relativity (GR) he wondered how quantum theory (QT) would modify it.[1] Yet, over a hundred years later, there is still no consensus on how these two fundamental theories should be unified. The conventional approach is to apply the principles of QT to gravity, resulting in a quantum gravity (QG) theory, such as string theory or loop QG. However, since it is not as straightforward to apply QT as compared to the other fundamental forces, an alternative class of unifying theories has been developed, classical gravity (CG) theories, where matter is quantized but gravity remains fundamentally classical.

The hope has been that theoretical study alone would lead us to how GR and QT are unified in nature. However, the fact that there are several proposals illustrates that this is unlikely to happen and that experimental intervention is required. Until recently, the common view was that there is little hope of laboratory tests of QG since we need to probe GR near a small length scale, the Planck length, where QT effects of spacetime become relevant, but for which we would likely need to build a Milky-Way-sized particle accelerator.[2,3]

However, there is another important scale, the Planck mass scale, where gravitational effects of massive quantum systems become relevant, allowing us, in particular, to distinguish QG from CG.[4] This mass scale should be within reach soon in laboratory settings due to the rapidly developing field of quantum technology, which is based on concepts of quantum information theory (QIT). This has led to promising proposals being recently developed by Bose et al.,[5] Marletto and one of us,[6] for an experiment to detect evidence of QG using techniques of QIT and quantum technology. In this Bose-Marletto-Vedral (BMV) proposal, the creation of entanglement between two microspheres, each in a superposition of two locations, is used as a witness of QG. Due to the strength of this effect, and the promise of mesoscopic superposition states,[7,8] it is anticipated that this QIT-inspired experiment will be possible in the near future.

The cornerstone of QIT is quantum computing, and a necessary condition for universal quantum computation is that there is entanglement.[9–12,13] The BMV proposal, therefore, suggests that only a quantum, not classical theory of gravity could be used to carry out universal quantum computation, as we might naturally anticipate.

Conventionally, quantum information is encoded in quantum systems with a discrete, finite number of degrees of freedom, such as qubits. However, as well as discrete variables, it is also possible to encode quantum information in degrees of freedom with a continuous spectrum, such as the quantized modes of a quantum field. This latter approach is known as continuous-variable QIT (CVQIT),[14,15] and is the "analog" version of QIT compared to the more conventional "digital" approach. In CVQIT, a necessary condition for universal quantum computation is that there are non-Gaussian states or operations.[16,17] It is possible, therefore, that when working in CVQIT, only QG and not CG can induce non-Gaussianity in the quantum field of matter. We will show that this is indeed the case and that non-Gaussianity can be used as a witness of QG.

This new witness has a number of advantages over an entanglement witness. For example, while entanglement requires tests based on multi-partite systems, the non-Gaussianity witness allows for tests of QG that are based on just a single quantum system, potentially simplifying experimental tests. Furthermore, while entanglement can be created by a classical interaction if this involves the quantum systems directly interacting with each other,[5,6,18] we will show that this restriction does not apply to non-Gaussianity of quantum fields. Using these advantages, we will describe a simple test of QG that is based on just a single quantum system and which is not in a superposition of locations.

Consider a free, real scalar quantum field. The Hamiltonian of this system can be written as a collection of quantum simple harmonic oscillators: $\hat{H} = \sum_k \hbar\omega_k[\hat{a}_k^\dagger \hat{a}_k + 1/2]$,[19] where $\hat{a}_k^\dagger$ and $\hat{a}_k$ are creation and annihilation operators of mode k; ω_k are the angular frequencies; and we have assumed a discrete mode spectrum for simplicity. For each oscillator we can associate position and momentum-like operators, $\hat{x}_k := \hat{a}_k + \hat{a}_k^\dagger$ and $\hat{p}_k := i(\hat{a}_k^\dagger - \hat{a})$, known as quadrature operators, which are observables with a continuous eigenspectra: $\hat{x}_k|x\rangle_k = x_k|x\rangle_k$ and $\hat{p}_k|p\rangle_k = p_k|p\rangle_k$. The quadrature eigenvalues, x_k and p_k, can be used as continuous variables to describe the entire quantum field system, and we can view this as a continuous phase space on which we encode our quantum information.[20] This approach to encoding quantum information can also be straightforwardly extended to general bosonic and fermionic quantum fields.[21–24]

Rather than describing this system using a density operator $\hat{\rho}$, an equivalent representation is provided by the Wigner function,[25] which is a quasi-probability distribution defined over phase space, analogous to probability distributions used in classical statistical mechanics. For example, for a single-mode, the Wigner function can be obtained through:[26]

$$W_{\hat{\rho}}(x,p) = \frac{1}{2\pi}\int dy\, e^{-iyp}\langle x+y|\hat{\rho}|x-y\rangle. \quad (1)$$

This is a quasi-probability distribution since, although it takes on real values and is normalized to unity, it can also take on negative values. The states for which the Wigner function takes on negative values, therefore, have no classical counterpart, and are considered to be highly non-classical states.[27]

The only states that have negative Wigner functions are non-Gaussian states, such as Fock states or Schrödinger cat states.[28] Gaussian states, on the other hand, such as coherent states, squeezed states and thermal states, have only positive Wigner functions.[20,29] Here a Gaussian state of a quantum field is defined as one for which its Wigner function is a Gaussian distribution.[20] Such a state is fully characterized by the first and second moments of the quadrature operators, or, equivalently, by the one and two-point correlation functions of the quantum field.[20,30,31]

The classification of Gaussian and non-Gaussian states is very important in CVQIT. For example, universal quantum computation with pure states is only possible with non-Gaussian states or operations,[16,17] while Gaussian states and operations can be efficiently simulated on a classical computer.[32–35] Furthermore, non-Gaussian states or operations are required for violation of Bell inequalities.[36–43] These, and additional examples, such as implementing entanglement distillation,[35] have led to non-Gaussianity being classified as a QIT resource for which measures and witnesses have been derived,[44–53] just as for entanglement.

Given the significance of Gaussian and non-Gaussian states in CVQIT, it is important to distinguish the type of Hamiltonians that can create such states: a Hamiltonian that is at most quadratic in quadratures, or equivalently in annihilation and creation operators, can only ever map a Gaussian state to another Gaussian state.[23,54–58] That is, the Hamiltonian must be of the form:

$$\hat{H} = \sum_k \boldsymbol{\lambda}_k(t)\hat{\boldsymbol{x}}_k + \sum_{k,l}\hat{\boldsymbol{x}}_k^T\boldsymbol{\mu}_{kl}(t)\hat{\boldsymbol{x}}_l, \quad (2)$$

where $\hat{\boldsymbol{x}}_k^T := (\hat{x}_k, \hat{p}_k)$, and $\boldsymbol{\lambda}_k(t)$ and $\boldsymbol{\mu}_{kl}(t)$ are 2×1 and 2×2 real-valued matrices of arbitrary functions of time. Although we have assumed a discrete, finite mode spectrum here for simplicity, the extension to infinite and continuous modes is straightforward.[30,31,59,60]

The Hamiltonian (2) preserves Gaussianity since it is associated with a general Bogoliubov transformation, which is a linear transformation of the quadratures (and, therefore, phase space) that preserves their commutation relations.[20] Any other Hamiltonian, i.e. one that is cubic or higher-order in quantum operators, will in general create non-Gaussianity.[16,17,20]

Note that a free quantum field has a Hamiltonian that is of the form (2) since it only contains the kinetic and mass terms, and so is necessarily quadratic in the field. For example, the Hamiltonian for a real scalar quantum field $\hat{\phi}$ is:[19]

$$\hat{H} = \frac{1}{2}\int d^3\boldsymbol{r}\left[(\partial_t\hat{\phi})^2 + (\nabla\hat{\phi})^2 + m^2\hat{\phi}^2\right] \quad (3)$$

where m is the mass of the field. Expanding the field in annihilation and creation operators $\hat{\phi} = \sum_k[u_k(t)\hat{a}_k + v(t)\hat{a}_k^\dagger]$, results in a Hamiltonian of the form (2).[19]

Now consider interacting this quantum field with a classical entity $\mathcal{G}$, which could depend on space and time. Taking the classical interaction to not induce quantum self-interactions of $\hat{\phi}$, then $\mathcal{G}$ and $\hat{\phi}$ can only interact through Hamiltonian terms that are linear or quadratic in $\hat{\phi}$.[61] For example, the classical interaction could occur through a Hamiltonian term such as $(\nabla\hat{\phi})^2 f[\mathcal{G}]$, where f is a real functional of $\mathcal{G}$. Then, expanding $\hat{\phi}$ in annihilation and creation operators, we would still find a Hamiltonian that is of the form (2), with $\mathcal{G}$ just absorbed into the time-dependent coupling constants. That is, the Hamiltonian of the classical interaction preserves Gaussianity, and this would apply to a classical interaction with any type of quantum field, not just a real scalar field $\hat{\phi}$.

In contrast, if we quantize $\mathcal{G}$, such that we interact $\hat{\phi}$ (or any other type of quantum field) with a quantum entity, then it is possible for the resulting Hamiltonian to be higher order than quadratic in quantum operators, and thus induce non-Gaussianity. Therefore, as long as we can neglect all other interactions, any sign of the creation of non-Gaussianity in the state of a quantum field would be evidence of a quantum interaction.

Due to the universal coupling of gravity, we can apply this argument to determine whether gravity obeys a quantum or classical theory. This is because, if we are working in a situation where all other interactions can be neglected, then the matter Hamiltonian only contains the kinetic and mass terms of the matter quantum field, to which gravity couples. For example, if, for simplicity, we used a real scalar field $\hat{\phi}$ to describe matter and ignored a possible quadratic Ricci scalar coupling term,[62] then the Hamiltonian of CG would be (3) but with $\sqrt{\mathfrak{g}}$ multiplying each term, where $\mathfrak{g}$ is the determinant of the gravitational 3-metric.[63–65,66] This Hamiltonian would preserve Gaussianity and, therefore, any sign of non-Gaussianity would be evidence of a quantum theory of gravity. Clearly this argument also applies in the relativistic weak-field limit of gravity, as well as the non-relativistic Newtonian limit.[67] In fact, since all known fundamental interactions with matter have interaction Hamiltonians with terms that are quadratic in matter fields,[19,64,68] non-Gaussianity could also be used to evidence that these are indeed quantum interactions.

We now consider an experiment of QG that uses our non-Gaussianity witness. This experiment is based on a single Bose-Einstein condensate (BEC) that is in a single location, and is an experiment to which an entanglement witness of QG could not be applied.

A Bose gas can be described by a non-relativistic scalar quantum field $\hat{\Psi}(\boldsymbol{r})$, which creates an atom at position $\boldsymbol{r}$.[69] Assuming that we are working at low enough temperatures such that the ground-state is macroscopically occupied, we neglect the thermal component of the gas and take $\hat{\Psi}(\boldsymbol{r}) \approx \psi(\boldsymbol{r})\hat{a}$,[69] where $\psi(\boldsymbol{r})$ is the wave-function of a condensed atom, and $\hat{a}$ is the annihilation operator for the condensate. The identical atoms are then all in the same state, have the same wavefunction, and are equally delocalized across the BEC.

These atoms will interact gravitationally with each other, and in the appropriate non-relativistic (Newtonian) limit of gravity, the interaction Hamiltonian density contains the terms $\hat{\Psi}^\dagger\hat{\Psi}\hat{\Phi}$ or $\hat{\Psi}^\dagger\hat{\Psi}\Phi$, where Φ is Newton's gravitational potential, depending on whether we have QG or CG. Solving the quantized version of Poisson's equation, we have $\hat{\Phi}(\boldsymbol{r}) = -Gm\int d\boldsymbol{r'}\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}^\dagger(\boldsymbol{r})/|\boldsymbol{r}-\boldsymbol{r'}|$, where G is the gravitational constant and m is the mass of the atoms. In contrast, depending on the chosen CG theory, Φ is a certain quantum average of this expression (for example, in the Schrödinger-Newton equations $\Phi = \langle\hat{\Phi}\rangle$[12,70,71]). The corresponding interaction Hamiltonians can then be written as:[72]

$$\hat{H}_{QG} = \frac{1}{2}\lambda_{QG}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}, \tag{4}$$

$$\hat{H}_{CG} = \lambda_{CG}[\Psi]\hat{a}^\dagger\hat{a}, \tag{5}$$

where:

$$\lambda_{QG} := -Gm^2\int d^3\boldsymbol{r}\, d^3\boldsymbol{r'}\frac{|\psi(\boldsymbol{r'})|^2|\psi(\boldsymbol{r})|^2}{|\boldsymbol{r}-\boldsymbol{r'}|}, \tag{6}$$

$$\lambda_{CG}[\Psi](t) := Gm\int d^3\boldsymbol{r}|\psi(\boldsymbol{r})|^2\Phi[\Psi](t,\boldsymbol{r}), \tag{7}$$

and we have made explicit that the classical potential Φ can be a functional of the quantum state Ψ of the BEC.[73,74,75] The QG interaction Hamiltonian (4) can also be derived as the non-relativistic limit of linearized QG where we start with an interaction Hamiltonian density of the form $\hat{\Psi}^\dagger\hat{\Psi}\hat{h}_{\mu\nu}$, with $\hat{h}_{\mu\nu}$ the quantized gravitational perturbation, consider the four-point Feynman diagram with a single virtual graviton propagator, and then effectively integrate out gravitational degrees of freedom.[76–80]

From the Hamiltonians (4)-(5), it is clear that only QG can induce non-Gaussianity in the quantum state of the BEC field and that, in contrast, entanglement cannot be used as a witness since this is just a single-mode system.[81,82] In fact, the QG Hamiltonian is analogous to the Kerr interaction, which induces non-Gaussianity in quantum optics.[83]

In order to detect evidence of non-Gaussianity in the system, we consider measurements of high-order cumulants.[84] For a Gaussian distribution, all cumulants higher than second order vanish and, therefore, a non-zero value of such cumulants is a signature of non-Gaussianity. Here we concentrate on the fourth-order cumulant κ_4, sine κ_3 is also zero for a symmetric non-Gaussian distribution. Defining a generalized quadrature as $\hat{q}(\varphi) = \hat{a}e^{-i\varphi} + \hat{a}^\dagger e^{i\varphi}$, we have:

$$\kappa_4 := \langle\hat{q}^4\rangle - 4\langle\hat{q}\rangle\langle\hat{q}^3\rangle - 3\langle\hat{q}^2\rangle^2 + 12\langle\hat{q}^2\rangle\langle\hat{q}\rangle^2 - 6\langle\hat{q}\rangle^4. \tag{8}$$

In an experiment, only a finite sample can be used to estimate κ_4 and we desire unbiased estimators, which are the k statistics: $\langle k_n\rangle = \kappa_n$.[85] The noise in the estimation of κ_4 is then the standard deviation of k_4,[86] such that the signal-to-noise ratio (SNR) for the measurement is $\mathrm{SNR} = |\kappa_4|/\sqrt{\mathrm{Var}(k_4)}$, where, for a large number of independent measurements $\mathcal{M}$, $\mathrm{Var}(k_4) \propto 1/\mathcal{M}$.

In order to make the SNR as large as possible, we use quantum metrology, where highly quantum states can improve the estimation of parameters that are not associated with observables.[87] This is also effectively used in the BMV proposal where the initial quantum states are N00N-like states.[88–91] However, rather than using a N00N state, here we consider a squeezed state for the BEC, which is a Gaussian state that often provides similar performance in quantum metrology to N00N states. In this case, in the limit that $\chi := |\lambda_{QG}|/\hbar$ is small and that the number of atoms N of the BEC is large, the SNR can be of order $\chi t N^2\sqrt{\mathcal{M}}$, where t is the interaction time.[92] Assuming a spherical BEC of mass M and radius R, we have:[93,94]

$$\chi t N^2 \equiv \sqrt{\frac{2}{\pi}}\frac{GM^2 t}{\hbar R}, \tag{9}$$

which is $t/\hbar$ times the gravitational self-energy of the BEC.[93] Note that, with the replacement of R with d, and neglecting the numerical factor, this expression is the same as the relative phase generated in the BMV proposal between the two microspheres that are separated by the smallest possible distance d that, when ignoring all other distances, leads to the entanglement.[5,6] It is demonstrated in the BMV proposal that a value of order one for this phase is achieved when $d = 200\,\mu$m, $t = 2\,$s and $M = 10^{-14}\,$kg, which is considered to be not too far away from current experiments.[5] However, since the SNR here scales with $\sqrt{\mathcal{M}}$, we can lower the total mass required by increasing the number of measurements. For example, to achieve an SNR of 5 for a ^{133}Cs BEC, we could use $R = 200\,\mu$m, $t = 2\,$s and $M = 10^{-15}\,$kg with around 40,000 measurements. Such a mass corresponds to around 4×10^9 atoms, which is a little larger than what has been achieved so far: in 1998 a ^{1}H BEC was created with over 10^9 atoms,[95] and in 2006 a ^{23}Na BEC had over 10^8 atoms.[96] However, the number of atoms required can be reduced by further increasing $\mathcal{M}$.

An experimental implementation of this scheme would be to use a spin-1 BEC where the $m_F = \pm1$ states are prepared in large coherent states and then a magnetic field is used to drive spin-mixing collisions to generate a quadrature squeezed state in the $m_F = 0$ condensate.[97,98,99] Then, after the system has evolved for a time t, we could apply the reverse squeezing process and measure the non-Gaussianity of the BEC field. In order to detect non-Gaussianity, a homodyne or heterodyne scheme could be used,[100–103] where moments up to fourth order are looked for in the intensity difference, providing a direct map for obtaining κ_4.[83,104] This would require single-atom counting in a quantum gas with high efficiency on small length scales.[105] Alternatively, the Wigner function of the BEC could be determined either through full state tomography with projective measurements,[100,106–109] or through 'direct' measurement with weak measurements of the position quadrature and projective measurements of the momentum quadrature[110–113] (this has so far been achieved with photons,[110,114–117] but could be extended to atoms [110]).

Generating the highly non-classical initial state, would be a great challenge, similar to the challenge required in creating the N00N states of the BMV proposal. Other experimental challenges would include suppressing all possible (non-Gaussian) noise. In particular, we would have to make sure that we can neglect or distinguish the electromagnetic interactions between the atoms compared to gravity. A BEC is very dilute and the atoms are neutral overall, but there are still, in general, weak electromagnetic interactions between the atoms. However, depending on the species of atom, these can generically be suppressed using optical or magnetic Feshbach resonances.[69] Here we have assumed a ^{133}Cs BEC since it has the broadest and strongest magnetic Feshbach resonance for bosonic atoms, with which we could achieve, in principle, zero electromagnetic interactions.[118] As well as being suppressible, electromagnetic interactions could also be distinguished from gravity through, for example, the fact that the effective strength of the electromagnetic interaction scales as $1/R^3$, whereas gravity scales as $1/R$.[119]

Discussion. We have argued that the production or change in non-Gaussianity in the state of the quantum field of matter would be sufficient evidence of QG, and have illustrated how this could be used in a test that is based on just a single-well BEC. The size of the effect in the BEC experiment appears to be similar to that observed in the BMV proposal, see (9). This illustrates how the experiment is related to the Planck mass since, using (9), we can write the SNR for one measurement as:[4,120]

$$\frac{M}{M_P}\frac{\delta\tau}{t_P}, \tag{10}$$

where M_P is the Planck mass, t_P is the Planck time, and $\delta\tau := \sqrt{2/\pi}GMt/(Rc^2)$. This expression can also be derived by dividing the BEC it in two halves, considering the gravitational interaction of one with the other and the time dilation $\delta\tau$ induced in GR in the centre of each half. If we fix the SNR of one measurement, then (10) illustrates that as M gets closer to M_P, it seems that we can probe more minute gravitational field intensities and thus further access its possible quantum properties.

As well as proving that gravity obeys a quantum theory if non-Gaussianity is observed, the BEC experiment could also teach us something about QG. For example, the Newtonian interaction used to predict the size of the non-Gaussianity signal derives from the gravity-matter interaction of GR.[64,121] A measurement of non-Gaussianity would therefore, for instance, provide evidence that the determinant $\mathfrak{g}$ of the 3-metric is quantized and that spatial volume is a quantum variable. This of course would not, however, point us to the appropriate quantization procedure to be followed. Note that in loop QG, volume is a quantum variable with discrete spectrum.

As mentioned above, a classical interaction can in fact create entanglement if this involves the quantum systems directly interacting with each other.[5,6,18] For example, consider two BECs that are in the two spatial arms of a double-well potential. In the two-mode approximation, we can write the full quantum field of the atoms as $\hat{\Psi}(\boldsymbol{r}) = \psi_L(\boldsymbol{r})\hat{a}_L + \psi_R(\boldsymbol{r})\hat{a}_R$ where $\hat{a}_L$ and $\hat{a}_R$ respectively destroy an atom in the left and right well, and ψ_L and ψ_R are the corresponding wavefunctions.[69,122] In the case of CG, and taking the Newtonian approximation for simplicity, there will, in principle, be terms of the form $\lambda_{LR}\hat{a}_L^\dagger\hat{a}_R + h.c.$, in the Hamiltonian, where

$\lambda_{LR} := m \int d^3\boldsymbol{r} \psi_L^*(\boldsymbol{r})\psi_R(\boldsymbol{r})\Phi[\Psi](\boldsymbol{r}, t)$. These are beam-splitting terms such that, if λ_{LR} is non-zero due to, for example, the wavefunctions overlapping, and either BEC is in a non-classical state, then the terms will induce entanglement between the BECs.[123] However, since these are quadratic terms, they will not induce non-Gaussianity, and so, although a direct classical interaction with matter can create entanglement, it cannot create non-Gaussianity in the quantum field of matter.

Note that here we are working with "mode" entanglement, i.e. entanglement between modes of a quantum field. If instead we attempted to use a first-quantization picture and describe the full system using a many-body wave-function, then it is possible to argue that the initial state of the full system is already entangled and that CG is not creating entanglement in this picture.[124–126] This is because there is so-called "particle" entanglement before and after the effective CG beam splitter.[127,128] For example, the initial state could be $|\alpha\rangle_L|\xi\rangle_R$, with $|\alpha\rangle$ a coherent state and $|\xi\rangle$ a squeezed state, which, in a quasi first-quantized picture, is particle entangled but not mode entangled.[128] In contrast, in this first-quantized picture, which can only be consistently applied in the case of Newtonian gravity, it is possible for CG to create non-Gaussian "particle" Wigner functions. For example, the many-body wavefunction of our single-well BEC experiment could start of Gaussian but become non-Gaussian under CG.[129] Interestingly then it would seem that, in a first-quantized Newtonian picture, CG could not be used to carry out universal quantum computation because it cannot create particle entanglement, whereas, in the second quantization picture with CVQIT this is, most generally, because it cannot create "mode" non-Gaussianity.

Above we have defined a classical interaction as an interaction with an entity $\mathcal{G}$ that takes on real and well-defined values, such as the gravitational field of GR. We now consider whether non-Gaussianity can also be used to distinguish other, more general, non-quantum interactions from their quantized counterparts. First we consider that $\mathcal{G}$ takes on complex values. This allows for the possibility that, most generally, the interaction can give rise to a Hamiltonian of the form (2) but where now the coupling constants $\boldsymbol{\lambda}_k$ and $\boldsymbol{\mu}_k$ are complex-valued. Although this, in general, leads to a non-Hermitian Hamiltonian, a matter state with a Gaussian Wigner function will continue to have a Wigner function of Gaussian form,[130–132,133] and so non-Gaussianity can also distinguish this interaction from a quantum interaction.

Another possibility is that $\mathcal{G}$ could be a non-quantum but stochastic quantity. For example, a relativistic theory of gravity coupled to matter has been proposed where the non-quantum gravitational field is stochastic.[134] It is found that gravity and matter interact through a Gaussian completely-positive (CP) channel, and so non-Gaussianity should also rule out this non-quantum theory of gravity.[135] More generally, interacting a stochastic entity $\mathcal{G}$ with a quantum field will still result in a Gaussian state of the quantum field remaining Gaussian if we now broaden our definition of a Gaussian state to include states that are a statistical mixture of pure states with Gaussian Wigner functions (the so-called Gaussian convex hull [52,136]). This is because a Gaussian state evolves to a state in the Gaussian convex hull if there is a combination of Gaussian operations and statistical randomization, i.e. stochasticity.[53,137]

The preservation of this broader definition of Gaussianity also applies if the entity $\mathcal{G}$ is both stochastic and complex-valued. However, in this case the norm will not, in general, be preserved, and so we have a mixture of unnormalized states with Wigner functions of Gaussian form.[130–132] To make sure that the theory is norm-preserving, the physical state vector can be redefined as $|\Psi\rangle/||\Psi\rangle|$, which then allows for a convex mixture of properly normalized Gaussian states. However, this, in general, results in a theory that is non-linear in the density matrix, leading to superliminal signalling.[138] Such an issue is also found in objective-collapse theories and, to rectify it, a new higher-order process is applied to the evolution of the quantum system, which would here be associated with a quantum (self) interaction of matter i.e., a new force.[139,140,137] This new quantum process can, in general, induce non-Gaussianity. However, in the conventional case that the noise term of the objective-collapse theory has a Gaussian profile and is anti-Hermitian (equivalent here to only the imaginary component of $\mathcal{G}$ being stochastic), Gaussianity in the matter field is still preserved. This is also analogous to a continuous-time measurement being performed on matter by the stochastic entity $\mathcal{G}$, which could be a stochastic gravitational field, and the new quantum self-interaction.[141,142,143,144]

As well as being used in continuous-variable systems, the Wigner function can also be defined for discrete systems.[145–148] The fact that negative Wigner functions are required for universal quantum computation,[149,150] might imply that such a property of quantum matter could be a more general indicator of QG than non-Gaussianity. This is also related to contextuality, suggesting that this property of quantum theory could be being tested by our non-Gaussianity witness in determining whether gravity is classical or quantum.

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3 Rather than using laboratory settings, such as a particle accelerator, it may also be possible to probe general Planck length/energy scale effects of QG through cosmological and astrophysical studies.[151] However, these lack the control of laboratory settings and, because of this, the predicted QG effects tend to also be explainable by classical theories of gravity.[152] There are also proposals for tests of specific phenomenological models of QG, rather than fundamental theories of QG, that are designed to show possible Planck length/energy scale effects, such as violations of Lorentz-invariance, at accessible levels in particular experiments [153].

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13 Note that, although entanglement is a necessary condition, it is not sufficient for universal computation, or to guarantee a computational speedup over classical computation [149,150,154].

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75 Note that, in the Newtonian limit, the QG Hamiltonian (4) appears as an effective quantum self-interaction of matter. In contrast, the CG Hamiltonian cannot induce quantum self-interactions of matter (5). This is because, in the absence of all other interactions, the CG Hamiltonian only contains the gravitational field coupled to the kinetic and mass terms of matter, which are quadratic in the matter field.

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82 As indicated in (7), CG theories can be non-linear in the evolution of the state vector.[73] However, even if it is non-linear, it is still a Gaussian process since a CG theory must be quadratic in matter fields: the non-linearity means that, although we know a Gaussian state will remain a Gaussian state, we may not be able to analytically determine the specific evolution (see Appendix E for more detail).

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When both the real and imaginary components are stochastic and there is a process related to quantum self-interaction, then the resulting theory is closely related to a continuous-time measurement being performed by the two interactions but now with a feedback mechanism.[141] Weak measurements with local feedback operations are also known to induce entanglement if the measurement is a joint measurement [175,176].

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$$-i\hat{\mathcal{H}}[\tilde{\mathcal{G}}] = \sqrt{\lambda}(\hat{A} - \langle\hat{A}\rangle)\tilde{\mathcal{G}} - \frac{1}{2}\lambda(\hat{A} - \langle\hat{A}\rangle)^2, \qquad (11)$$

which is closely connected to the evolution of the state vector found in objective-collapse theories such as CSL and Diósi-Penrose [139,177,178].

Appendix A: Non-Gaussianity in quantum gravity

The way in which matter and gravity interact in GR is described by the matter action S, which can be derived from the specific Lagrangian density $\mathcal{L}(x)$ for the matter field:

$$S = \int d^4x \mathcal{L}(x). \tag{A1}$$

For example, neglecting all other interactions (which automatically includes any self-interactions), then in the metric or tetrad formulations of GR, the respective Lagrangian densities for a real scalar ϕ, spin-1/2 ψ, and spin-1 field A_μ, are:[64,179]

$$\mathcal{L}_\phi = \frac{1}{2}\sqrt{g}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - (m^2 + \varepsilon R)\phi^2] \tag{A2}$$

$$\equiv \frac{1}{2}e[\eta^{ab}e^\mu_a\partial_\mu\phi e^\nu_b\partial_\nu\phi - (m^2 + \varepsilon R)\phi^2], \tag{A3}$$

$$\mathcal{L}_\psi = \sqrt{g}\left(\frac{1}{2}i[\overline{\psi}\gamma^\mu\nabla_\mu\psi - (\nabla_\mu\overline{\psi})\gamma^\mu\psi] - m\overline{\psi}\psi\right), \tag{A4}$$

$$\mathcal{L}_A = -\frac{1}{4}\sqrt{g}g^{\mu\nu}g^{\nu\sigma}F_{\mu\rho}F_{\nu\sigma}, \tag{A5}$$

where $e^\mu_\alpha(x)$ are tetrads, the 'matrix square root' of the metric tensor: $g^{\mu\nu}(x) =: e^\mu_\alpha(x)e^\nu_\beta(x)\eta^{\alpha\beta}$, with μ labelling the general spacetime coordinate, a the local Lorentz spacetime, and $\eta^{\alpha\beta}$ is the Lorentz metric. Furthermore, $F_{\mu\nu} := \partial_\nu\mathcal{A}_\mu - \partial_\mu\mathcal{A}_\nu$ is the electromagnetic tensor; $\mathcal{A}_\mu$ is the electromagnetic four-potential; R is the Ricci scalar; ∇_μ is the covariant derivative; $\gamma^\mu := e^\mu_\alpha\gamma^\alpha$ are the curved space counterparts of the gamma (Dirac) γ matrices, which satisfy $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$; ε is a numerical factor which we set to zero for the rest of this Appendix for simplicity; and the chosen metric signature is $(-,+,+,+)$. Note that for a complex rather than real scalar field, we just replace terms with two copies of ϕ by one copy of ϕ^* and ϕ, e.g. $\partial_\mu\phi\partial_\nu\phi$ becomes $\partial_\mu\phi^*\partial_\nu\phi$.

We can also write corresponding Hamiltonian (constraint) densities for the above Lagrangian densities:[63–65]

$$\mathcal{H}_\phi = \frac{1}{2}\left(\frac{\pi^2}{\sqrt{\mathfrak{g}}} + \sqrt{\mathfrak{g}}\mathfrak{g}^{ab}\partial_a\phi\partial_b\phi + \sqrt{\mathfrak{g}}m^2\phi^2\right), \tag{A6}$$

$$\mathcal{H}_\psi = \frac{1}{2\sqrt{\mathfrak{g}}}E^a_j\left[i\zeta\tau^j\mathcal{D}_a\xi + \mathcal{D}_a(\zeta\tau^j\xi) + \frac{1}{2}iK^j_a\sigma\xi + c.c.\right], \tag{A7}$$

$$\mathcal{H}_A = \frac{1}{2\sqrt{\mathfrak{g}}}\mathfrak{g}_{ab}\left[\mathcal{E}^a\mathcal{E}^b + \mathcal{B}^a\mathcal{B}^b\right]. \tag{A8}$$

Here spacetime has been split into spatial slices and a time axis $M = \mathbb{R} \times \sigma$. Taking n^μ to be the normal vector field of the time slices σ, the tetrad can be written as $e^\mu_\alpha = E^\mu_\alpha - n^\mu n_\alpha$, with $\eta^{\alpha\beta}n_\alpha n_\beta = -1$ an internal unit timelike vector (which we may choose to be $n_\alpha = -\delta_{\alpha,0}$), so that E^μ_α is a triad, where $E^\mu_\alpha = (0, E^\mu_i)$ and we further define $E^\mu_i = (0, E^a_i)$ with $i, a = 1, 2, 3$. The conjugate momenta to the triad E^a_i is the chiral spin connection $A^i_a := \Gamma^i_a + K^i_a$, where $\Gamma^i_a = \Gamma_{ajk}\epsilon^{jki}$ and $K^i_a = K_{ab}E^{bi}$, with Γ_{ajk} the spin-connection and K_{ab} the extrinsic curvature. In (A6)-(A8), $\mathfrak{g}$ is then the determinant of the induced 3-metric $\mathfrak{g}_{ab} \equiv |\det(E)|E^j_aE^j_b$ on the spatial slices; $\pi := \sqrt{\mathfrak{g}}n^\mu\partial_\mu\phi$ is the momentum conjugate to ϕ; $\mathcal{E}^a := \sqrt{\mathfrak{g}}\mathfrak{g}^{ab}n^\mu F_{\mu b}$ is the electric field; $\mathcal{B}^a := \epsilon^{abc}F_{bc}$ is the magnetic field; τ_i are the generators of the Lie algebra su(2) with the convention $[\tau_i, \tau_j] = \epsilon_{ijk}\tau_k$; $\xi = \sqrt{\mathfrak{g}}\psi$, with ψ a Grassman-valued fermion field; ζ is the momentum conjugate to ξ; and $\mathcal{D}_a\xi := (\partial_a + \tau_jA^j_a)\xi$. For simplicity, we have also assumed that the scalar and fermionic fields are singlets under any internal group symmetry.

Since we have neglected all other interactions, the above Lagrangian and Hamiltonian densities are all necessary quadratic in matter fields as they then only consist of kinetic term and mass terms. This quadratic scaling of course applies to any spin field not just those considered above.[180–188] Therefore, if we quantize the matter fields but leave the gravitational degrees of freedom classical, we have a theory that preserves Gaussianity. However, if gravity obeys a quantum theory, then there must be some quantum operator associated with it, and, therefore, we must have a theory that has interactions involving three or more quantum operators and that thus induces non-Gaussianity. In the next two sections we also consider this argument in the weak-field and non-relativistic limits of gravity.

1. Weak-field limit

In the weak-field limit of gravity, we write $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $h_{\mu\nu}$ is a perturbation around a space-time background $\eta_{\mu\nu}$. In this case, the GR matter-gravity interaction Hamiltonian is:[189]

$$H_{int} = -\frac{1}{2}\int d^3\boldsymbol{r} T^{\mu\nu} h_{\mu\nu}, \tag{A9}$$

where

$$\Box h_{\mu\nu} = \frac{16\pi G}{c^4}\Big(\frac{1}{2}\eta_{\mu\nu}\eta^{\sigma\rho}T_{\sigma\rho} - T_{\mu\nu}\Big), \tag{A10}$$

with $\Box$ the d'Alembert operator, and $T^{\mu\nu}$ the stress-energy tensor for matter. The stress-energy tensor for a field of arbitrary spin in curved spacetime can be obtained by variation of the action with respect to the metric:[179]

$$T_{\mu\nu}(x) = \frac{2}{\sqrt{-g}}\frac{\partial S}{\partial g^{\mu\nu}(x)} \equiv \frac{e_{a\mu}(x)}{e}\frac{\delta S}{\delta e^{\mu}_{a}(x)}. \tag{A11}$$

For example, when neglecting all other interactions, for a real scalar, spin-1/2 and spin-1 field, the curved space stress-energy tensors are (before taking a weak-field limit):[179]

$$T^{\phi}_{\mu\nu} = (1-2\xi)\partial_\mu\phi\partial_\nu\phi + (2\xi - \frac{1}{2})g_{\mu\nu}g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi - 2\xi(\nabla_\mu\partial_\nu\phi)\phi + \frac{1}{2}\xi g_{\mu\nu}\phi\Box\phi \tag{A12}$$

$$- \xi\Big[R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}(1-\frac{3}{2}\xi)\Big]\phi^2 + \frac{1}{2}\Big[1-3\xi\Big]m^2 g_{\mu\nu}\phi^2, \tag{A13}$$

$$T^{\psi}_{\mu\nu} = \frac{1}{2}i[\overline{\psi}\gamma_{(\mu}\nabla_{\nu)}\psi - [\nabla_{(\mu}\overline{\psi}]\gamma_{\nu)}\psi], \tag{A14}$$

$$T^{A}_{\mu\nu} = \frac{1}{4}g_{\mu\nu}F^{\rho\sigma}F_{\rho\sigma} - F^{\rho}_{\mu}F_{\rho\nu}, \tag{A15}$$

where we have ignored any gauge fixing or ghost terms in $T^{A}_{\mu\nu}$.[179] Since we have neglected all other interactions, all stress-energy tensors are necessarily just quadratic in matter fields.

In a QG theory we add a hat to both $T_{\mu\nu}$ and $h_{\mu\nu}$. This then results in an interaction Hamiltonian that is cubic in field operators. For example, for a complex scalar field we have terms of the form $\hat{\phi}^\dagger\hat{\phi}\hat{h}_{\mu\nu}$, where we have suppressed any derivatives. On the other hand, for a CQ theory, the interaction Hamiltonian contains terms only quadratic in quantum field operators. For example, in the semi-classical theory of gravity,[190,191] with complex scalar matter fields, we have terms of the form $\hat{\phi}^\dagger\hat{\phi}h_{\mu\nu}$, where $h_{\mu\nu}$ is given by the expectation value of the right-hand side of (A10). Therefore, this weak-field limit of CG cannot produce or change quantum non-Gaussianity in the state of matter, whereas QG can, as expected from the general discussion of GR and QG in the previous section.

2. Newtonian limit

We now consider a Newtonian theory of gravity with matter quantized. This can be obtained by starting from Newton's theory and quantizing matter or from taking the non-relativistic limit of the above weak-field theories. For the latter, we consider only the components T_{00} and h_{00} in (A9) and (A10). This results in Poisson's equation:

$$\nabla^2\Phi(\boldsymbol{r}) = 4\pi G\rho(\boldsymbol{r}) \tag{A16}$$

$$\implies \Phi(\boldsymbol{r}) = -G\int d^3\boldsymbol{r'}\frac{\rho(\boldsymbol{r'})}{|\boldsymbol{r}-\boldsymbol{r'}|}, \tag{A17}$$

and the Newtonian interaction Hamiltonian:

$$H_{int} = \frac{1}{2}\int d^3\boldsymbol{r}\rho(\boldsymbol{r})\Phi(\boldsymbol{r}), \tag{A18}$$

where $\Phi := c^2 h_{00}/2$ is the Newtonian potential, and $\rho := T_{00}/c^2$ is the matter density. Irrespective of the spin of the field, ρ again contains two copies of the matter field, e.g., for a single non-relativistic scalar matter field Ψ, $\rho = m\Psi^*\Psi$.

The interaction Hamiltonians for quantum and classical Newtonian gravity (with quantized scalar matter fields) are then:

$$\hat{H}^{int}_{QG} = \frac{1}{2}m\int d^3\boldsymbol{r} : \hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r})\hat{\Phi}(\boldsymbol{r}) : \tag{A19}$$
$$= -\frac{1}{2}Gm^2\int d^3\boldsymbol{r'}d^3\boldsymbol{r}\frac{\hat{\Psi}^\dagger(\boldsymbol{r'})\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r'})\hat{\Psi}(\boldsymbol{r})}{|\boldsymbol{r}-\boldsymbol{r'}|},$$
$$\hat{H}^{int}_{CG} = m\int d^3\boldsymbol{r}\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r})\Phi[\Psi](t,\boldsymbol{r}), \tag{A20}$$

where :: refers to normal ordering, and we have made explicit that Φ may depend on the quantum state of matter Ψ in a CG theory. For example, for the Schrödinger-Newton equations (the non-relativistic limit of semi-classical gravity), Φ is given by the expectation value of the right-hand side of the quantized version of (A17). Expanding the non-relativistic field in annihilation operators, $\hat{\Psi}(\boldsymbol{r}) = \sum_k \psi_k(\boldsymbol{r})\hat{a}_k$, we again find CG is only quadratic in quantum operators and so cannot change the degree of quantum non-Gaussianity in the state of matter, whereas QG can.

a. First quantization

The interaction Hamiltonian of classical Newtonian gravity is given by (A18). The Hamiltonian of QG and CG in the Newtonian limit can then be derived by quantizing the matter density $\rho(\boldsymbol{r})$ and, in the QG case, the gravitational potential $\Phi(\boldsymbol{r})$. In the previous section we took matter to obey a non-relativistic quantum field $\hat{\Psi}$, such that $\hat{\rho} = m\hat{\Psi}^\dagger\hat{\Psi}$, assuming a single type of matter. Since $\hat{\Psi}$ is linear in annihilation operators, and so also in quadratures, the interaction Hamiltonian for CG is at most quadratic, such that an initial Gaussian state of the matter field will always remain Gaussian. However, in the case that we always have definite particle number, which can only be possible in the Newtonian approximation of the respective theories not the full relativistic theories, we could also view QG and CG in a first-quantized form.[192] In this case, assuming a single type of particle, we quantize $\rho(\boldsymbol{r})$ through:

$$\hat{\rho}(\boldsymbol{r}) = m\sum_{i=1}^N \delta^{(3)}(\boldsymbol{r}-\hat{\boldsymbol{r}}_i), \tag{A21}$$

where N is the total number of particles in the matter system. The respective QG and CG Hamiltonians are then:

$$\hat{H}^{int}_{QG} = \frac{1}{2}\sum_{i=1}^N\int d^3\boldsymbol{r}\,\hat{\Phi}(\hat{\boldsymbol{r}}_i), \tag{A22}$$
$$\hat{H}^{int}_{CG} = \sum_{i=1}^N\int d^3\boldsymbol{r}\,\Phi(\hat{\boldsymbol{r}}_i). \tag{A23}$$

Sine $\Phi(\boldsymbol{r})$ does not need to be a quadratic function of $\boldsymbol{r}$, it is possible for CG to create non-Gaussianity in the first quantized picture. For example, in the Shrödinger-Newton equations, where $\Phi(\boldsymbol{r}) = \langle\hat{\Phi}(\boldsymbol{r})\rangle$ with $\hat{\Phi}(\boldsymbol{r})$ obeying Poisson's equation (A16), the many-body wavefunction of N massive particles would evolve as:[193]

$$i\hbar\partial_t\psi_N(t;\boldsymbol{r}_1,\ldots,\boldsymbol{r}_N) = \Big(-\frac{\hbar^2}{2m}\sum_{i=1}^N\nabla_i^2 + V(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_N)$$
$$- Gm^2\sum_{i,j=1}^N\int d^3\boldsymbol{r}'_1\cdots d^3\boldsymbol{r}'_N\frac{|\psi_N(t;\boldsymbol{r}'_1,\ldots,\boldsymbol{r}'_N)|^2}{|\boldsymbol{r}_i-\boldsymbol{r}'_j|}\Big)\psi_N(t;\boldsymbol{r}_1,\ldots,\boldsymbol{r}_N), \tag{A24}$$

where V is a trapping potential. Although a Gaussian approximation is expected to be very good for table-top experiments,[102,194] the evolution of ψ_N (and hence its corresponding Wigner function) can, in principle, be non-Gaussian. Therefore, in the BEC experiment proposed in the main text, although the state of the BEC in the second quantized picture must stay Gaussian under CG, its many-body wave-function need not.

This of course does not just apply to a BEC: CG can lead to a non-Gaussian Wigner function of the first quantization picture in general systems. However, note that, unlike the quadrature operators of the modes in the second quantized picture, the position operator of each particle only appears by itself in the Hamiltonian of CG (A23). This means that it is not possible for entanglement to be generated by Newtonian CG in the first quantized picture,[195] even though, in principle, it can in the second quantized picture, as illustrated in the main text. Therefore, it would seem that, either in the first or second quantized picture, it is not possible for CG to be used to carry out universal quantum computation (if all other matter interactions are neglected). This is, however, for different reasons in each picture: in the second quantized picture it is possible for CG to create "mode" entanglement but not "mode" non-Gaussianity, whereas, in the first quantized picture (which can only be properly applied in the non-relativistic limit of gravity), it is possible for CG to create "particle" non-Gaussianity but not "particle" entanglement. Note also that, just as particles tend to automatically get "entangled" in the first quantized picture when we have identical particles, the particle system also tends to become automatically non-Gaussian. That is, if we have two identical particles in positions $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ and two different states a and b, then the many-body wavefunction is $\psi_N = [\phi_a(\boldsymbol{r}_1)\phi_b(\boldsymbol{r}_2) \pm \phi_a(\boldsymbol{r}_2)\phi_b(\boldsymbol{r}_1)]/\sqrt{2}$, depending on whether the particles are bosons or fermions. The system looks entangled just because of the exchange symmetry of the identical particles (it is so-called "particle" entangled). Similarly, even if each single-particle wavefunction ϕ_a and ϕ_b is Gaussian, ψ_N will, in general, be non-Gaussian due to the exchange symmetry (and the corresponding Wigner function will be non-Gaussian also[196]). However, there has been much discussion on whether this "particle" entanglement is really physical.[197–202]

3. Quantum and classical gravity in a single BEC

Using the Newtonian limit of gravity, the QG and CG interaction Hamiltonians for a BEC are given by (A19)-(A20) with $\hat{\Psi}(\boldsymbol{r})$ representing the field of the BEC. Taking the limit of zero temperature as in the main text and neglecting any explicit time dependence of the density of the trapped BEC due to gravity, we can set $\hat{\Psi}(\boldsymbol{r}) = \psi(\boldsymbol{r})\hat{a}$, where $\psi(\boldsymbol{r})$ is the condensate wavefunction and $\hat{a}$ is its annihilation operator. This then results in equations (4)-(5) used in the main text for the interaction Hamiltonians of QG and CG in a single BEC.

Appendix B: Experimental details of the Bose-Einstein condensate test

1. Electromagnetic interactions

As derived in the previous section, the Hamiltonian for the gravitational interaction in a BEC at very low temperatures can be approximated by (4):

$$\hat{H}_{QG} = \frac{1}{2}\lambda_{QG}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a} \tag{B1}$$

where:

$$\lambda_{QG} := -Gm^2 \int d^3\boldsymbol{r} d^3\boldsymbol{r'} \frac{|\psi(\boldsymbol{r'})|^2|\psi(\boldsymbol{r})|^2}{|\boldsymbol{r}-\boldsymbol{r'}|}. \tag{B2}$$

Assuming a spherical trap and taking all interaction terms to be much smaller than the kinetic part of the full Hamiltonian, the wavefunction of the BEC is a Gaussian function of $\boldsymbol{r}$:[69,93]

$$|\psi(\boldsymbol{r})|^2 = \frac{4}{3\sqrt{\pi}}\rho_0 e^{-r^2/R^2}, \tag{B3}$$

where $\rho_0 := 1/((4/3)\pi R^3)$, and $R := \sqrt{\hbar/m\omega_0}$ is the effective radius of the spherical BEC, with ω_0 the trapping frequency and m the mass of the BEC atoms. The coupling constant λ_{QG} is then found to be:[93]

$$\lambda_{QG} = -\sqrt{\frac{2}{\pi}}\frac{Gm^2}{R}. \tag{B4}$$

In comparison, at low temperatures, the Hamiltonian for the electromagnetic interactions between the atoms is given by:[69]

$$\hat{H}_{EM} = \frac{1}{2}g \int d^3\boldsymbol{r} \hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r}), \tag{B5}$$

where the coupling constant $g := 4\pi\hbar^2 a/m$, with a the s-wave scattering length of the BEC. As with the gravitational interactions, we assume that the temperature (and interactions) are small enough such that we can ignore all energy levels except for the lowest one, which contains the condensate, and take $\hat{\Psi}(\boldsymbol{r}) \approx \psi(\boldsymbol{r})\hat{a}$. The Hamiltonian (B5) is then:

$$\hat{H}_{EM} = \frac{1}{2}\lambda_{EM}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}, \tag{B6}$$

where:

$$\lambda_{EM} := g\int d^3\boldsymbol{r}|\psi(\boldsymbol{r})|^4. \tag{B7}$$

Assuming a spherical BEC with a Gaussian wavefunction as above:

$$\lambda_{EM} = \frac{g}{2\sqrt{2}\pi^{3/2}R^3}. \tag{B8}$$

Including both the electromagnetic and gravitational interactions of the atoms, as well as the trapping potential $V(\boldsymbol{r})$, the full Hamiltonian of the BEC, at low temperatures, is then:

$$\hat{H} = \int d^3\boldsymbol{r}\Big[-\frac{\hbar^2}{2m}\psi^*(\boldsymbol{r})\nabla^2\psi(\boldsymbol{r}) + V(\boldsymbol{r})|\psi(\boldsymbol{r})|^2\Big]\hat{a}^\dagger\hat{a} + \frac{1}{2}\lambda_{EM}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a} + \frac{1}{2}\lambda_{QG}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}. \tag{B9}$$

For a spherical trapping potential with the Gaussian wavefunction approximation, this Hamiltonian reduces to:

$$\hat{H} = \hbar\omega\hat{a}^\dagger\hat{a} + \frac{1}{2}\lambda_{EM}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a} + \frac{1}{2}\lambda_{QG}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}, \tag{B10}$$

where $\hbar\omega := \hbar\omega_0 + (3/4)m\omega_0^2R^2$. Since the electromagnetic interaction Hamiltonian (B6) will induce non-Gaussianity, we need to make sure that we can distinguish it from the gravitational Hamiltonian (B1). For example, depending on the species of atom(s) or molecules used for the BEC, the electromagnetic interactions can be generically suppressed using optical or magnetic Feshbach resonances.

Other techniques can also be used to distinguish the gravitational and electromagnetic interactions. For example, within our above approximations, the gravitational coupling constant λ_{QG} scales inversely with the radius of the BEC (B4), whereas, the electromagnetic coupling constant λ_{EM} scales as $1/R^3$ (B8). Therefore, we could also distinguish between the two interactions by considering how the predicted effects scale with the size of the BEC.

2. Creating the non-classical initial states

a. Gaussian squeezed state

In a spin-1 BEC, the interaction Hamiltonian is:[97,98,203]

$$\hat{H} = \hbar\kappa\left[\hat{a}_0^2\hat{a}_+^\dagger\hat{a}_-^\dagger + \left(\hat{a}_0^\dagger\right)^2\hat{a}_+\hat{a}_-\right] + \hbar\kappa\left(\hat{a}_0^\dagger\hat{a}_0 - \frac{1}{2}\right)\left(\hat{a}_+^\dagger\hat{a}_+ + \hat{a}_-^\dagger\hat{a}_-\right) + \hbar q\left(\hat{a}_+^\dagger\hat{a}_+ + \hat{a}_-^\dagger\hat{a}_-\right), \tag{B11}$$

where $\hat{a}_0$ is the annihilation operator of the $m_F = 0$ mode and $\hat{a}_\pm$ are the annihilation operators of the $m_F = \pm$ modes. By dynamically tuning q with a magnetic field, the quadratic Zeeman shift (third term) cancels collisional shifts due to s-wave scattering of the three modes (second term).[203,204]

Usually it is assumed that the $m_F = 0$ mode is a very large coherent state (i.e. a coherent state with $N_0 \gg 1$) so that we can approximate $\hat{a}_0 \approx \sqrt{N_0}$ and then (B11) is a two-mode squeezing Hamiltonian for $\hat{a}_\pm$. This results in modes $m_F = \pm 1$ being (approximately) in a two-mode vacuum squeezed state. Here, instead we assume that the $m_F = \pm 1$ modes are in large coherent states ($N_\pm \gg 1$) so that $\hat{a}_\pm \approx \sqrt{N_\pm}$ and we can approximate (B11) by:

$$\hat{H}_{\mathrm{SMD}} = \hbar N\kappa\left[\hat{a}_0^2 + \left(\hat{a}_0^\dagger\right)^2\right], \tag{B12}$$

where $N := \sqrt{N_+N_-}$. This is a single-mode squeezing Hamiltonian, resulting in $\hat{a}_0$ approximately being in a quadrature squeezed vacuum state.

b. Single-mode cat state

Approximating the quantum field of a Bose gas by $\hat{\Psi} = \psi(\boldsymbol{r})\hat{a}$, where ψ is the condensate wavefunction and $\hat{a}$ is the annihilation operator for the condensate, the Hamiltonian for the electromagnetic interactions between the atoms is:

$$\hat{H} = \hbar\kappa\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}, \tag{B13}$$

where $\kappa := \lambda_{EM}/(2\hbar)$ and λ_{EM} is defined in (B7). This Hamiltonian is the Kerr interaction of quantum optics, which has been considered in BECs (see e.g. [102,103]). It is known that this Hamiltonian can, in principle, create a Yurke-Stoler state $|\psi\rangle = (|\alpha\rangle + i|-\alpha\rangle)/\sqrt{2}$ from an initial coherent state $|\alpha\rangle$.[83] The evolution of such a state under QG in a BEC is considered in the subsequent section.

3. Measuring non-Gaussianity

Measuring quadrature non-Gaussianity with homodyne or heterodyne detection requires single-atom detection in a quantum gas with high efficiency on small length scales. Recent advances have opened up three promising approaches to this:

1. After the interaction time t, the atomic evolution can be frozen by quickly ramping up a far-detuned optical lattice that confines atoms with a spatial resolution of the lattice wavelength, after which fluorescence-imaging light emitted by the atoms upon exposure to near-resonant light fields can be detected to achieve single atom, high spatial resolution imaging. Single-atom resolved imaging of a quantum gas in a two-dimensional optical lattice with sub-micrometer lattice spacing has been first demonstrated in [205–207].

2. A related optical fluorescence technique follows a similar working principle measuring the transit of single atoms through a light sheet that is located below the atomic sample. While the atoms are falling through the light sheet, a CCD camera records the fluorescence traces. This has been used to measure Hanbury Brown and Twiss correlations across the Bose-Einstein condensation threshold.[208]

3. Alternatively, a high finesse cavity can be used where the transit of single atoms through the cavity will cause detectable shifts in the cavity resonance. While this technique does not allow the detection of individual atoms, the emerging photons from the cavity can be used to probe the system, revealing atom number fluctuations in real-time.[209,210] Such techniques have been used to demonstrate many-body entanglement.[109,211]

Appendix C: Fourth-order cumulant for a single-mode bosonic system

The fourth-order cumulant k_4 is given by (8) for the generalised quadrature $\hat{q} = \hat{a}e^{-i\varphi} + \hat{a}^\dagger e^{i\varphi}$. This requires the determination of various expectation values of combinations of $\hat{a}$ and $\hat{a}^\dagger$:

$$\langle\hat{q}^4\rangle = \frac{1}{4}(3 + \langle\hat{a}^4\rangle\, e^{-4i\varphi} + 4\,\langle\hat{a}^\dagger\hat{a}^3\rangle\, e^{-2i\varphi} + 6\langle\hat{a}^2\rangle e^{-2i\varphi} + 12\,\langle\hat{a}^\dagger\hat{a}\rangle + 6\,\langle\hat{a}^{\dagger 2}\hat{a}^2\rangle + h.c.). \tag{C1}$$

The QG Hamiltonian for a single-mode BEC with electromagnetic inter-atomic interactions neglected is given by (4):

$$\hat{H}_{QG} = \hbar\omega\hat{a}^\dagger\hat{a} + \frac{1}{2}\lambda_{QG}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}, \tag{C2}$$

where we have also included the free Hamiltonian term $\hbar\omega\hat{a}^\dagger\hat{a}$, which derives from the kinetic and (time-independent) trapping potential terms of the BEC Hamiltonian (see (B9)). Working in the Heisenberg picture, the evolution of $\hat{a}$ is:

$$\frac{d\hat{a}(t)}{dt} = -\frac{i}{\hbar}[\hat{a}, \hat{H}] \tag{C3}$$

$$= -i(\omega - \chi\hat{N})\hat{a}(t), \tag{C4}$$

where $\hat{N} := \hat{a}^\dagger\hat{a}$ and $\chi := |\lambda_{QG}|/\hbar$. Since $\hat{N}$ is a constant of motion, this can be solved as:

$$\hat{a}(t) = e^{-i\omega t}e^{i\chi\hat{N}t}\hat{a}, \tag{C5}$$

where $\hat{a} := \hat{a}(t=0)$. From now on we will ignore the phase ω of the free evolution since this can just be absorbed into the φ angle of the quadrature $\hat{q}(\varphi) = \hat{a}e^{-i\varphi} + \hat{a}^\dagger e^{i\varphi}$. Then, $\hat{a}^n$ evolves as:

$$\hat{a}^n(t) = e^{i\frac{n}{2}(n-1)\chi t} e^{in\chi\hat{N}t}\hat{a}^n, \tag{C6}$$

and therefore:

$$\begin{aligned}\hat{a}^{\dagger m}\hat{a}^n(t) &= e^{i\frac{1}{2}(n-m)(m+n-1)\chi t}\hat{a}^{\dagger m} e^{i(n-m)\chi\hat{N}t}\hat{a}^n, &\text{(C7)}\\ &\equiv e^{i\frac{1}{2}(n-m)(m-n+1)\chi t} e^{i(n-m)\chi\hat{N}t}\hat{a}^{\dagger m}\hat{a}^n. &\text{(C8)}\end{aligned}$$

We could now assume $4\chi N t \ll 1$, with $N := \langle\hat{N}\rangle$, and expand the exponentials in (C1), i.e. take:

$$e^{in\chi\hat{N}t} = 1 + in\chi\hat{N}t + \frac{1}{2!}n^2\chi^2\hat{N}^2t^2 + \cdots \tag{C9}$$

to calculate the expectation value of $\hat{a}^n$ etc. In this case, taking an initial squeezed coherent state $|\xi,\alpha\rangle$ (which is a general pure Gaussian state), κ_4 is initially vanishing and remains zero if CG acts (see (5)), whereas, under QG (see (4) and (C2) above), κ_4 evolves as:

$$\begin{aligned}\kappa_4(t) = -3\chi t \sin\nu \, \sinh^2(2r)\,\eta_1(r,\nu) + \frac{3}{8}\chi^2t^2\Big[\sinh^2(2r)\eta_2(r,\nu) + 2|\alpha|^2\Big(2\sinh^2(2r)\eta_3(r,\nu) + 2\sinh 2r\,\eta_4(r,\nu)\\ + 8\sinh 4r\cos 2\nu\cos\psi - 5\sinh 6r \sin 2\nu\sin\psi\Big)\Big] + \cdots \quad \text{(C10)}\end{aligned}$$

where:

$$\begin{aligned}\xi &:= re^{i\vartheta}, &\text{(C11)}\\ \nu &:= 2\varphi - \vartheta, &\text{(C12)}\\ \eta_1(r,\nu) &:= \sinh 2r - \cos\nu\cosh 2r, &\text{(C13)}\\ \eta_2(r,\nu) &:= 6\sinh^2(2r) + 8\cos\nu\sinh 2r(5\cosh 2r - 2) - \cos 2\nu(23\cosh 4r - 16\cosh 2r + 9)\\ \eta_3(r,\nu) &:= 2\sinh 4r[\cos\psi(8\cos 2\nu - 3) + 5\cos\nu] + 3\cos\psi\cos\nu - \cos 2\nu, &\text{(C14)}\\ \eta_4(r,\nu) &:= \sinh 6r(3 - 8\cos 2\nu - 5\cos\nu\cos\psi) - \sin\nu\sin\psi(\cos\nu - 10\sinh 4r). &\text{(C15)}\end{aligned}$$

In the limit of a coherent state and $N \gg 1$, we obtain the same scaling found in [51] at χ^4 with $\varphi = \pi/2$, whereas, in the opposite limit of full squeezing, κ_4 limits to $24\chi t N^3$ when $\nu = \pi/2$, illustrating that the small value of χ can be compensated for by a large number of atoms.

If, on the other hand, we had chosen an initial Yurke-Stoler state, $|\psi\rangle := (|\alpha\rangle + i\,|{-\alpha}\rangle)/\sqrt{2}$, then κ_4 at time t is:

$$\kappa_4(t) = -\,8|\alpha|^4(\cos^4\varphi + 3\sin^4\varphi\, e^{-8|\alpha|^2}) - 16\chi t|\alpha|^6\sin 2\varphi\Big[\cos^2\varphi - e^{-4|\alpha|^2}[3 + \sin^2\varphi(2 - 3e^{-4|\alpha|^2})]\Big] + \cdots.$$

In the limit $N \gg 1$, the first order term scales as $6\sqrt{3}\chi t N^3$ at $\varphi = \pi/6$, similar to when the initial state is $|\xi\rangle$ as above.

1. Non-perturbative approach

We now pursue a non-perturbative approach to how κ_4 evolves with time. For the Yurke-Stoler state $|\psi\rangle := (|\alpha\rangle + i\,|{-\alpha}\rangle)/\sqrt{2}$, we can use:

$$\langle\alpha|\, e^{in\chi\hat{N}t}\,|\alpha\rangle \equiv \langle\alpha|: e^{(\cos[n\chi t] + i\sin[n\chi t] - 1)\hat{N}} :|\alpha\rangle = e^{(\cos[n\chi t] + i\sin[n\chi t] - 1)|\alpha|^2}, \tag{C16}$$

and (C6) and (C7). For a squeezed coherent state $|\xi,\alpha\rangle$, with $\xi := re^{i\vartheta}$, we can use (C6) and (C8) with:

$$\langle\alpha,\xi|\, e^{in\chi\hat{N}t}\,|\xi,\alpha\rangle \equiv \frac{1}{\sqrt{z}} e^{\frac{1}{2}in\chi t} G_0 \langle 0|\, \hat{G}_+\hat{G}_{2+}\hat{G}_3\hat{G}_{2-}\hat{G}_-\,|0\rangle\,, \tag{C17}$$

where:

$$G_0 := \exp(\beta|\alpha|^2 - \frac{1}{2}\Lambda_+\alpha^{*2} - \frac{1}{2}\Lambda_-\alpha^2), \tag{C18}$$

$$\hat{G}_+ := \exp([\beta\alpha - \Lambda_+\alpha^*]\hat{a}), \tag{C19}$$

$$\hat{G}_- := \exp([\beta\alpha^* - \Lambda_-\alpha]\hat{a}^\dagger), \tag{C20}$$

$$\hat{G}_{2+} := \exp(-\frac{1}{2}\Lambda_+\alpha^{*2}\hat{a}^{\dagger 2}), \tag{C21}$$

$$\hat{G}_{2-} := \exp(-\frac{1}{2}\Lambda_-\alpha^2\hat{a}^2), \tag{C22}$$

$$\hat{G}_3 :=: \exp(\beta\hat{a}^\dagger\hat{a}) :, \tag{C23}$$

$$\beta := (1-z)/z, \tag{C24}$$

$$\Lambda_+ := i\sinh(2r)\sin(n\chi t)e^{i\vartheta}/z, \tag{C25}$$

$$\Lambda_+ := i\sinh(2r)\sin(n\chi t)e^{-i\vartheta}/z, \tag{C26}$$

$$z := \cos(n\chi t) - i\cosh(2r)\sin(n\chi t) \tag{C27}$$

Here we have used the identities $\exp(\theta[A+B]) \equiv \exp(\theta B)\exp([e^\theta - 1]A) \equiv \exp([1-e^{-\theta}]A)\exp(\theta B)$ when $[A, B] = A$, as well as:[212]

$$\exp\left(\gamma_+\hat{K}_+ + \gamma_-\hat{K}_- + \gamma_3\hat{K}_3\right) = \exp\left(\Gamma_+\hat{K}_+\right)\exp\left[(\ln\Gamma_3)\,\hat{K}_3\right]\exp\left(\Gamma_-\hat{K}_-\right) \quad , \tag{C28}$$

with:

$$\Gamma_3 = \left(\cosh\beta - \frac{\gamma_3}{2\beta}\sinh\beta\right)^{-2} \tag{C29}$$

$$\Gamma_\pm = \frac{2\gamma_\pm\sinh\beta}{2\beta\cosh\beta - \gamma_3\sinh\beta} \tag{C30}$$

$$\beta^2 = \frac{1}{4}\gamma_3^2 - \gamma_+\gamma_-, \tag{C31}$$

and $[\hat{K}_3, \hat{K}_\pm] = \pm\hat{K}_\pm$, $[\hat{K}_+, \hat{K}_-] = -2\hat{K}_3$. For example, using (C17), $\langle\xi|\,\hat{a}^4(t)\,|\xi\rangle$ under $\hat{H}_{QG}$ can be shown to be:

$$\langle\xi|\,\hat{a}^4(t)\,|\xi\rangle = \frac{3e^{-4i\chi t+2i\vartheta}\sinh^2(2r)}{2^2[\cos(4\chi t) - i\cosh(2r)\sin(4\chi t)]^{5/2}}, \tag{C32}$$

where we can use $\sqrt{z} \equiv \sqrt{|z|}(z+|z|)/|z+|z||$ to remove the square root of the complex number.

2. Including the reverse process

Above we have considered the evolution of κ_4 under the QG Hamiltonian $\hat{H}_{QG}$ and assuming that the BEC is prepared in either a squeezed coherent state or a Yurke-Stoler state. In the main text, we also considered a measurement protocol where we first prepare the BEC state that is required, let the BEC evolve under QG, and then measure κ_4 after we have applied the reverse process to that we used to create the initial BEC state. In the Heisenberg picture, $\hat{a}$ then undergoes the following evolutions:

1. $\hat{a} \to \hat{a}' = \hat{U}_\Psi^\dagger\hat{a}\hat{U}_\Psi$ at $t = 0$,
2. $\hat{a} \to \hat{a}''(t) = e^{i\chi\hat{N}t}\hat{a}'$ for $0 < t < \tau$,
3. $\hat{a} \to \hat{a}'''(\tau) = \hat{U}_{-\Psi}^\dagger\hat{a}_2(\tau)\hat{U}_{-\Psi}$ at $t = \tau$,

where $\hat{U}_\Psi$ refers to the unitary that creates the initial state, and $\hat{U}_{-\Psi}$ is the reverse process. For example, if a squeezed vacuum state is prepared then $\hat{U}_\Psi = \exp(r[e^{i\vartheta}\hat{a}^{\dagger 2} - e^{-i\vartheta}\hat{a}^2]/2)$ and $\hat{U}_{-\Psi} = \exp(-r[e^{i\vartheta}\hat{a}^{\dagger 2} - e^{-i\vartheta}\hat{a}^2]/2)$. In this case, in the limit that $\chi \ll 1$, κ_4 at the end of the process is:

$$\kappa_4(\tau) = \frac{3}{2}\chi\tau\sin[2\nu]\sinh(2r)^2 + \cdots, \tag{C33}$$

with ν given by (C12). In the limit of large N, this scales as $\chi\tau N^2$ in contrast to the N^3 scaling for the process considered previously, see (C10). However, the SNR scaling is the same.

Appendix D: Estimation of the fourth-order cumulant

The SNR for measuring the fourth-order cumulant κ_4 is given by:

$$\mathrm{SNR} = \frac{|\kappa_4|}{\sqrt{\mathrm{Var}(k_4)}}, \tag{D1}$$

where k_4 is the fourth k-statistic. The variance of k_4 is given by:[85]

$$\mathrm{Var}(k_4) = \frac{\kappa_8}{\mathcal{M}} + \frac{16\kappa_2\kappa_6}{\mathcal{M}-1} + \frac{48\kappa_3\kappa_5}{\mathcal{M}-1} + \frac{34\kappa_4^2}{\mathcal{M}-1} + \frac{72\mathcal{M}\kappa_2^2\kappa_4}{(\mathcal{M}-1)(\mathcal{M}-2)} + \frac{144\mathcal{M}\kappa_2\kappa_3^2}{(\mathcal{M}-1)(\mathcal{M}-2)} + \frac{24\mathcal{M}(\mathcal{M}+1)\kappa_2^4}{(\mathcal{M}-1)(\mathcal{M}-2)(\mathcal{M}-3)},$$

where $\mathcal{M}$ is the number of independent estimations. In the limit $\mathcal{M} \gg 1$, $\mathrm{Var}(k_4)$ becomes:

$$\mathrm{Var}(k_4) \approx \frac{1}{\mathcal{M}}\Big[\kappa_8 + 16\kappa_2\kappa_6 + 48\kappa_3\kappa_5 + 34\kappa_4^2 + 72\kappa_2^2\kappa_4 + 144\kappa_2\kappa_3^2 + 24\kappa_2^4\Big].$$

The nth-order cumulant κ_n, can be found using:

$$\kappa_n = \mu_n - \sum_{m=1}^{n-1}\binom{n-1}{m-1}\mu_{n-m}\kappa_m, \tag{D2}$$

where $\mu_n := \langle \hat{q}^n \rangle$ is the nth moment.

In the limit that $\chi \ll 1$, the SNR for the estimation of κ_4 for a squeezed vacuum state $|\xi\rangle$ is:

$$\mathrm{SNR} = \sqrt{6\mathcal{M}}t\chi\sinh^2(2r)\frac{|\sin\nu\,(\sinh 2r - \cos\nu\,\cosh 2r)|}{(\cosh 2r - \cos\nu\sinh 2r)^2} + \cdots \tag{D3}$$

This is maximized at the angles:

$$\varphi = \frac{1}{2}\Big[\vartheta \pm \frac{1}{2}\cos^{-1} y\Big], \tag{D4}$$

where:

$$y := \frac{\sinh^2 2r(\sinh^2 2r - 2) \pm 2\sqrt{2}\sinh 4r}{(\sinh^2 2r + 2)^2}, \tag{D5}$$

which results in the above SNR being approximately $4.9\chi t N^2\sqrt{\mathcal{M}}$ for $N \gg 1$. When $\chi N^2 t$ is not small, this SNR approximation is not so accurate, and instead the results of the previous section can be used to find a non-perturbative solution to SNR. For example, for the BMV proposal values $d = 200\,\mu\mathrm{m}$, $t = 2\,\mathrm{s}$ and $M = 10^{-14}\,\mathrm{kg}$, we find that the maximum SNR for a spherical $^{133}\mathrm{Cs}$ BEC is approximately $0.3\sqrt{\mathcal{M}}$ (with the value of d being used for the radius R). At these values, $\chi N^2 t = \sqrt{2/\pi}\phi \approx 0.5$, where $\phi = 0.6$ is the relative phase expected in the BMV experiment when all distances between the microspheres other than d, the smallest possible distance, are ignored. Therefore, the SNR is still of order $\chi t N^2\sqrt{\mathcal{M}}$ in this case. If instead the mass is lowered to $M = 10^{-15}\,\mathrm{kg}$ then we can use the approximation that $\mathrm{SNR} = 4.9\chi t N^2\sqrt{\mathcal{M}}$.

For the protocol where we reverse the squeezing operation before the measurement, the SNR is given by:

$$\mathrm{SNR} = \sqrt{\frac{3}{2}}\chi\tau|\sin 2\nu|\,\sinh^2(2r) + \cdots, \tag{D6}$$

in the limit that $\chi \ll 1$.

Appendix E: Evolution under classical gravity

Here we consider how a single BEC evolves under CG compared to QG. We start with the general Newtonian expressions (A19) and (A20). Working in the Schrödinger picture, for QG the evolution of our state vector $|\Psi\rangle$ is given by:

$$i\hbar\frac{d\,|\Psi(t)\rangle}{dt} = \hat{H}_{QG}^{BEC}\,|\Psi(t)\rangle, \tag{E1}$$

where:

$$\hat{H}_{QG}^{BEC} := \int d^3\boldsymbol{r}\Big[-\frac{\hbar^2}{2m}\hat{\Psi}^\dagger(\boldsymbol{r})\nabla^2\hat{\Psi}(\boldsymbol{r}) + V(\boldsymbol{r})\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r}) + \frac{1}{2}m:\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r})\hat{\Phi}(\boldsymbol{r}):\Big] \tag{E2}$$

$$= \int d^3\boldsymbol{r}\Big[-\frac{\hbar^2}{2m}\hat{\Psi}^\dagger(\boldsymbol{r})\nabla^2\hat{\Psi}(\boldsymbol{r}) + V(\boldsymbol{r})\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r}) - \frac{1}{2}Gm^2\int d^3\boldsymbol{r'}\frac{\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}^\dagger(\boldsymbol{r'})\hat{\Psi}(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r'})}{|\boldsymbol{r}-\boldsymbol{r'}|}\Big], \tag{E3}$$

with $V(\boldsymbol{r})$ the trapping potential. In contrast, for CG, we have:

$$i\hbar\frac{d\,|\Psi(t)\rangle}{dt} = \hat{H}_{CG}^{BEC}[\Psi](t)\,|\Psi(t)\rangle\,, \tag{E4}$$

where:

$$\hat{H}_{CG}^{BEC}[\Psi](t) := \int d^3\boldsymbol{r}\Big[-\frac{\hbar^2}{2m}\hat{\Psi}^\dagger(\boldsymbol{r})\nabla^2\hat{\Psi}(\boldsymbol{r}) + V(\boldsymbol{r})\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r}) + m\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r})\Phi[\Psi(t)](\boldsymbol{r})\Big]. \tag{E5}$$

In the Schrödinger-Newton example of CG, this is:

$$\hat{H}_{CG}^{BEC}[\Psi](t) = \int d^3\boldsymbol{r}\Big[-\frac{\hbar^2}{2m}\hat{\Psi}^\dagger(\boldsymbol{r})\nabla^2\hat{\Psi}(\boldsymbol{r}) + V(\boldsymbol{r})\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r}) \tag{E6}$$

$$- Gm^2\int d^3\boldsymbol{r'}\frac{\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r})\,\langle\Psi(t)|\,\hat{\Psi}^\dagger(\boldsymbol{r'})\hat{\Psi}(\boldsymbol{r'})\,|\Psi(t)\rangle}{|\boldsymbol{r}-\boldsymbol{r'}|}\Big]. \tag{E7}$$

Note that the evolution of $|\Psi\rangle$ in CG is, in general, 'non-linear' in that $|\Psi\rangle$ is needed to determine Φ. This is often referred to as a wavefunction 'self-interaction' since, in the first quantization picture, the wavefunction of a single-particle will now interact with itself, something that can never occur in a quantum theory of gravity, where (E1) is said to be 'linear'.

Neglecting any explicit time dependence, the evolution of $|\Psi\rangle$ in QG can, in principle, be solved as:

$$|\Psi(t)\rangle = e^{-i\hat{H}_{QG}^{BEC}t/\hbar}\,|\Psi(0)\rangle\,. \tag{E8}$$

In contrast, it may not be possible to find an analytic solution in CG due to the potential non-linearities. However, the evolution will still take the form:

$$|\Psi(t)\rangle = \hat{T}\left\{e^{-\frac{i}{\hbar}\int_0^t d\tau \hat{H}_{CG}^{BEC}[\Psi](\tau)}\right\}|\Psi(0)\rangle\,, \tag{E9}$$

where $\hat{T}$ is the time-ordering operator. Despite the potential non-linearity, since $\hat{H}_{CG}^{BEC}$ is quadratic in matter field operators, it is still a Gaussian process. For example, consider the single-mode BEC experiment introduced in the main text where we assume $\hat{\Psi}(\boldsymbol{r}) = \psi(\boldsymbol{r})\hat{a}$. Neglecting the trapping potential and free dynamics, we then have:

$$|\Psi(t)\rangle = \hat{T}\left\{e^{-\frac{i}{\hbar}\int_0^t d\tau\lambda_{CG}[\Psi](t)\hat{a}^\dagger\hat{a}}\right\}|\Psi(0)\rangle\,, \tag{E10}$$

with:

$$\lambda_{CG}[\Psi](t) = m\int d^3\boldsymbol{r}|\psi(\boldsymbol{r})|^2\Phi[\Psi](t,\boldsymbol{r}). \tag{E11}$$

Equation (E10) can be written as:[54]

$$|\Psi(t)\rangle = e^{-\frac{i}{\hbar}\Lambda_{CG}[\Psi](t)\hat{a}^\dagger\hat{a}}\,|\Psi(0)\rangle\,, \tag{E12}$$

where:

$$\Lambda_{CG}[\Psi](t) := \int_0^t d\tau\lambda_{CG}[\Psi](\tau). \tag{E13}$$

The evolution of $|\Psi\rangle$ in this case is then, in general, a non-linear Gaussian process. However, it need not always be non-linear. For instance, in the Schrödinger-Newton case we have:

$$\lambda_{CG}[\Psi](t) = -Gm^2\,\langle\Psi(t)|\,\hat{N}\,|\Psi(t)\rangle\int d^3\boldsymbol{r}d^3\boldsymbol{r'}\frac{|\psi(\boldsymbol{r'})|^2|\psi(\boldsymbol{r})|^2}{|\boldsymbol{r}-\boldsymbol{r'}|}, \tag{E14}$$

where $\hat{N} := \hat{a}^\dagger\hat{a}$. Since $\hat{N}$ is a constant of motion (it commutes with $\hat{H}^{BEC}_{CG}$), we have:

$$\lambda_{CG} = -Gm^2N\int d^3\boldsymbol{r}d^3\boldsymbol{r'}\frac{|\psi(\boldsymbol{r'})|^2|\psi(\boldsymbol{r})|^2}{|\boldsymbol{r}-\boldsymbol{r'}|}, \tag{E15}$$

where $N := \langle\hat{N}\rangle$. Therefore, $|\Psi(t)\rangle$ evolves as:

$$|\Psi(t)\rangle = e^{-\frac{i}{\hbar}\gamma_{CG}\hat{a}^\dagger\hat{a}t}|\Psi(0)\rangle, \tag{E16}$$

where:

$$\gamma_{CG} := \int d^3\boldsymbol{r}\Big[-\frac{\hbar^2}{2m}\psi^*(\boldsymbol{r})\nabla^2\psi(\boldsymbol{r}) + V(\boldsymbol{r})|\psi(\boldsymbol{r})|^2\Big] - \lambda_{CG}, \tag{E17}$$

such that $|\Psi(t)\rangle$ evolves under a Gaussian phase-shift channel. For example, if the BEC were initially in a coherent state $|\alpha\rangle$, it would then stay a coherent state but with just a time-dependent phase:

$$|\Psi(t)\rangle = |\alpha e^{-i\gamma_{CQ}t/\hbar}\rangle, \tag{E18}$$

with $N = |\alpha|^2$.

Appendix F: Stochastic and complex interactions

Here we consider matter interacting with a complex, stochastic non-quantum field (non-operator-valued distribution), and why this interaction cannot, in the absence of all other interactions, turn a Gaussian state into a non-Gaussian state, where the latter is defined as any state that does not belong to the Gaussian convex hull.[136]

In the main text, we considered interacting matter with a classical entity $\mathcal{G}$ (a quantity that takes on real and well-defined values) and how this can be distinguished from the quantum version of the interaction. Taking, for simplicity, matter to be described by a real scalar quantum field $\hat{\phi}$ then, as long as we do not allow the classical interaction to induce quantum self-interactions of matter, $\mathcal{G}$ and $\hat{\phi}$ can only interact through Hamiltonian terms that are linear or quadratic in $\hat{\phi}$. That is, the Hamiltonian density of the interaction must be of the form:

$$\hat{\mathcal{H}} = s[\hat{\phi}]f[\mathcal{G}] + t[\hat{\phi}]h[\mathcal{G}], \tag{F1}$$

where s and t are respectively linear and quadratic real functionals of $\hat{\phi}$; and f and h are general real functionals of $\mathcal{G}$. It was shown in the main text that a Hamiltonian density of the form (F1) preserves the Gaussianity of the matter field, and we can use this fact to distinguish it from a quantum interaction.

We now, in contrast to the main text, allow $\mathcal{G}$, or f and h, to be complex-valued. Expanding $\hat{\phi}$ in creation and annihilation operators, $\hat{\phi} = \sum_k[u_k(t)\hat{a}_k + v(t)\hat{a}_k^\dagger]$, the corresponding Hamiltonian will be of the form of (2) except that now $\boldsymbol{\lambda}_k(t), \boldsymbol{\mu}_{kl}(t) \in \mathbb{C}$ so that the Hamiltonian is, in general, non-Hermitian. Despite this, the quadratic nature of the non-Hermitian Hamiltonian means that it still preserves the Gaussian form of the Wigner function for an initial Gaussian state.[130–132] For example, consider the Hamiltonian $\hat{H} = \lambda\hat{a}^\dagger\hat{a}$, where $\lambda := \lambda_R - i\lambda_I$. Under this Hamiltonian, an initial coherent state $|\alpha\rangle$ will evolve to $\exp\{-|\alpha|^2(1-\exp\{-2\lambda_I t\})/2\}|\alpha\exp\{-i\lambda t\}\rangle$, which is just an unnormalized, damped coherent state with a time-dependent phase (note that we have taken $\hbar = 1$ here and do so throughout the rest of this Appendix). In fact, in general, a non-Hermitian Hamiltonian will lead to an unnormalized state. To rectify this, the physical state vector can be defined as $|\psi^{\mathcal{N}}\rangle := |\psi\rangle/||\psi\rangle|$. For the above example, this would mean that an initial coherent state evolves to a damped coherent state with a time-dependent phase: $|\alpha'(t)\exp\{-\lambda_I t\}\rangle$, where $\alpha'(t) := \alpha\exp\{-i\lambda_R t\}$.

We now take $\mathcal{G}$ to be a stochastic field, which we denote as $\tilde{\mathcal{G}}$, and keep f and h complex-valued. The interaction Hamiltonian density (F1) can then be written as:

$$\hat{\mathcal{H}}[\tilde{\mathcal{G}}] = s[\hat{\phi}]f[\tilde{\mathcal{G}}] + t[\hat{\phi}]h[\tilde{\mathcal{G}}]. \tag{F2}$$

In the interaction picture, an out state $|\psi_{out}[\tilde{\mathcal{G}}]\rangle$ of the quantum field $\hat{\phi}$ is now given by a stochastic S-matrix $\hat{S}[\tilde{\mathcal{G}}]$ acting on the in state $|\psi_{in}\rangle$.[141] That is:

$$|\psi_{out}[\tilde{\mathcal{G}}]\rangle = \hat{S}[\tilde{\mathcal{G}}]|\psi_{in}\rangle, \tag{F3}$$

where:

$$\hat{S}[\tilde{\mathcal{G}}] := Te^{-i\int d^4x(\hat{\mathcal{H}}_0+\hat{\mathcal{H}}[\tilde{\mathcal{G}}])}, \tag{F4}$$

with T the time-ordering operator; x a four-coordinate; and $\hat{\mathcal{H}}_0$ the free (non-stochastic) Hamiltonian density.

Since the Hamiltonian may not be Hermitian, the out state may not be normalized, but we can define a normalized out state as:

$$|\psi^{\mathcal{N}}_{out}[\tilde{\mathcal{G}}]\rangle := \mathcal{N}^{-1/2}|\psi_{out}[\tilde{\mathcal{G}}]\rangle \tag{F5}$$

where:

$$\mathcal{N} := \langle\psi_{out}[\tilde{\mathcal{G}}]|\psi_{out}[\tilde{\mathcal{G}}]\rangle. \tag{F6}$$

The density matrix corresponding to a particular out state $|\psi_{out}[\tilde{\mathcal{G}}]\rangle$ can be defined as usual:

$$\hat{\rho}_{out}[\tilde{\mathcal{G}}] := |\psi_{out}[\tilde{\mathcal{G}}]\rangle\langle\psi_{out}[\tilde{\mathcal{G}}]|, \tag{F7}$$

or the normalized version:

$$\hat{\rho}^{\mathcal{N}}_{out}[\tilde{\mathcal{G}}] := \mathcal{N}^{-1}\hat{\rho}_{out}[\tilde{\mathcal{G}}]. \tag{F8}$$

From (F3), the density matrix $\hat{\rho}_{out}[\tilde{\mathcal{G}}] := |\psi_{out}[\tilde{\mathcal{G}}]\rangle\langle\psi_{out}[\tilde{\mathcal{G}}]|$ can be found through:

$$\hat{\rho}_{out}[\tilde{\mathcal{G}}] = \hat{S}[\tilde{\mathcal{G}}]\hat{\rho}_{in}\hat{S}^{\dagger}[\tilde{\mathcal{G}}], \tag{F9}$$

where $\hat{\rho}_{in} := |\psi_{in}\rangle\langle\psi_{in}|$. The above density matrix corresponds to a particular stochastic out state $|\psi_{out}[\tilde{\mathcal{G}}]\rangle$. However, the quantity that provides the correct expectation values of operators ($\langle\hat{A}\rangle = \mathrm{Tr}[\hat{\rho}_{out}\hat{A}]$) is the average density matrix (averaged over $\tilde{\mathcal{G}}$) $\hat{\rho}_{out}$.[173,213] That is, $\hat{\rho}_{out}$ is given by:[141]

$$\begin{aligned}
\hat{\rho}_{out} &:= \int \mathcal{D}\tilde{\mathcal{G}}\, P[\tilde{\mathcal{G}}]\,(\hat{\rho}_{out}[\tilde{\mathcal{G}}]) && \text{(F10)}\\
&= \int \mathcal{D}\tilde{\mathcal{G}}\, P[\tilde{\mathcal{G}}]\,(\hat{S}[\tilde{\mathcal{G}}]\,\hat{\rho}_{in}\,\hat{S}^{\dagger}[\tilde{\mathcal{G}}]) && \text{(F11)}\\
&:= \int \mathcal{D}\tilde{\mathcal{G}}\, P[\tilde{\mathcal{G}}]\,(\hat{\mathcal{S}}_S[\tilde{\mathcal{G}}]\,\hat{\rho}_{in}) && \text{(F12)}\\
&:= \hat{\mathcal{S}}_{av}\hat{\rho}_{in} && \text{(F13)}
\end{aligned}$$

where $P[\tilde{\mathcal{G}}]$ is the probability distribution functional of $\tilde{\mathcal{G}}$; $\hat{\mathcal{S}}_S[\tilde{\mathcal{G}}]$ is the scattering superoperator; and $\hat{\rho}_{in}$ now, in general, corresponds to a general initial mixed state.[138,214]

Taking $\hat{\rho}_{in}$ to be a pure Gaussian state, then since each $\hat{S}[\tilde{\mathcal{G}}]$ is associated with a Gaussian transformation (i.e. (F1)), (F10) is just the stochastic quantum field theory generalization of a state, $\hat{\rho}_{Ch}$, in the Gaussian convex hull of quantum optics:[136]

$$\hat{\rho}_{Ch} = \int d\boldsymbol{g}\, P(\boldsymbol{g})\,\hat{\rho}_G(\boldsymbol{g}), \tag{F14}$$

where $\boldsymbol{g}$ is a set of complex numbers, $P(\boldsymbol{g})$ is a probability distribution, and $\hat{\rho}_G(\boldsymbol{g}) = |\psi_G(\boldsymbol{g})\rangle\langle\psi_G(\boldsymbol{g})|$ is a pure Gaussian density matrix. Defining a Gaussian state as a pure state with Gaussian Wigner function or a mixture of pure states with Gaussian Wigner functions, represents a broader definition of a Gaussian state compared to the more conventional definition of any state with a Gaussian Wigner function that is used in the main text.[52,136] A non-Gaussian state (also sometimes referred to as a 'quantum' non-Gaussian state to distinguish it from the more conventional definition of a non-Gaussian state [52]) can then be defined as any state that lives outside the convex hull of Gaussian states.[215]

As shown and discussed in the main text, to rule out a classical interaction (defined as an interaction with a non-quantum field that takes on real and well-defined values, such as the classical electromagnetic or gravitational fields) any detection of a non-Gaussian state as it is conventionally defined (any state with a non-Gaussian Wigner function), is sufficient as long as all other interactions can be neglected. As shown above, this also applies when the field takes on complex values. However, to rule out a stochastic interaction (defined as an interaction with a non-quantum field that is fundamentally stochastic, sometimes referred to as a 'post-quantum' interaction), we must appeal to the detection of a non-Gaussian state (or 'quantum' non-Gaussian state) in its broader definition as any state that sits outside the Gaussian convex hull.[216] This is to be expected since a Gaussian state evolves to a state in the Gaussian convex hull if there is a combination of Gaussian operations and statistical randomization.[53]

1. Example

We now consider a specific example of a stochastic and complex interaction. This is described by the following interaction Hamiltonian density:

$$\hat{\mathcal{H}} = \hat{\phi}^\dagger \hat{\phi}\, \tilde{h}[\tilde{\mathcal{G}}], \tag{F15}$$

where $\hat{\phi}$ is a complex relativistic scalar field and $\tilde{h}[\tilde{\mathcal{G}}]$ is defined as:

$$\tilde{h}[\tilde{\mathcal{G}}(x)] := \int d^4x' \Lambda(x,x')\tilde{\mathcal{G}}(x'), \tag{F16}$$

with $\Lambda(x,x') := \Lambda_R(x,x') - i\Lambda_I(x,x')$; $\Lambda_R(x,x')$ a real kernel; $\Lambda_I(x,x')$ a positive definite kernel; and $\tilde{\mathcal{G}}(x)$ a real stochastic field. The stochastic (Gaussian) scattering matrix $\hat{S}[\tilde{\mathcal{G}}]$ is then (ignoring $\hat{\mathcal{H}}_0$ for simplicity):

$$\hat{S}[\tilde{\mathcal{G}}] = Te^{-i\int d^4xd^4x'\Lambda_R(x,x')\hat{\phi}^\dagger(x')\hat{\phi}(x')\tilde{\mathcal{G}}(x')}e^{-\int d^4xd^4x'\Lambda_I(x,x')\hat{\phi}^\dagger(x')\hat{\phi}(x')\tilde{\mathcal{G}}(x')}, \tag{F17}$$

such that the (Gaussian) stochastic scattering superoperator is:

$$\hat{\mathcal{S}}_S[\tilde{\mathcal{G}}] = \hat{T}\exp\left\{-i\int d^4xd^4x'\Lambda_R(x,x')\hat{A}_\Delta(x')\tilde{\mathcal{G}}(x') - \int d^4xd^4x'\Lambda_I(x,x')\hat{A}_\Sigma(x')\tilde{\mathcal{G}}(x')\right\}, \tag{F18}$$

where $\hat{T}$ is the time-ordering superoperator; $\hat{A} := \hat{\phi}^\dagger\hat{\phi}$; $\hat{A}_\Delta = \hat{A}_+ - \hat{A}_-$ and $\hat{A}_\Sigma = \hat{A}_+ + \hat{A}_-$, with $\hat{A}_+$ acting on $\hat{\rho}_{in}$ from the left, and $\hat{A}_-$ from the right. Taking, for convenience, the probability distribution functional to be Gaussian:

$$P[\tilde{\mathcal{G}}] = (\det\Gamma)^{1/2}\, e^{-\int d^4xd^4x'\Gamma(x,x')\tilde{\mathcal{G}}(x)\tilde{\mathcal{G}}(x')}, \tag{F19}$$

with $\Gamma(x,x')$ a positive-definite symmetric kernel, then we can perform Gaussian functional integration (F10) over $\tilde{\mathcal{G}}$ to obtain $\hat{\rho}_{out} = \hat{\mathcal{S}}_{av}\hat{\rho}_{in}$, with $\hat{\mathcal{S}}_{av}$:

$$\hat{\mathcal{S}}_{av} = \hat{T}e^{\int d^4xd^4x'[-\beta_{RR}(x,x')\hat{A}_\Delta(x)\hat{A}_\Delta(x')+i\beta_{IR}(x,x')\hat{A}_\Delta(x)\hat{A}_\Sigma(x')+i\beta_{IR}(x,x')\hat{A}_\Sigma(x)\hat{A}_\Delta(x')+\beta_{II}(x,x')\hat{A}_\Sigma(x)\hat{A}_\Sigma(x')]}, \tag{F20}$$

where:

$$\beta_{RR}(x,x') := \frac{1}{4}\int d^4x''d^4x'''\Lambda_R(x,x'')\Gamma^{-1}(x'',x''')\Lambda_R(x',x'''), \tag{F21}$$

$$\beta_{IR}(x,x') := \frac{1}{4}\int d^4x''d^4x'''\Lambda_R(x,x'')\Gamma^{-1}(x'',x''')\Lambda_I(x',x'''), \tag{F22}$$

$$\beta_{II}(x,x') := \frac{1}{4}\int d^4x''d^4x'''\Lambda_I(x,x'')\Gamma^{-1}(x'',x''')\Lambda_I(x',x'''). \tag{F23}$$

We now take a Markovian approximation and define Λ_R, Λ_I and Γ as:[141]

$$\Lambda_{R,I}(x,x') = \lambda_{R,I}(x_0,\boldsymbol{r},\boldsymbol{r'})\delta(x_0 - x_0'), \tag{F24}$$

$$\Gamma(x,x') = \gamma(x_0,\boldsymbol{r},\boldsymbol{r'})\delta(x_0 - x_0'). \tag{F25}$$

The superoperator $\hat{S}_{av}$ can then be written as:

$$\hat{S}_{av} = \hat{T}\exp\left\{\int_{-\infty}^{\infty} dt\hat{\mathcal{L}}(t)\right\}, \tag{F26}$$

where $\hat{\mathcal{L}}(t)$ is the linear evolution superoperator and $\hat{T}$ the time-ordering superoperator.[141] The averaged density matrix $\hat{\rho}$ at a time t can now be obtained through:

$$\hat{\rho}(t) = \hat{T}\exp\left\{\int_0^t d\tau\hat{\mathcal{L}}(\tau)\right\}\hat{\rho}(0). \tag{F27}$$

The superoperator $\hat{\mathcal{L}}$ acts on $\hat{\rho}$ as:

$$\hat{\mathcal{L}}\hat{\rho} = \int d\boldsymbol{r}d\boldsymbol{r'}\Big[2ib_{IR}(t,\boldsymbol{r},\boldsymbol{r'})[\hat{A}(t,\boldsymbol{r})\hat{A}(t,\boldsymbol{r'}),\hat{\rho}] - b_{RR}(t,\boldsymbol{r},\boldsymbol{r'})[\hat{A}(t,\boldsymbol{r}),[\hat{A}(t,\boldsymbol{r'}),\hat{\rho}]] + b_{II}(t,\boldsymbol{r},\boldsymbol{r'})\{\hat{A}(t,\boldsymbol{r}),\{\hat{A}(t,\boldsymbol{r'}),\hat{\rho}\}\}\Big] \tag{F28}$$

where:

$$b_{RR}(t,\boldsymbol{r},\boldsymbol{r'}) := \frac{1}{4}\int d\boldsymbol{r''}\boldsymbol{r'''}\lambda_R(t,\boldsymbol{r},\boldsymbol{r''})\gamma^{-1}(t,\boldsymbol{r''},\boldsymbol{r'''})\lambda_R(t,\boldsymbol{r'},\boldsymbol{r'''}), \tag{F29}$$

$$b_{IR}(t,\boldsymbol{r},\boldsymbol{r'}) := \frac{1}{4}\int d\boldsymbol{r''}\boldsymbol{r'''}\lambda_I(t,\boldsymbol{r},\boldsymbol{r''})\gamma^{-1}(t,\boldsymbol{r''},\boldsymbol{r'''})\lambda_R(t,\boldsymbol{r'},\boldsymbol{r'''}), \tag{F30}$$

$$b_{II}(t,\boldsymbol{r},\boldsymbol{r'}) := \frac{1}{4}\int d\boldsymbol{r''}\boldsymbol{r'''}\lambda_I(t,\boldsymbol{r},\boldsymbol{r''})\gamma^{-1}(t,\boldsymbol{r''},\boldsymbol{r'''})\lambda_I(t,\boldsymbol{r'},\boldsymbol{r'''}), \tag{F31}$$

and we have used $[\hat{A}^2,\hat{\rho}] \equiv \{\hat{A},[\hat{A},\hat{\rho}]\} \equiv [\hat{A},\{\hat{A},\hat{\rho}\}]$. Therefore, $\hat{\rho}(t)$ obeys the following master equation:

$$\frac{d\hat{\rho}(t)}{dt} = \int d\boldsymbol{r}d\boldsymbol{r'}\Big[2ib_{IR}(t,\boldsymbol{r},\boldsymbol{r'})[\hat{A}(t,\boldsymbol{r})\hat{A}(t,\boldsymbol{r'}),\hat{\rho}] - b_{RR}(t,\boldsymbol{r},\boldsymbol{r'})[\hat{A}(t,\boldsymbol{r}),[\hat{A}(t,\boldsymbol{r'}),\hat{\rho}]] + b_{II}(t,\boldsymbol{r},\boldsymbol{r'})\{\hat{A}(t,\boldsymbol{r}),\{\hat{A}(t,\boldsymbol{r'}),\hat{\rho}\}\}\Big]. \tag{F32}$$

Finally, we take the non-relativistic limit and replace $\hat{\phi}$ with the non-relativistic scalar field $\hat{\Psi}$. Assuming also that β_{RR}, β_{IR} and β_{RR} are time-independent, we end up with:

$$\frac{d\hat{\rho}(t)}{dt} = \int d\boldsymbol{r}d\boldsymbol{r'}\Big(2ib_{IR}(\boldsymbol{r},\boldsymbol{r'})[\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r})\hat{\Psi}^\dagger(\boldsymbol{r'})\hat{\Psi}(\boldsymbol{r'}),\hat{\rho}(t)] - b_{RR}(\boldsymbol{r},\boldsymbol{r'})[\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r}),[\hat{\Psi}^\dagger(\boldsymbol{r'})\hat{\Psi}(\boldsymbol{r'}),\hat{\rho}(t)]] + b_{II}(\boldsymbol{r},\boldsymbol{r'})\{\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r}),\{\hat{\Psi}^\dagger(\boldsymbol{r'})\hat{\Psi}(\boldsymbol{r'}),\hat{\rho}(t)\}\}\Big), \tag{F33}$$

where we have ignored the time dependence of $\hat{\Psi}$ for simplicity. The first term is of the same form as that which would be induced by the Newtonian limit of QG (see Appendix A 2). However, despite Newtonian QG inducing non-Gaussianity (and negative Wigner functions), the other two terms conspire with the first to reduce the full process to a channel that keeps a Gaussian state in the (unnormalized) Gaussian convex hull. That is, despite the appearance of the first term, this master equation cannot turn a Gaussian state into a non-Gaussian state (defined as a state that lives outside the Gaussian convex hull). This is clear from our starting point (F10) for the averaged density matrix $\hat{\rho}$.

We can, in fact, write the solution of (F33) as a state in the standard quantum optics definition of the Gaussian convex hull by, for example, dropping the temporal and spatial dependence of $\tilde{\mathcal{G}}$, γ, λ_R and λ_I: the above master equation can then be written as:

$$\frac{d\hat{\rho}(t)}{dt} = -i\kappa_{IR}[(\hat{a}^\dagger\hat{a})^2,\hat{\rho}(t)] - \kappa_{RR}[\hat{a}^\dagger\hat{a},[\hat{a}^\dagger\hat{a},\hat{\rho}(t)]] + \kappa_{II}\{\hat{a}^\dagger\hat{a},\{\hat{a}^\dagger\hat{a},\hat{\rho}(t)\}\}, \tag{F34}$$

where we have also taken the single-mode approximation $\hat{\Psi}(\boldsymbol{r}) = \psi(\boldsymbol{r})\hat{a}$ found in the main text, and defined $\kappa_{RR} := \frac{1}{4}\kappa^2\lambda_R^2$; $\kappa_{IR} := \frac{1}{2}\kappa^2\lambda_I\lambda_R$; $\kappa_{II} := \frac{1}{4}\kappa^2\lambda_I^2$; $\kappa := \int d\boldsymbol{r}|\psi(\boldsymbol{r})|^2$ and $\gamma = \delta^{(3)}(\boldsymbol{r}-\boldsymbol{r'})$ for convenience. Using (F10), the solution to (F34) can be written as:

$$\hat{\rho}(t) = \int dg P(g,t) e^{-i\kappa\lambda_R\hat{a}^\dagger\hat{a}gt - \kappa\lambda_I\hat{a}^\dagger\hat{a}gt}\hat{\rho}(0)e^{i\kappa\lambda_R\hat{a}^\dagger\hat{a}gt - \kappa\lambda_I\hat{a}^\dagger\hat{a}gt}, \tag{F35}$$

with:

$$P(g,t) := \sqrt{\frac{t}{\pi}}e^{-g^2 t}, \tag{F36}$$

where $g \in \mathbb{R}$ is a dummy variable used in place of $\tilde{\mathcal{G}}$. If $\hat{\rho}(0)$ in (F35) is a pure Gaussian state, the density matrix $\hat{\rho}(t)$ of (F35), which solves (F34), is then part of the (in general, non-normalized) Gaussian convex hull (F14).

The master equations (F32) and (F33) (and so also (F34)) do not preserve the norm of the state. As detailed above, in order to preserve the norm, each stochastic density matrix can be redefined through (F8) and we can then take these as the physical stochastic density matrices. However, this results in a non-linear evolution of the new averaged density matrix $\hat{\rho}$, which can lead to superliminal signalling.[138,141] This issue can also be found in objective-collapse theories where matter is coupled to a stochastic field through an anti-Hermitian term involving a particular matter operator $\hat{A}$.[139,140] In these models a term of the form $\hat{A}^2$ is included in the evolution of the stochastic state vector in order to eliminate the problematic non-linear terms in the evolution of the averaged density matrix.[139–141,177,178,217–219] Such higher-order terms can also be used to eliminate the non-norm preserving terms in the evolution of the non-normalized density matrix.[141] For example, to our Hamiltonian density (F15), we can include a term of the form $\hat{A}^2$:

$$\hat{\mathcal{H}}[\tilde{\mathcal{G}}(x)] := \int d^4x' \Lambda(x,x')\tilde{\mathcal{G}}(x')\hat{A}(x) - 2i \int d^4x' \beta_{II}(x,x')\hat{A}(x')\hat{A}(x), \tag{F37}$$

with $\hat{A}(x) := \hat{\phi}^\dagger(x)\hat{\phi}(x)$ and β_{II} defined in (F23). Taking the Markovian limit and assuming a Gaussian profile for $\tilde{\mathcal{G}}$ as above, the new term turns the non-norm preserving term in (F32) into a norm-preserving term:

$$\frac{d\hat{\rho}(t)}{dt} = \int d\boldsymbol{r}d\boldsymbol{r'} \Big(2ib_{IR}[\hat{A}(t,\boldsymbol{r})\hat{A}(t,\boldsymbol{r'}), \hat{\rho}(t)] - (b_{RR} + b_{II})[\hat{A}(t,\boldsymbol{r}), [\hat{A}(t,\boldsymbol{r'}), \hat{\rho}(t)]] \Big). \tag{F38}$$

Since $\hat{A} := \hat{\phi}^\dagger\hat{\phi}$, the new term in (F37) is an (anti-Hermitian) quantum self-interaction of matter. That is, we have effectively introduced a new force. This new quantum force will, in general, induce non-Gaussianity. However, if we take $\Lambda_R = 0$ in (F37) so that the stochastic interaction is anti-Hermitian (as is usually the case in objective-collapse theories), then the master equation simplifies to:

$$\frac{d\hat{\rho}(t)}{dt} = - \int d\boldsymbol{r}d\boldsymbol{r'} b_{II}(t,\boldsymbol{r},\boldsymbol{r'})[\hat{A}(t,\boldsymbol{r}), [\hat{A}(t,\boldsymbol{r'}), \hat{\rho}(t)]]. \tag{F39}$$

which is a master equation that preserves the Gaussian convex hull since such a master equation is also derived when taking $\Lambda_I = 0$ in the original theory without the new quantum self-interaction (see (F32) with $b_{IR} = b_{II} = 0$). When taking the non-relativistic limit $\hat{\phi} \to \hat{\Psi}$, (F39) is of the form of the master equation found in objective-collapse theories such as CSL and Diósi-Penrose.[177,178,220] It is also the master equation of continuous-time measurements in the basis $\hat{A}$, such that we can essentially consider the stochastic field $\tilde{\mathcal{G}}$ and new quantum self-interaction $\beta_{II}\hat{A}^2$ working together to perform continuous measurements of matter (that preserve the Gaussian convex hull).

If, however, both Λ_R and Λ_I are non-zero (see e.g. [139,141] for similar models), then, in general, the non-Gaussian character of the new quantum self-interaction $\beta_{II}\hat{A}^2$ is preserved, and we have a channel that can induce non-Gaussianity. Even so, in the asymptotic limit, the state will become a state of the Gaussian convex hull rather than a non-Gaussian state.

When both Λ_R and Λ_I are non-zero (and we also have the new quantum self-interaction $\beta_{II}\hat{A}^2$), the theory is closely related to a continuous-time measurement being performed by the two interactions as above but now with a feedback mechanism.[141] Note that weak measurements with local feedback operations can also induce entanglement in the case of joint measurements.[175,176]