Pregeometry and euclidean quantum gravity

Christof Wetterich
Institut für Theoretische Physik
Universität Heidelberg
Philosophenweg 16, D-69120 Heidelberg

Einstein's general relativity can emerge from pregeometry, with the metric composed of more fundamental fields. We formulate euclidean pregeometry as a $SO(4)$ - Yang-Mills theory. In addition to the gauge fields we include a vector field in the vector representation. The gauge - and diffeomorphism - invariant kinetic terms for these fields permit a well-defined euclidean functional integral, in contrast to metric gravity with the Einstein-Hilbert action. The propagators of all fields are well behaved at short distances, without tachyonic or ghost modes. The long distance behavior is governed by the composite metric and corresponds to general relativity. In particular, the graviton propagator is free of ghost or tachyonic poles despite the presence of higher order terms in a momentum expansion of the inverse propagator. This pregeometry seems to be a valid candidate for euclidean quantum gravity, without obstructions for analytic continuation to a Minkowski signature of the metric.

I. Introduction

In a modern view, quantum field theories are defined by functional integrals. For many models of particle physics one can employ an euclidean functional integral and continue analytically to Minkowski signature. No well defined functional integral is known for a continuum formulation of quantum gravity based on the metric field. Such a functional integral would require the explicit specification of the microscopic or classical action. For a formulation in terms of the metric any action involving only a finite number of derivatives is problematic, however. The Einstein-Hilbert action with a cosmological constant involves up to two derivatives of the metric. The restriction to two derivatives is not compatible with a renormalizable theory. Furthermore, the euclidean action is not bounded from below, such that no corresponding euclidean functional integral can be defined. Adding invariants with up to four derivatives of the metric leads to a renormalizable theory [1–3]. Also the euclidean action can be bounded from below for appropriate signs of the couplings, such that an euclidean functional integral can be defined. The problem arises now from the presence of instabilities in the propagators in flat or weakly curved geometries. In flat space, ghosts or tachyons cannot be avoided for any finite polynomial momentum expansion of the inverse propagator beyond quadratic order in momenta. This implies that the addition of terms with a finite number of derivatives cannot cure the problem.

Quantum gravity based on the metric may be a nonperturbatively renormalizable quantum field theory. This is the case if the functional flow of a scale-dependent effective action [4] admits an ultraviolet fixed point. Asymptotically safe quantum gravity [5–10] is defined by choosing this ultraviolet fixed point for a definition of the short distance behavior of quantum gravity. The fixed point behavior permits to extrapolate this short distance behavior to arbitrarily short distances or high momenta.

A fixed point corresponds to a scaling solution of an exact functional flow equation [4]. Such scaling solutions have been found for many approximated or "truncated" forms of the scale-dependent effective action [11]. This makes it plausible that an ultraviolet fixed point indeed exists, rendering metric quantum gravity non- perturbatively renormalizable.

A functional integral can be defined formally as a solution of the exact flow equation. In practice, this is of little help if the solution gets too complicated. This is presently the case for the functional renormalization group approach. The fixed point involves many invariants, and it is not clear what approximation is needed in order to obtain a well defined propagator for the graviton without ghost or tachyonic instabilities [12]. This explains why so far no proposal for a simple classical action defining the functional integral for quantum gravity has been formulated in this approach.

One possibility to avoid these problems is a formulation of the functional integral in terms of fields different from the metric. This approach stays within the setting of quantum field theories. In analogy to quantum chromodynamics (QCD) for the strong interactions in particle physics, the short distance behavior - gluons and quarks in QCD - is described by fields different from the metric. The metric arises as a collective or composite field in an effective low energy theory, in analogy to mesons and baryons in QCD. This type of quantum field theory with an emergent metric may be called "pregeometry" [13]. The most radical approach suggests to use only fermions as fundamental degrees of freedom [13–15]. In spinor gravity a formulation with local Lorentz symmetry and diffeomorphism invariance has indeed been achieved [16–18]. (For first formulations with global Lorentz symmetry see ref. [19, 20].) The vierbein and the metric have been constructed as composites of the fermions. While conceptually rather attractive, practical computations in this purely fermionic setting have not yet advanceed very much, due to the structure of the action containing only invariants with a rather high number of fermions.

The present work proposes a formulation of euclidean pregeometry as a $SO(4)$ - Yang-Mills theory. Besides fermions, which we do not discuss explicitly in the present paper, the degrees of freedom of pregeometry are the six gauge fields A_μ of the $SO(4)$ - gauge symmetry, and four additional vector fields e_μ which belong to the vector representation of $SO(4)$. A diffeomorphism invariant action is based on the gauge invariant kinetic terms of the two sorts of vector fields A_μ and e_μ. This model has been discussed earlier [21, 22], see also ref [23] for related ideas.

In the present paper we demonstrate that the euclidean action involving the sum of the invariant kinetic terms for A_μ and e_μ is bounded from below. The euclidean functional integral is well defined for this type of pregeometry. The short distance propagators show neither ghosts nor tachyons. These stability properties may be expected since only terms with two derivatives of the fields are included in the effective action.

The metric is a composite field defined as a $SO(4)$ - invariant bilinear of the vector fields e_μ. The vector fields e_μ play the role of generalized vierbeins. In contrast to Cartan's geometry [24] the covariant derivative of e_μ does not vanish, however. This distinguishes the present approach from many formulations where the gauge fields are associated to the spin connection which is a functional of the vierbein [24–35]. In our approach both the vierbein and the gauge fields contain propagating degrees of freedom beyond the ones contained in the metric. They are crucial for the different short distance behavior. (For first functional renormalization group investigations of Cartan geometry see refs [36–38], and for alternative ideas on diffeomorphism invariant gauge theories cf. ref. [39, 40].)

The classical field equations have a solution with vanishing gauge fields A_μ and constant vierbeins e_μ, leading for the composite metric $g_{\mu\nu}$ to a geometry of flat space, $g_{\mu\nu} = \delta_{\mu\nu}$. This solution entails a "spontaneous symmetry breaking" of the $SO(4)$ - gauge symmetry. As a consequence of this "generalized Higgs mechanism" the gauge bosons acquire a mass term and mix with the other vector field e_μ. Linear combinations of both fields decouple from the effective low energy theory, which can be defined for momenta smaller than the gauge boson mass.

Field equations and effective low energy theory should be discussed in terms of the quantum effective action which includes all effects of quantum fluctuations. We will not perform here a computation of the quantum effective action. We rather make the assumption that the quantum fluctuations generate for the effective action all terms allowed by diffeomorphism and gauge symmetries. In particular, this includes a term linear in the derivatives of the gauge fields, and a cosmological constant. There are also additional terms with two derivatives of A_μ, as well as terms with more than two derivatives that we neglect here.

The effective low energy theory is dominated by the terms with a small number of derivatives. In leading order this turns out to be general relativity for the metric, with an Einstein-Hilbert action and a possible cosmological constant that we assume to be very small, as required by observation. General relativity emerges naturally from the present formulation of pregeometry. Beyond the leading Einstein-Hilbert term the effective low energy theory contains terms with more than two derivatives for the metric. An expansion in the number of derivatives yields at fourth order an effective theory of the type of ref [1]. One may therefore wonder what happens to the ghost instability in the graviton propagator for this fourth order gravity.

In our formulation of pregeometry the graviton fluctuation is a linear combination of fields contained in A_μ and e_μ. As a result of the diagonalization of the propagator matrix we find that the dependence of the inverse graviton propagator on the squared momentum q^2 is given by a function

$$G^{-1}_{\text{grav}}(q^2) = \frac{m^2}{8}\Big\{(Z+1)q^2 + m^2 - M^2 - \sqrt{\big[(Z-1)q^2 + m^2 - M^2\big]^2 + 4\frac{q^2}{m^2}\big(m^2 - M^2\big)^2}\Big\} \,. \tag{1}$$

It involves three model parameters, namely m^2 which multiplies the kinetic term for the vierbein and determines the effective mass for the gauge bosons, the coupling Z which multiplies the gauge boson kinetic term and equals the inverse squared gauge coupling, and M^2 which multiplies the term linear in derivatives and can be associated with the squared Planck mass. The absence of ghost or tachyonic instabilities is guaranteed for the parameter range

$$0 < M^2 < m^2 \,, \qquad 0 < Z < Z_c \,, \tag{2}$$

with

$$Z_c = \frac{M^2}{m^2}\left(1 - \frac{M^2}{m^2}\right)^{-1} \,. \tag{3}$$

For this parameter range the graviton propagator is well behaved with a single pole at $q^2 = 0$ and without any instability. The ghost in fourth order gravity is an artefact of the truncated polynomial expansion [12].

The euclidean classical action can be continued analitically to Minkowski signature. The same holds for the graviton propagator (1). So far we have not found any obstruction to analytic continuation. We may therefore consider our model of pregeometry as a proposal for a functional integral for quantum gravity.

Our model of pregeometry is introduced in sect. II, where we also show that flat space solves the classical field equations. Sect. III discusses the generalized Higgs mechanism. It proceeds to a decomposition of the fluctuations around flat space according to different representations of its symmetry. This is the basis for the stability analysis of the classical theory which we address in sect. IV. There we establish that no ghosts or tachyons are present in our model. In sect. V we turn to the quantum effective action and the emergence of general relativity as an effective theory for small momenta and curvature. The graviton propagator is discussed in detail in sect. VI. Together with a stable graviton our model of pregeometry also contains a stable massive spin-two particle. Sect. VII turns to the sector of scalar fields. Flat space is not the absolute minimum of the euclidean action in the space of all possible field configurations. It may, however, be the minimum in the space of solutions to the field equations. In sect. VIII we add the effects of a cosmological constant. Sect. IX finally discusses our findings and possible extensions in various directions.

II. Pregeometry

Similar to the electroweak or strong interactions our model is a non-abelian local gauge theory or Yang-Mills theory. For our euclidean setting the gauge group is $SO(4)$. The particularity as compared to the gauge theories in particle physics is the presence of an additional vector field $e_\mu{}^m(x)$ in the vector representation of $SO(4)$. We restrict the space of field configurations in the functional integral to $e = \det(e_\mu{}^m) > 0$, where $e_\mu{}^m$ is considered as a 4×4 - matrix. This allows for the definition of the inverse matrix $e_m{}^\mu$, and the construction of diffeomorphism invariant kinetic terms for both the gauge fields A_μ and the vector fields e_μ.

1. Diffeomorphism invariant Yang-Mills theory

Our starting point is a SO(4)-gauge theory with gauge fields

$$A_{\mu mn} = -A_{\mu nm}, \qquad n,\ m = 0, \cdots, 3\,, \tag{4}$$

and field strength

$$F_{\mu\nu mn} = \partial_\mu A_{\nu mn} - \partial_\nu A_{\mu mn} + A_{\mu m}{}^p A_{\nu pn} - A_{\nu m}{}^p A_{\mu pn}\,. \tag{5}$$

Here $z = (m, n)$ is a double-index labeling the six gauge bosons in the adjoint representation of $SO(4)$. We further include vector fields $e_\mu{}^m$ in the vector representation of $SO(4)$. Accordingly, the covariant derivative reads

$$U_{\mu\nu}{}^m = D_\mu e_\nu{}^m = \partial_\mu e_\nu{}^m - \Gamma_{\mu\nu}{}^\sigma e_\sigma{}^m + A_\mu{}^m{}_n e_\nu{}^n\,. \tag{6}$$

These vector fields are restricted to obey

$$e = \det(e_\mu{}^m) > 0\,, \tag{7}$$

where the components $e_\mu{}^m$ are considered as elements of a regular 4×4 matrix. This permits the definition of inverse vector fields $e_m{}^\mu$, with

$$e_\mu{}^m e_m{}^\nu = \delta_\mu^\nu\,, \qquad e_m{}^\mu e_\mu{}^n = \delta_m^n\,. \tag{8}$$

We introduce a composite metric $g_{\mu\nu}$ as a bilinear in the vector field and note that for $e \neq 0$ its inverse $g^{\mu\nu}$ exists,

$$g_{\mu\nu} = e_\mu{}^m e_\nu{}^n \delta_{mn}, \qquad g^{\mu\nu} g_{\nu\rho} = \delta^\mu_\rho\,. \tag{9}$$

The connection $\Gamma_{\mu\nu}{}^\sigma$ in Eq. (6) is the Levi-Civita connection formed with the composite metric $g_{\mu\nu}$. Inserting eq. (9) it can be expressed in terms of the vector field and its derivatives. For $D_\mu e_\nu{}^m = 0$ the vector field $e_\mu{}^m$ can play the role of the usual vierbein. We will not impose this constraint here such that the vector field contains additional degrees of freedom.

The field strengh (5) and the covariant derivative (6) transform as tensors. These tensors can be used as basic building blocks for the construction of invariant kinetic terms for the gauge fields $A_{\mu mn}$ and the vector field $e_\mu{}^m$. We define a diffeomorphism invariant action

$$S = \int \mathrm{d}^4x\, e(L_F + L_U)\,, \tag{10}$$

with gauge field kinetic term

$$L_F = \frac{Z}{8} F_{\mu\nu mn} F^{\mu\nu mn}\,, \tag{11}$$

and vector field kinetic term

$$L_U = \frac{m^2}{4} U_{\mu\nu m} U^{\mu\nu m}\,. \tag{12}$$

Here world (space-time) indices μ, ν are raised and lowered with $g^{\mu\nu}$ and $g_{\mu\nu}$ given by eq. (9), while "Lorentz indices" m, n are raised and lowered with δ^{mn} and δ_{mn}. We can convert Lorentz indices to world indices by multiplication with $e_\mu{}^m$ or $e_m{}^\mu$,

$$U_{\mu\nu\rho} = U_{\mu\nu}{}^m e_{\rho m}\,. \tag{13}$$

For $U_{\mu\nu}{}^m \neq 0$ the covariant derivatives commute with the raising and lowering of world indices or Lorentz indices, but not with the conversion from world to Lorentz indices,

$$D_\sigma U_{\mu\nu\rho} = (D_\sigma U_{\mu\nu}{}^m)\, e_{\rho m} + U_{\mu\nu}{}^m U_{\sigma\rho m}\,. \tag{14}$$

Because of their analogous role to the vierbein in Cartan's geometry [24] we will call the vector fields $e_\mu{}^m$ "generalized vierbeins", or simply "vierbeins".

Two different connections are present in our setting, the gauge connection $A_{\mu mn}$ and the geometric connection $\Gamma_{\mu\nu}{}^\rho$. The tangent bundle and the gauge bundle are, a priori, not related. For example, one can have the metric corresponding to a sphere and nevertheless vanishing gauge fields, $A_{\mu mn} = 0$. This permits, for example, to have massless fermions on a sphere [21]. A relation between the connections can be formulated by the identity

$$U_{\mu\nu\rho} = \omega_{\mu\nu\rho} - A_{\mu\nu\rho}\,, \tag{15}$$

with $\omega_{\mu\nu\rho}$ related to the "spin connection" formed from the vector field

$$\begin{aligned}\omega_{\mu\nu\rho} = \frac{1}{2}\Big\{ & e_{\mu m}\,(\partial_\rho e_\nu{}^m - \partial_\nu e_\rho{}^m) + e_{\nu m}\,(\partial_\rho e_\mu{}^m - \partial_\mu e_\rho{}^m) \\ & + e_{\rho m}\,(\partial_\mu e_\nu{}^m - \partial_\nu e_\mu{}^m) \Big\} = e_\nu{}^m e_\rho{}^n \omega_{\mu mn}\,.\end{aligned} \tag{16}$$

This expresses the Levi-Civita connection in terms of the vierbein by

$$\omega_{\mu\nu\rho} = \Gamma_{\mu\rho}{}^\sigma g_{\sigma\nu} - e_{\nu m}\partial_\mu e_\rho{}^m\,. \tag{17}$$

We observe the antisymmetry in the last two indices

$$\omega_{\mu\rho\nu} = -\omega_{\mu\nu\rho}\,,\ \ A_{\mu\rho\nu} = -A_{\mu\nu\rho}\,,\ \ U_{\mu\rho\nu} = -U_{\mu\nu\rho}\,. \tag{18}$$

For a covariantly conserved vector field, $U_{\mu\nu\rho} = 0$, the gauge field equals the spin connection formed from the vierbein. This is the setting of Cartan's geometry [24].

2. Boundedness of the action and euclidean functional integral

The action (10) is bounded from below, $S \geqslant 0$, as long as $m^2 \geqslant 0$, $Z \geqslant 0$. In order to see this, we first show that the composite metric has only positive eigenvalues. Since $g_{\mu\nu}$ is a real symmetric matrix, it can be diagonalized by an orthogonal transformation O,

$$\begin{aligned}\tilde{g}_{\rho\sigma} =& O_\rho{}^\mu O_\sigma{}^\nu g_{\mu\nu} = O_\rho{}^\mu e_\mu{}^m O_\sigma{}^\nu e_\nu{}^n \delta_{mn} \\ =& \tilde{e}_\rho{}^m \tilde{e}_\sigma{}^n \delta_{mn} ,\end{aligned} \tag{19}$$

with $\tilde{e}_\rho{}^m = O_\rho{}^\mu e_\mu{}^m$. The diagonal elements are indeed positive (no sum over ρ)

$$\tilde{g}_{\rho\rho} = \sum_m (\tilde{e}_\rho{}^m)^2 > 0 . \tag{20}$$

For $e > 0$ one can exclude vanishing eigenvalues $\tilde{g}_{\rho\rho}$.

We next write for $z = (m, n)$

$$F_{\mu\nu mn} = F_{\alpha z} , \qquad F_{\mu\nu mn} F^{\mu\nu mn} = \sum_z F_{\alpha z} G^{\alpha\beta} F_{\beta z} , \tag{21}$$

with $\alpha = (\mu, \nu)$, $\beta = (\rho, \sigma)$ double indices, and real symmetric matrix

$$G^{\alpha\beta} = g^{\mu\rho} g^{\nu\sigma} = G^{\beta\alpha} . \tag{22}$$

The expression L_F is positive semidefinite if $G^{\alpha\beta}$ has only positive eigenvalues. This is indeed the case, as can be seen by diagonalizing $G^{\alpha\beta}$ by an orthogonal transformation. We can chose the required orthogonal matrix as the direct product of the matrices that diagonalize $g^{\mu\nu}$, resulting in $\tilde{G}^{\alpha\alpha} = \tilde{g}^{\mu\mu}\tilde{g}^{\nu\nu} > 0$. The same argument shows $L_U \geqslant 0$, with $z = m$ in this case. With both L_U and L_F positive semidefinite and $e > 0$, the action is indeed positive semidefinite, $S \geq 0$. The action vanishes precisely if both L_F and L_U vanish.

With positive semidefinite classical action S an euclidean functional integral can be defined by integrating over unconstrained gauge fields and vector fields obeying the constraint (7). The constraint $e > 0$ is invariant under local $SO(4)$ - gauge transformations and diffeomorphisms. In presence of these two local symmetries the action needs a regularization, either by the usual gauge fixing procedure for a continuous formulation, or perhaps by a lattice formulation that still has to be found. We consider the action (10) as a well defined and simple starting point for an investigation of quantum fluctuactions in euclidean gravity. The analytic continuation to a Minkowski signature will be discussed in sect. VI.

Configurations with a constant vierbein and vanishing gauge fields include the "flat space configuration"

$$e_\mu{}^m = \delta_\mu^m , \quad g_{\mu\nu} = \delta_{\mu\nu} , \quad A_{\mu mn} = 0 . \tag{23}$$

These configurations yield $U_{\mu\nu}{}^m = 0$ and are therefore minima of the action (10). They are solutions of the corresponding field equations that we discuss in more detail in sect. V.

III. Generalized Higgs mechanism

In order to understand the physical content of our model at short distances we decompose the fluctuations of the vector field and the gauge fields around flat space and zero gauge fields. We work in euclidean flat space. Analytic continuation to Minkowski space can be performed at the end. For sufficiently short distances one can neglect any effects of a given geometry, as encoded in $e_\mu{}^m$ or $g_{\mu\nu}$, or of a given gauge field configuration $A_{\mu mn}$. For the short distance limit the flat space configuration $e_\mu{}^m = \delta_\mu^m$, $A_{\mu mn} = 0$ is always an appropriate starting point for a field-expansion.

The decomposition involves different representations of the "euclidean Lorentz group" $SO(4)$ that cannot mix in quadratic order. Mixing is observed, however, between modes belonging to the same representation. This renders the discussion of stability somewhat complex. In the present section we discuss the general structure of the mixing and focus subsequently on the graviton propagator and the scalar propagator in the next section. We will find a stable graviton propagator without ghosts or tachyons.

The discussion will be extended in sect. V to the quantum effective action by including a further invariant that dominates a low momenta. The graviton propagator for this extended setting is discussed in sect. VI, and scalars are addressed in sect. VII.

Due to spontaneous symmetry breaking by the nonvanishing value of the vierbein $e_\mu{}^m = \delta_\mu^m$ mass terms are generated for the gauge bosons. This "generalized Higgs mechanism" differs from the usual Higgs mechanism since the agent of spontaneous symmetry breaking is a vector field and not a scalar field. The mode decomposition reveals the detailed structure of the generalized Higgs mechanism by focusing on the effects for the propagators of the various fluctuation modes. The mode mixing is a characteristic feature of this generalized Higgs mechanism.

1. Fluctuations in flat space

The field equations derived from the action (10) admit as a solution flat space, $e_\mu{}^m = \delta_\mu^m$, $g_{\mu\nu} = \delta_{\mu\nu}$, with vanishing gauge fields $A_{\mu mn} = 0$. This solution will be found to be stable, as may be expected since it corresponds to the minimum of the action. For a more detailed investigation of the stability for various modes we expand in quadratic order in $H_{\mu\nu}$ and $A_{\mu mn}$,

$$e_\mu{}^m = \delta_\mu^m + \frac{1}{2} H_{\mu\nu} \delta^{\nu m} . \tag{24}$$

The standard kinetic term for the transverse gauge fields reads in momentum space

$$\int_x e L_F = \frac{Z}{4} \int_q A_{\mu mn}(-q) \left(q^2 \delta^{\mu\nu} - q^\mu q^\nu \right) A_\nu{}^{mn}(q) . \tag{25}$$

This dominates the high momentum behavior and ensures stability in this range. For the discussion of fluctuations world indices are raised and lowered with $\delta^{\mu\nu}$ and $\delta_{\mu\nu}$.

In quadratic order the covariant kinetic term L_U for the vierbein contains three pieces $L_U = L_U^{(1)} + L_U^{(2)} + L_U^{(3)}$. The first piece is a kinetic term which reads, with $\partial^2 = \partial^\rho\partial_\rho$,

$$\int_x eL_U^{(1)} = -\frac{m^2}{16}\int_x \Big\{ H_{\mu\nu}^{(\mathrm{A})}\partial^2 H^{(\mathrm{A})\mu\nu} + 2H_{\mu\nu}^{(\mathrm{S})}(\partial^2\delta^\nu_\rho - \partial^\nu\partial_\rho)H^{(\mathrm{S})\rho\mu} + 4H_{\mu\nu}^{(\mathrm{A})}\partial^\nu\partial_\rho H^{(\mathrm{A})\rho\mu}\Big\} . \tag{26}$$

Here $H_{\mu\nu}^{(\mathrm{S})} = (H_{\mu\nu} + H_{\nu\mu})/2$ and $H_{\mu\nu}^{(\mathrm{A})} = (H_{\mu\nu} - H_{\nu\mu})/2$ are the symmetric and antisymmetric parts of the fluctuation $H_{\mu\nu}$. The second piece acts as a mass term for the gauge bosons. Without mixing effects (see below), it would generate an equal mass for all gauge bosons

$$eL_U^{(2)} = \frac{m^2}{4}A_{\mu mn}A^{\mu mn} . \tag{27}$$

Finally, the third piece is a type of source term for the gauge bosons,

$$eL_U^{(3)} = -J_{\mu\nu\rho}A^{\mu\nu\rho} , \tag{28}$$

with

$$J_{\mu\nu\rho} = \frac{m^2}{2}\left(\partial_\mu H_{\nu\rho}^{(\mathrm{A})} + \partial_\rho H_{\mu\nu}^{(\mathrm{S})} - \partial_\nu H_{\mu\rho}^{(\mathrm{S})}\right) . \tag{29}$$

The field value $e_\mu{}^m = \delta_\mu^m$ breaks the SO(4)-gauge symmetry spontaneously. Similarly to the Higgs mechanism, this spontaneous symmetry breaking generates a mass term for the gauge bosons. In contrast to the Higgs mechanism the field $e_\mu{}^m$ is a vector, not a scalar. Flat space preserves a global SO(4)-"Lorentz" symmetry of simultaneous SO(4)-gauge rotations and SO(4)-coordinate transformations. It is this residual symmetry that we employ for the mode expansion.

2. Decomposition of vierbein fluctuations

For an analysis of the kinetic term (27) for the vierbein we employ the decomposition which orders the scalar fluctuations into a physical and a gauge degree of freedom [41],

$$H_{\mu\nu}^{(\mathrm{S})} = t_{\mu\nu} + \partial_\mu\kappa_\nu + \partial_\nu\kappa_\mu + \frac{1}{3}\left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)\sigma + \frac{\partial_\mu\partial_\nu}{\partial^2}u , \tag{30}$$

and

$$H_{\mu\nu}^{(\mathrm{A})} = b_{\mu\nu} + \partial_\mu\gamma_\nu - \partial_\nu\gamma_\mu . \tag{31}$$

Here $t_{\mu\nu}$, $b_{\mu\nu}$, κ_μ and δ_μ are all transversal,

$$\partial^\mu t_{\mu\nu} = \partial^\nu t_{\mu\nu} = \partial^\mu b_{\mu\nu} = \partial^\nu b_{\mu\nu} = 0 , \quad \partial^\mu\kappa_\mu = \partial^\mu\gamma_\mu = 0 , \qquad t_\mu{}^\mu = 0 . \tag{32}$$

The kinetic piece reads in momentum space

$$\int_x eL_U^{(1)} = \frac{m^2}{16}\int_q \Big\{ 2t_{\mu\nu}(-q)q^2 t^{\mu\nu}(q) + b_{\mu\nu}(-q)q^2 b^{\mu\nu}(q) + 2\left(\kappa_\mu(-q) - \gamma_\mu(-q)\right)q^4\left(\kappa^\mu(q) - \gamma^\mu(q)\right) + \frac{2}{3}\sigma(-q)q^2\sigma(q)\Big\} . \tag{33}$$

The factor q^4 for $\kappa_\mu - \gamma_\mu$ is a consequence of the particular normalization of these fields. Indeed, we can absorb a factor $4q^2$ in a dimensionless normalization of κ_μ and γ_μ. All kinetic terms are then positive and increase $\sim q^2$. The kinetic term $L_U^{(1)}$ does not involve the combination $\kappa_\mu + \gamma_\mu$ and the scalar u. Similarly, L_F does not involve the longitudinal components $\partial^\mu A_{\mu mn}$ of the gauge fields. These fields correspond to gauge degrees of freedom.

3. Decomposition of gauge field fluctuations

In flat space we need no longer to distinguish between world and Lorentz indices. Covariant derivatives become partial derivatives. We decompose the linear gauge fields $A_{\mu\nu\rho}$ into transversal modes $B_{\mu\nu\rho}$ and longitudinal modes $L_{\nu\rho}$,

$$A_{\mu\nu\rho} = B_{\mu\nu\rho} + \partial_\mu L_{\nu\rho} , \quad \partial^\mu B_{\mu\nu\rho} = 0 . \tag{34}$$

Using the projector

$$P_\mu^\nu = \delta_\mu^\nu - \frac{\partial_\mu\partial^\nu}{\partial^2} , \quad \partial^\mu P_\mu^\nu = 0 , \quad P_\mu^\nu\partial_\nu = 0 , \tag{35}$$

we can write

$$B_{\mu\nu\rho} = P_\mu^\sigma A_{\sigma\nu\rho} = P_\mu^\sigma B_{\sigma\nu\rho} . \tag{36}$$

The transversal fluctuations can be decomposed as

$$B_{\mu\nu\rho} = \frac{1}{4}\varepsilon_{\nu\rho}{}^{\sigma\tau}(P_{\mu\sigma}v_\tau - P_{\mu\tau}v_\sigma) + \frac{1}{3}(P_{\mu\nu}w_\rho - P_{\mu\rho}w_\nu) + D_{\mu\nu\rho} , \tag{37}$$

where v_μ and w_μ are (reducible) four-vectors. The decomposition of the remaining part $D_{\mu\nu\rho}$ into irreducible representations is given by

$$D_{\mu\nu\rho} = \frac{1}{2}(\partial_\nu E_{\mu\rho} - \partial_\rho E_{\mu\nu}) + C_{\mu\nu\rho} . \tag{38}$$

The transversal traceless symmetric tensor $E_{\mu\nu}$ obeys

$$\partial^\mu E_{\mu\nu} = 0 , \quad \delta^{\mu\nu}E_{\mu\nu} = 0 , \quad E_{\mu\nu} = E_{\nu\mu} , \tag{39}$$

and $C_{\mu\nu\rho}$ is subject to the constraints

$$C_{\mu\nu\rho} = -C_{\mu\rho\nu} , \quad \partial^\mu C_{\mu\nu\rho} = 0 , \quad \varepsilon^{\mu\nu\rho\sigma}C_{\mu\nu\rho} = 0 , \quad \delta^{\mu\nu}C_{\mu\nu\rho} = \delta^{\mu\rho}C_{\mu\nu\rho} = 0 , \quad \tilde{P}_{\sigma\tau}{}^{\nu\rho}C_{\mu\nu\rho} = 0 . \tag{40}$$

We have introduced here a further projector

$$\tilde{P}_{\sigma\tau}{}^{\nu\rho} = \frac{1}{2\partial^2}(\partial_\sigma\partial^\nu\delta_\tau^\rho - \partial_\tau\partial^\nu\delta_\sigma^\rho - \partial_\sigma\partial^\rho\delta_\tau^\nu + \partial_\tau\partial^\rho\delta_\sigma^\nu) , \tag{41}$$

which obeys

$$\tilde{P}_{\sigma\tau}{}^{\alpha\beta}\tilde{P}_{\alpha\beta}{}^{\nu\rho} = \tilde{P}_{\sigma\tau}{}^{\nu\rho} , \quad \tilde{P}_{\nu\rho}{}^{\nu\rho} = 3 , \quad \partial^\tau\tilde{P}_{\sigma\tau}{}^{\nu\rho} = \frac{1}{2}(\partial^\rho\delta_\sigma^\nu - \partial^\nu\delta_\sigma^\rho) . \tag{42}$$

This results in the identities

$$\tilde{P}_{\sigma\tau}{}^{\nu\rho}D_{\mu\nu\rho} = (\partial_\sigma E_{\mu\tau} - \partial_\tau E_{\mu\sigma}) , \tag{43}$$

and

$$\tilde{P}_{\sigma\tau}{}^{\nu\rho}(\partial_\nu E_{\mu\rho} - \partial_\rho E_{\mu\nu}) = (\partial_\sigma E_{\mu\tau} - \partial_\tau E_{\mu\sigma}) \,. \tag{44}$$

We can therefore see $C_{\mu\nu\rho}$ as a projection,

$$C_{\mu\nu\rho} = D_{\mu\nu\rho} - \tilde{P}_{\nu\rho}{}^{\sigma\tau} D_{\mu\sigma\tau} \,. \tag{45}$$

Out of the 24 components $A_{\mu\nu\rho}$ six degrees of freedom $L_{\nu\rho} = -L_{\rho\nu}$ are longitudinal degrees of freedom. Thus there are 18 transversal modes $B_{\mu\nu\rho}$. The four modes v_μ correspond to the totally antisymmetric part $A_{[\mu\nu\rho]}$, while the four modes w_ρ account for the trace $A_{\mu\nu\rho}\delta^{\mu\nu}$. There remain 10 modes for $D_{\mu\nu\rho}$. The projector $\tilde{P}$ eliminates half of them, and the traceless transversal symmetric tensor $E_{\mu\nu}$ accounts indeed for five modes. The other five modes correspond to $C_{\mu\nu\rho} = -C_{\mu\rho\nu}$, which indeed is subject to 19 constraints. The vectors v_μ and w_μ can be decomposed into transversal vectors and scalars

$$\begin{aligned} &v_\mu = v_\mu^{(t)} + \partial_\mu \tilde{v} \,, \quad w_\mu = w_\mu^{(t)} + \partial_\mu \tilde{w} \,, \\ &\partial^\mu v_\mu^{(t)} = 0 \,, \quad \partial^\mu w_\mu^{(t)} = 0 \,. \end{aligned} \tag{46}$$

The irreducible representations of the transversal modes $B_{\mu\nu\rho}$ are $2\times5+2\times3+2\times1$. In particular, the scalar part of $B_{\mu\nu\rho}$ reads

$$B^{(s)}_{\mu\nu\rho} = \frac{1}{2}\varepsilon_{\mu\nu\rho}{}^{\tau}\partial_\tau\tilde{v} + \frac{1}{3}(\delta_{\mu\nu}\partial_\rho\tilde{w} - \delta_{\mu\rho}\partial_\nu\tilde{w}) \,. \tag{47}$$

Finally, the six longitudinal modes are two triplets

$$L_{\nu\rho} = M_{\nu\rho} + \partial_\nu l_\rho - \partial_\rho l_\nu \,, \tag{48}$$

with

$$\partial^\nu M_{\nu\rho} = \partial^\rho M_{\nu\rho} = 0 \,, \quad \partial^\nu l_\nu = 0 \,. \tag{49}$$

4. Generalized Higgs mechanism

In contrast to spontaneous symmetry breaking by a scalar field the expectation value of a vector field leads to a mixing of the gauge field fluctuations and the vierbein fluctuations. This particular feature of the generalized Higgs mechanism is due to the "source term" $L_U^{(3)}$. It is our aim to separate the various physical modes. For this purpose we have to diagonalize the inverse propagator which corresponds to the second functional derivative of the action. In other words, we have to find the field combinations for which the quadratic expansion of the action in the fluctuations becomes diagonal. We end this section by writing the quadratic expansion of the action in terms of the various fields appearing in our decomposition. In the next section we will proceed to the diagonalization.

The source term $L_U^{(3)}$ mixes the gauge fields $A_{\mu\nu\rho}$ and the vierbein fluctuations $H_{\mu\nu}$. In quadratic order one finds from eqs. (28)(29)

$$\begin{aligned} \int_x \bar{e} L_U^{(3)} &= -\frac{m^2}{4}\int_x A^{\mu\nu\rho}\Big(\partial_\mu H^{(A)}_{\nu\rho} + \partial_\rho H^{(S)}_{\mu\nu} - \partial_\nu H^{(S)}_{\mu\rho}\Big) \\ &= \frac{m^2}{4}\int_x \Big\{ L^{\nu\rho}\Big(\partial^2 H^{(A)}_{\nu\rho} + \partial_\rho\partial^\mu H^{(S)}_{\mu\nu} - \partial_\nu\partial^\mu H^{(S)}_{\mu\rho}\Big) \\ &\qquad + 2\partial_\rho B^{\mu\nu\rho} H^{(S)}_{\mu\nu}\Big\} = \frac{m^2}{4}\int_x (Y_l + Y_t) \,. \end{aligned} \tag{50}$$

The longitudinal part Y_l concerns the mixing effects for the longitudinal gauge bosons. By use of the decomposition (31),(48), this results in

$$Y_l = M^{\nu\rho}\partial^2 b_{\nu\rho} + 2l^\mu\partial^4(\kappa_\mu - \gamma_\mu) \,. \tag{51}$$

The transversal part Y_t accounts for the mixing of the transversal gauge bosons. For its evaluation we decompose the tensor $\partial_\rho B^{\mu\nu\rho}$ into its traceless symmetric, antisymmetric, and (modified) trace parts

$$\partial_\rho B^{\mu\nu\rho} = \tilde{B}^{(S)\mu\nu} + \tilde{B}^{(A)\mu\nu} + \frac{1}{3}P^{\mu\nu}\tilde{b} \,, \tag{52}$$

with

$$\begin{aligned} &\tilde{B}^{(S)\mu\nu} = \tilde{B}^{(S)\nu\mu}, \quad \tilde{B}^{(S)\mu\nu}\delta_{\mu\nu} = 0, \quad \tilde{B}^{(A)\mu\nu} = -\tilde{B}^{(A)\nu\mu} \,, \\ &\partial_\mu\tilde{B}^{(S)\mu\nu} + \partial_\mu\tilde{B}^{(A)\mu\nu} = 0 \,. \end{aligned} \tag{53}$$

Comparing with the expansion (37) we identify

$$\begin{aligned} &\tilde{B}^{(S)\mu\nu} = -\frac{1}{2}\partial^2 E^{\mu\nu} \,, \quad \tilde{B}^{(A)\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\tau}\partial_\rho v_\tau^{(t)} \,, \\ &\tilde{b} = \partial^2\tilde{w} \,. \end{aligned} \tag{54}$$

The components $C_{\mu\nu\rho}$ do not appear due to the identity

$$\partial^\rho C_{\mu\nu\rho} = 0 \,, \tag{55}$$

which follows from eqs. (42) (43).

Inserting also the expansion (31) for $H^{(S)}_{\mu\nu}$, the transversal part of eq. (50) reads

$$Y_t = \Big(2\tilde{B}^{(S)\mu\nu} + \frac{2}{3}P^{\mu\nu}\tilde{b}\Big)H^{(S)}_{\mu\nu} = -E^{\mu\nu}\partial^2 t_{\mu\nu} + \frac{2}{3}\tilde{w}\partial^2\sigma \,. \tag{56}$$

We observe that the twelve transversal gauge field fluctuations $v^{(t)}$, $w^{(t)}$, $\tilde{v}$, $C_{\mu\nu\rho}$ do not appear in $L_U^{(3)}$. They do not take part in the mixing and simply acquire a mass from $L_U^{(2)}$. Also the four gauge modes $\kappa_\mu + \gamma_\mu$ and u are absent.

For a discussion of the inverse propagator we also have to decompose the gauge boson mass term $L_U^{(2)}$,

$$\begin{aligned} L_U^{(2)} &= \frac{m^2}{4}A^{\mu\nu\rho}A_{\mu\nu\rho} = \frac{m^2}{4}(B^{\mu\nu\rho}B_{\mu\nu\rho} - L^{\nu\rho}\partial^2 L_{\nu\rho}) \\ &= \frac{m^2}{4}\Big\{-\frac{1}{2}E^{\nu\rho}\partial^2 E_{\nu\rho} + C^{\mu\nu\rho}C_{\mu\nu\rho} + v^{(t)\mu}v_\mu^{(t)} - \frac{3}{2}\tilde{v}\partial^2\tilde{v} \\ &\quad + \frac{4}{9}w^{(t)\mu}w_\mu^{(t)} - \frac{2}{3}\tilde{w}\partial^2\tilde{w} - M^{\nu\rho}\partial^2 M_{\nu\rho} + 2l^\mu\partial^4 l_\mu\Big\} \,. \end{aligned} \tag{57}$$

The corresponding expression for $L_U^{(1)}$ is given by eq. (33)

$$\begin{aligned} L_U^{(1)} = &\frac{m^2}{4}\Big\{-\frac{1}{2}t^{\mu\nu}\partial^2 t_{\mu\nu} - \frac{1}{4}b^{\mu\nu}\partial^2 b_{\mu\nu} \\ &+ \frac{1}{2}(\kappa^\mu - \gamma^\mu)\partial^4(\kappa_\mu - \gamma_\mu) - \frac{1}{6}\sigma\partial^2\sigma\Big\} \,. \end{aligned} \tag{58}$$

5. Mode mixing

Taking things together, L_U involves a couple of independent pieces

$$L_U = \frac{m^2}{4}\left\{L_{tE} + L_{\sigma\tilde{w}} + L_{bM} + L_{\kappa\gamma l} + L'_m\right\} . \tag{59}$$

They can be diagonalized separately. For the transversal traceless symmetric tensors t and E one has

$$L_{tE} = -\frac{1}{2}(t^{\mu\nu} + E^{\mu\nu})\partial^2(t_{\mu\nu} + E_{\mu\nu}) , \tag{60}$$

while in the scalar sector σ and $\tilde{w}$ are connected

$$L_{\sigma\tilde{w}} = -\frac{1}{6}(\sigma - 2\tilde{w})\partial^2(\sigma - 2\tilde{w}) . \tag{61}$$

The sector for the longitudinal gauge bosons involves

$$L_{bM} = -\frac{1}{4}(b^{\mu\nu} - 2M^{\mu\nu})\partial^2(b_{\mu\nu} - 2M_{\mu\nu}) , \tag{62}$$

and

$$L_{\kappa\gamma l} = \frac{1}{2}(\kappa^\mu - \gamma^\mu + 2l^\mu)\partial^4(\kappa_\mu - \gamma_\mu + 2l_\mu) . \tag{63}$$

The remaining part L'_m involves only mass terms for the gauge bosons and no mixing

$$L'_m = C^{\mu\nu\rho}C_{\mu\nu\rho} + v^{(t)\mu}v^{(t)}_\mu - \frac{3}{2}\tilde{v}\partial^2\tilde{v} + \frac{4}{9}w^{(t)\mu}w^{(t)}_\mu . \tag{64}$$

The kinetic term for the gauge bosons (25) contributes only for the transversal gauge bosons,

$$L_F = -\frac{Z}{4}A^{\mu\nu\rho}\partial^2 P_\mu{}^\sigma A_{\sigma\nu\rho} = -\frac{Z}{4}B^{\mu\nu\rho}\partial^2 B_{\mu\nu\rho} . \tag{65}$$

The decomposition is similar to the transversal part of eq. (57), replacing m^2 by $-Z\partial^2$,

$$\begin{aligned} L_F = \frac{Z}{4}\Big\{&\frac{1}{2}E^{\nu\rho}\partial^4 E_{\nu\rho} - C^{\mu\nu\rho}\partial^2 C_{\mu\nu\rho} - v^{(t)\mu}\partial^2 v^{(t)}_\mu \\ &+\frac{3}{2}\tilde{v}\partial^4\tilde{v} - \frac{4}{9}w^{(t)\mu}\partial^2 w^{(t)}_\mu + \frac{2}{3}\tilde{w}\partial^4\tilde{w}\Big\} . \end{aligned} \tag{66}$$

At this point we have expressed both L_U and L_F in terms of the fields of our decomposition. The transition to momentum space is straightforward. The pieces $\sim q^4$ are artefacts of the particular normalization of fields in our decomposition. They can be absorbed by a momentum dependent rescaling which makes all fields in the decomposition of the vierbein dimensionless, and gives all fields in the expansion of the gauge fields the canonical dimension of mass. The high momentum behavior for all modes is then $\sim q^2$, as appropriate for an action that contains only two derivatives. The different blocks can now be diagonalized separately.

IV. Stability of classical theory

In this section we diagonalize the propagator. The question of stability of the classical theory can be read off directly from the form of the propagators. Flat space is stable if there are neither ghosts nor tachyons and all modes are described by massive or massless particles. Stability of flat space extends to stability of the high-momentum modes for arbitrary "background configurations" of the gauge fields and vierbein.

1. High momentum limit

Before proceeding to the diagonalization of the inverse propagator it is instructive to discuss the limit $q^2 \to \infty$. In this limit $L_U^{(2)}$ and $L_U^{(3)}$ can be neglected. The infinite momentum limit becomes a good approximation for $q^2 \gg m^2$. In consequence, the leading short distance behavior is well described by an action based on $L_F + L_U^{(1)}$, evaluated for the flat space solution. This is a free theory with inverse propagators for the physical fluctuations proportional to q^2. No instability occurs. For this free theory the analytic continuation to Minkowski space is straightforward. We therefore consider our pregeometry based on a Yang-Mills theory as a candidate for an ultraviolet completion of quantum gravity.

Beyond the leading approximation the action (10) also entails interactions. First, there are the gauge interactions mediated by the gauge field A_μ. The effective gauge coupling g, with $\alpha = g^2/(4\pi)$, is given by

$$\alpha = \frac{4\pi}{Z} . \tag{67}$$

It vanishes in the limit $Z \to \infty$. For large Z a perturbative treatment of the gauge interactions becomes possible.

In contrast, the vierbein-mediated "gravitational interactions" cannot be switched off. We can use scaling fields [22]

$$\tilde{e}_\mu{}^m = k e_\mu{}^m , \quad \tilde{g}_{\mu\nu} = k^2 g_{\mu\nu} , \quad \tilde{g}^{\mu\nu} = k^{-2} g^{\mu\nu} , \tag{68}$$

with k some "renormalization scale". As a result one finds

$$S = \frac{1}{16\pi}\int_x \tilde{e}\left(\frac{1}{2\alpha}F_{\mu\nu mn}F_{\rho\sigma}{}^{mn} + \frac{1}{\gamma}\tilde{U}_{\mu\nu m}\tilde{U}_{\rho\sigma}{}^m\right)\tilde{g}^{\mu\rho}\tilde{g}^{\nu\sigma} , \tag{69}$$

with

$$\tilde{U}_{\mu\nu}{}^m = D_\mu \tilde{e}_\nu{}^m = kU_{\mu\nu}{}^m , \tag{70}$$

and dimensionless coupling

$$\gamma = \frac{k^2}{4\pi m^2} . \tag{71}$$

A multiplicative rescaling of $\tilde{e}_\mu{}^m$ does not change the Levi-Civita connection $\Gamma_{\mu\nu}$ in the covariant derivative (6). It only results in a multiplicative rescaling of $\tilde{U}_{\mu\nu}{}^m$. As a result, γ is not a free parameter - it can be set to any arbitrary value by a rescaling of $\tilde{e}_\mu{}^m$. In contrast to the gauge interactions, the gravitational interactions do not switch off in the limit $\gamma \to 0$. Using the freedom of field-rescaling the only remaining free parameter for the classical theory is the gauge coupling α.

2. Diagonalization of the propagator

Let us next consider the full classical action based on $L_F + L_U$. With an appropriate normalization, the inverse propagator for the gauge boson fluctuactions C, $v^{(t)}$, $w^{(t)}$ and $\tilde{v}$ is the standard one for massive particles in momentum space

$$P = G^{-1} = q^2 + \frac{m^2}{Z} \,. \tag{72}$$

This is the inverse propagator of a particle with mass $m/\sqrt{Z}$. No instability occurs in this sector.

For the traceless transversal tensors in the $t - E$ - sector the inverse propagator matrix takes the form

$$P = G^{-1} = \frac{q^2}{4} \begin{pmatrix} Zq^2 + m^2 & , m^2 \\ m^2 & , m^2 \end{pmatrix} \,. \tag{73}$$

The only zero eigenvalue of P occurs for $q^2 = 0$. The eigenvalues of P are given by

$$\lambda_\pm = \frac{q^2}{8}\Big(Zq^2 + 2m^2 \pm \sqrt{Z^2 q^4 + 4m^4}\Big) \,. \tag{74}$$

These eigenvalues have dimension mass^4. We can switch to the more usual normalization for bosonic fields for which the boson fields have dimension of mass and the inverse propagator has dimension mass squared. This can be achieved by a momentum dependent renormalization of the fields, $E_{R\mu\nu} = \frac{q}{2}E_{\mu\nu}$, $q = \sqrt{|q^2|}$, $t_{R\mu\nu} = \frac{m}{2}t_{\mu\nu}$, resulting in the renormalized inverse propagator matrix

$$P_R = \begin{pmatrix} Zq^2 + m^2 & , mq \\ mq & , q^2 \end{pmatrix} \,. \tag{75}$$

The q- dependence in the renormalization of E corresponds to the standard normalization of gauge fields A, noting $A \sim qE$, while the normalization of t provides a canonical mass dimension to this field.

Poles in the propagator correspond to vanishing eigenvalues of P_R. The condition for the existence of propagator poles reads

$$\det P_R = Zq^4 = 0 \,. \tag{76}$$

We conclude that the only possible poles of the propagator occur for $q^2 = 0$.

For the renormalized fields the eigenvalues of P_R for general values of q^2 are given by

$$\lambda_\pm = \frac{1}{2}\Big\{(Z+1)q^2 + m^2 \pm \sqrt{(Z-1)^2 q^4 + 2(Z+1)q^2 m^2 + m^4}\Big\} \,. \tag{77}$$

They correspond to the inverse propagators after diagonalization and describe the propagation of independent modes.

For the high momentum behavior, $q^2 \gg m^2$, this yields

$$\lambda_+ = \begin{cases} Z\Big(q^2 + \dfrac{m^2}{Z-1}\Big) & \text{for } Z > 1 \\ q^2 + \dfrac{m^2}{1-Z} & \text{for } Z < 1 \,, \end{cases} \tag{78}$$

and

$$\lambda_- = \begin{cases} q^2 - \dfrac{m^2}{Z-1} & \text{for } Z > 1 \\ Z\Big(q^2 - \dfrac{m^2}{1-Z}\Big) & \text{for } Z < 1 \,. \end{cases} \tag{79}$$

The apparent zeros in these approximate expression cannot correspond to poles in the propagators since eq. (76) tells us that all possible poles occur for $q^2 = 0$. Near the pole at $q^2 = 0$ one finds

$$\lambda_- = \frac{Zq^4}{m^2} \quad , \quad \lambda_+ = m^2 + (Z+1)q^2 \,. \tag{80}$$

Again, the apparent zero in λ_+ is a "fake pole", since the only possible poles for $q^2 = 0$ are associated to λ_-. The function $\lambda_+(q^2)$ has no zero. No propagating particle is associated to this mode.

The zero of the inverse propagator at $q^2 = 0$ corresponds to a double pole in the propagator. A propagator $1/q^4$ leads to a "secular instability" after analytic continuation to Minkowski space. It does not pose any problem for a well defined euclidean functional integral. A double pole can be considered as the boundary between stability and instability. The stability for low q^2 should be discussed within the quantum effective action once quantum fluctuation effects are included. We will see below that additional terms in the effective action modify the behavior for $q^2 \to 0$.

For the other limit at large q^2 one may assume that the short distance behavior of the effective action is close to the classical action. The short distance fluctuations in the t - E - sector are stable.

We next turn to the remaining part in the sector of physical scalar fluctuations. In the $\tilde{w} - \sigma$ - sector the inverse propagator matrix

$$P = \frac{q^2}{3} \begin{pmatrix} Zq^2 + m^2 & , -\dfrac{m^2}{2} \\ -\dfrac{m^2}{2} & , \dfrac{m^2}{4} \end{pmatrix} \tag{81}$$

has the same structure as eq. (73), up to an overall factor 4/3 and a rescaling of σ by a factor (-2). The poles in the propagators are therefore the same as in the sector of transverse traceless tensors (t, E). Together with eq. (72) for the scalar $\tilde{v}$ we observe in the scalar sector one massive particle and one massless particle with a double pole. The scalar $\tilde{u}$ is a gauge degree of freedom of diffeomorphisms and does not appear in the quadratic action.

The sector of transversal vectors contains several independent blocks. The vectors $v^{(t)}$ and $w^{(t)}$ from the expansion of the transversal gauge field fluctuations correspond to massive particles with propagators given by eq. (72). A third physical vector fluctuation is given by the combination

$$s_\mu = \frac{1}{2}(\kappa_\mu - \gamma_\mu) + l_\mu \ . \quad (82)$$

Its inverse propagator $P = m^2 q^4$ obtains from $L_{\kappa\gamma l}$ in L_U as given by eq. (63). A canonical normalization of this vector field absorbs in P a factor $m^2 q^2$. We end with a massless vector particle with normalized inverse propagator $P_R = q^2$. The transverse vector fluctuations contain also two gauge degrees of freedom, one from diffeomorphisms and the other from the $SO(4)$ - gauge symmetry. We may take them as κ_μ and l_μ at fixed s_μ. Only the physical mode s_μ appears in the quadratic action.

The fluctuations C describe one more massive particle with inverse propagator given by eq. (72). What remains is the sector of antisymmetric tensor fluctuations, consisting of the longitudinal gauge bosons M and the antisymmetric vierbein fluctuations b. For the longitudinal gauge bosons there is no contribution from L_F. The inverse propagator matrix in the $M - b$ - sector obtains from L_{bM} in eq. (62)

$$P = \frac{q^2 m^2}{8}\begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix} \ . \quad (83)$$

After proper renormalization of the fields this becomes

$$P_R = \begin{pmatrix} m^2 & -mq \\ -mq & q^2 \end{pmatrix} \ . \quad (84)$$

One eigenvalue λ_+ of this matrix corresponds to the physical fluctuation

$$r_{\mu\nu} = \frac{1}{2} b_{\mu\nu} - M_{\mu\nu} \ . \quad (85)$$

Only this physical mode appears in L_U through L_{bM} in eq. (62). It corresponds to a massive particle with inverse propagator

$$\lambda_+ = q^2 + m^2 \ . \quad (86)$$

The other eigenvalue λ_- is zero for all q^2, corresponding to $\det P = 0$ or $\det P_R = 0$. This is the gauge mode of local $SO(4)$ - gauge transformations. Indeed, these gauge transformations, applied to a "vacuum state" $A_{\mu mn} = 0$, $e_\mu^m = \delta_\mu^m$, do not only shift the longitudinal components of $A_{\mu mn}$, but also rotate e_μ^m, contributing infinitesimally to $H_{\mu\nu}^{(A)}$. For this reason the gauge modes can be taken as linear combinations of $L_{\mu\nu}$ and $b_{\mu\nu}$. We use the freedom in the precise parametrization of the gauge mode to take the fluctuation $L_{\mu\nu}$ at fixed physical fluctuation $s_{\mu\nu}$.

In total we observe 9 massless physical modes ($t-E$, $\tilde{w}-\sigma$, s), 15 massive physical modes ($v^{(t)}$, $w^{(t)}$, $\tilde{v}$, C, r) and 10 gauge degrees of freedom (κ, u, l, M). We recall that the gauge fluctuations have to be taken at fixed physical modes. One linear combination in the $t-E$-sector and in the $\tilde{w}-\sigma$-sector (6 modes) correspond to physical modes that do not describe a propagating particle. Similar to Einstein gravity one expects that due to non-local projectors not all of the modes ($t - E$, $\tilde{w} - \sigma$, s , $v^{(t)}$, $w^{(t)}$, $\tilde{v}$, C , r) are propagating particles [41]. This reduces the number of modes corresponding to physical propagating particles below 24.

3. Short distance completion for euclidean gravity

Taking things together, for the action $L_U + L_F$ the propagators of the physical fluctuations around euclidean flat space (excluding the gauge modes) have the following properties:

(i) In the complex q^2- plane all poles occur on the negative real axis, including $q^2 = 0$.

(ii) For the massive modes with location of the poles at $q_p^2 = -m_i^2$ the residuum is always positive, as required for standard stable massive particles.

(iii) For real $q^2 \geqslant 0$ the inverse propagator is positive for all modes, increasing $\sim q^2$ for $q^2 \to \infty$.

(iv) At $q^2 = 0$ there are degenerate poles, with eigenvalues of the inverse propagator $\sim q^4$. These massless modes are linear combinations of gauge boson - and vierbein-fluctuations. The behavior $\sim q^4$ can be considered as the boundary of the region of stability.

We can consider this model of $SO(4)$ - pregeometry as a valid candidate for the ultraviolet completion of euclidean gravity.

Furthermore, their is no obstruction for analytic continuation to a Minkowski signature. Investigating the propagator for a Minkowski signature in the complex q_0 - plane one finds besides the poles branch cuts for $q_0^2 > |q_c^2| + \vec{q}^{\,2}$, with positive non zero $|q_c|^2$ to be discussed in more detail in sect.IV. Such a setting is rather promising for a consistent description of quantum gravity.

V. Effective action and emergence of general relativity

The action (10) may be considered as the microphysical or classical action which is used to define a functional integral. Quantum fluctuations will lead to a macroscopic or effective action. The quantum effective action is the generating functional of the one-particle-irreducible Green's functions. It includes all fluctuation effects. The field equations obtained by variation of the effective action are exact. They replace the field equations obtained from the classical action. The second functional derivative of the effective action is the inverse propagator in an arbitrary background of fields.

With a proper definition (and in the absence of anomalies) the effective action has the same symmetries as the classical action. In general, this invariant functional of the fields contains infinitely many invariants. The arguments are now macroscopic fields, corresponding to expectation values of microscopic fields in the presence of arbitrary sources.

We do not attempt to compute the effective action Γ in this paper. We rather make an ansatz for its form which is consistent with the symmetries. In practice, we replace S by Γ and add further invariants. The parameters m^2 and Z are replaced by renormalized parameters which include the effects of the quantum fluctuations. For the additional invariants we investigate a derivative expansion up to second order in the derivatives. It may be possible to include some of the additional invariants into the classical action. We will leave it open here if they are also part of the classical action or if they are generated only by fluctuation effects.

1. New invariants

At low momenta a derivative expansion of the effective action is often a good guide. We will next add to the action (10) further invariants with up to two derivatives,

$$\Gamma = \int d^4x e(L_F + L_U + U + L_R + L_G) . \tag{87}$$

The term without derivatives is a cosmological constant U. We will first neglect it here for simplicity, such that flat space remains a solution of the field equations. This may be motivated by the observation that in the present universe flat space is a very good approximation. For early cosmology this may no longer hold. We discuss the effects of $U \neq 0$ in sect. VIII.

The particular field content with a vierbein permits an invariant that is linear in the fields

$$L_R = -\frac{M^2}{2} F_{\mu\nu}{}^{\mu\nu} . \tag{88}$$

Indeed, contraction of the field strength with the inverse vierbein yields a scalar which is invariant under $SO(4)$ - gauge transformations.

$$F = F_{\mu\nu}{}^{\mu\nu} = F_{\mu\nu mn} e^{m\mu} e^{n\nu} . \tag{89}$$

The presence of L_R in the effective action is a crucial ingredient for the emergence of general relativity.

Further invariants with two derivatives can be constructed by use of the contractions F and

$$F_{\mu\nu} = F_{\mu\rho\nu}{}^{\rho} = F_{\mu\rho mn} e_\nu{}^m \, e^{n\rho} , \tag{90}$$

namely

$$L_G = \frac{A}{2} F_{\mu\nu} F^{\mu\nu} + \frac{B}{2} F^2 . \tag{91}$$

We omit here L_G in order to keep the discussion simple. The invariants in L_G do not change the qualitative aspects for a suitable range of A and B. They may become important for quantitative early cosmology.

2. Field equations

The field equations obtain by variation of the effective action (87) with respect to $A_{\mu mn}$ and $e_\mu{}^m$. For a given effective action Γ they are exact, including all effects of quantum fluctuations. Approximations occur only because the effective action is not known precisely. For a model that is thought to describe quantum gravity these field equations have to lead, after analytic continuation to Minkowski signature, to the gravitational field equations that we use on Earth or for the description of black holes.

The field equation for the gauge fields is given by

$$Z D_\nu F^{\mu\nu mn} = J^{\mu mn} , \tag{92}$$

with "source" or "current"

$$\begin{aligned} J^{\mu mn} =& m^2 U^{\mu mn} - M^2 (U^{m\mu n} - U^{n\mu m} \\ & + U_\rho{}^{\rho m} e^{n\mu} - U_\rho{}^{\rho n} e^{m\mu}) . \end{aligned} \tag{93}$$

From the identity $D_\mu D_\nu F^{\mu\nu mn} = 0$ one infers that solutions of the field equations need a covariantly conserved current,

$$D_\mu J^{\mu mn} = 0 . \tag{94}$$

The field equation for the vierbein can be split into symmetric and antisymmetric parts. The one for the symmetric part reads

$$\frac{M^2}{2}(F_{\mu\nu} + F_{\nu\mu} - F g_{\mu\nu}) = T^{(U)}_{\mu\nu} + T^{(F)}_{\mu\nu} , \tag{95}$$

where the r.h.s. involves the symmetric tensors

$$\begin{aligned} T^{(U)}_{\mu\nu} =& \frac{m^2}{2}(D_\rho U_{\mu\nu}{}^\rho + D_\rho U_{\nu\mu}{}^\rho + U_\mu{}^{\tau\rho} U_{\nu\tau\rho} \\ & - \frac{1}{2} U^{\sigma\tau\rho} U_{\sigma\tau\rho} g_{\mu\nu}) , \end{aligned} \tag{96}$$

and

$$T^{(F)}_{\mu\nu} = \frac{Z}{2}(F_\mu{}^{\rho mn} F_{\nu\rho mn} - \frac{1}{4} F^{\sigma\rho mn} F_{\sigma\rho mn} g_{\mu\nu}) . \tag{97}$$

We observe that $T^{(F)}_{\mu\nu}$ is traceless

$$g^{\mu\nu} T^{(F)}_{\mu\nu} = 0 . \tag{98}$$

The antisymmetric part yields

$$m^2 D_\rho U^\rho{}_{\mu\nu} + M^2 (F_{\mu\nu} - F_{\nu\mu}) = 0 . \tag{99}$$

An interesting class of solutions is characterized by

$$U_{\mu\nu\rho} = 0 , \qquad D_\mu e_\nu{}^m = 0 . \tag{100}$$

In this case one has $F_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma}$. The field equations (92) and (95) become

$$\begin{aligned} & D_\nu R^{\mu\nu\rho\sigma} = 0 , \\ & M^2 (R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}) = T^{(F)}_{\mu\nu} , \\ & T^{(F)}_{\mu\nu} = \frac{Z}{2}(R_\mu{}^{\rho\sigma\tau} R_{\nu\rho\sigma\tau} - \frac{1}{4} R^{\sigma\rho\tau\eta} R_{\sigma\tau\rho\eta} g_{\mu\nu}) , \end{aligned} \tag{101}$$

while eq. (99) is obeyed identically. The second equation (101) is Einstein's equation with an effective energy momentum tensor reflecting the effects of higher curvature invariants. Taking the trace,

$$R = 0 \, , \tag{102}$$

the Bianchi identity implies

$$D^\nu T^{(F)}_{\mu\nu} = 0 \, , \tag{103}$$

or

$$\partial_\mu (R^{\sigma\tau\rho\eta} R_{\sigma\tau\rho\eta}) = 4(D^\nu R_\mu{}^{\sigma\tau\rho}) R_{\nu\sigma\tau\rho} \, . \tag{104}$$

A particular solution is flat space, $R_{\mu\nu\rho\sigma} = 0$.

For momenta and curvature much smaller than m^2 eq. (100) becomes a valid approximation, with corrections suppressed by factors R/m^2 or D^2/m^2. Furthermore, $T^{(F)}_{\mu\nu}$ can be neglected. One ends with Einstein's equation in vacuum. The coupling to elementary particles, that we do not discuss in the present paper, adds an energy momentum tensor as for general relativity.

3. Emergent general relativity

For a discussion of the low energy effective theory the invariant L_R will become dominant. For $U = 0$ this is the leading term in an expansion in derivatives. We will therefore consider an effective action based on $L_R + L_U + L_F$. This will reveal the emergence of general relativity with the Einstein-Hilbert action dominating the long distance physics.

For the construction of an effective theory at low momenta $q^2 \ll m^2$ we write

$$A_{\mu\nu\rho} = \omega_{\mu\nu\rho} - U_{\mu\nu\rho} \, , \tag{105}$$

and solve the field equation for the tensor $U_{\mu\nu\rho}$ as a functional of the vierbein $e_\mu{}^m$ which is kept fixed. If the invariant L_U dominates, the solution is simply $U_{\mu\nu\rho} = 0$. The vierbein is then covariantly conserved, $D_\mu e_\nu{}^m = 0$, and the gauge fields cease to be independent degrees of freedom, given by $A_{\mu mn} = \omega_{\mu mn}$.

For the field strength one finds

$$F_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - V_{\mu\nu\rho\sigma} \, , \tag{106}$$

with $R_{\mu\nu\rho\sigma}$ the curvature tensor constructed from the metric $g_{\mu\nu}$. It involves two derivatives of the vierbein. The tensor

$$\begin{aligned} V_{\mu\nu\rho}{}^\sigma =& e_m{}^\sigma \left(D_\mu U_{\nu\rho}{}^m - D_\nu U_{\mu\rho}{}^m \right) \\ =& D_\mu U_{\nu\rho}{}^\sigma - D_\nu U_{\mu\rho}{}^\sigma - U_\mu{}^{\sigma\tau} U_{\nu\rho\tau} + U_\nu{}^{\sigma\tau} U_{\mu\rho\tau} \end{aligned} \tag{107}$$

provides for a kinetic term for $U_{\mu\nu\rho}$ through L_F. In the presence of L_F the action contains a term linear in U and $U_{\mu\nu\rho} = 0$ is no longer an exact solution. This linear term arises from the second term in the expression

$$L_F = \frac{Z}{8} \left\{ R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 2 R_{\mu\nu\rho\sigma} V^{\mu\nu\rho\sigma} + V_{\mu\nu\rho\sigma} V^{\mu\nu\rho\sigma} \right\} . \tag{108}$$

After partial integration we can replace

$$2 R_{\mu\nu\rho\sigma} V^{\mu\nu\rho\sigma} \to -4 U_{\nu\rho}{}^m D_\mu R^{\mu\nu\rho}{}_m \, , \tag{109}$$

such that the term linear $U_{\mu\nu\rho}$ involves covariant derivatives of the curvature tensor. Similarly, a linear term arises from L_R,

$$L_R = -\frac{M^2}{2} \left(R - V_{\mu\nu}{}^{\mu\nu} \right) \, , \tag{110}$$

with R the curvature scalar.

The low momentum effective theory becomes valid if the derivatives of the curvature tensor are small as compared to m^3. In this case the quadratic term for $U_{\mu\nu\rho}$ is dominated by L_U and one finds for small M^2/m^2 the approximate solution

$$U_{\mu\nu\rho} = -\frac{Z}{m^2} D^\sigma R_{\sigma\mu\nu\rho} \, . \tag{111}$$

Insertion into L_F and L_U yields a term $\sim (Z^2/m^2)(DRDR)$ with six derivatives that we may neglect for the low momentum effective theory. For M^2 of the same order as m^2 the result changes quantitatively, but not qualitatively. As a result, we can use $U_{\mu\nu\rho} = 0$ and find the effective action for low momenta

$$S = \int_x e \left\{ -\frac{M^2}{2} R + \frac{Z}{8} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \right\} . \tag{112}$$

With $e = \sqrt{g}$, $g = \det(g_{\mu\nu})$, the low energy effective action only involves the metric. General relativity emerges in this limit.

Indeed, the first term in eq. (112) constitutes the Einstein-Hilbert action and we can associate M with the (reduced) Planck mass. The second term involves the squared Riemann tensor. Since it contains four derivatives of the metric its influence at momenta $q^2 \ll M^2/Z$ will be suppressed. We conclude that for most practical purposes, except perhaps for very early cosmology or the vicinity of the singularity in black holes, our model predicts precisely the field equations for general relativity. Since general relativity is compatible with all present tests of the gravitational interactions, the same holds for our model of pregeometry.

4. Stability for low momenta

The issue of stability arises on two different levels. The first concerns the properties of the classical action. This topic is related to the question if the euclidean functional integral is well defined. It typically concerns the microscopic or high momentum physics. We have discussed this issue in sect. IV. The second topic concerns the stability of solutions of the field equations. Since the relevant field equations obtain by variation of the effective action this part of the stability discussion concerns the effective action. In particular, for the stability at momenta below the Plank mass the term L_R will play an important role. We therefore extend the stability discussion of sect. IV to an effective action based on $L_R + L_U + L_F$.

In contrast to L_F and L_U the invariant L_R is not a square and can therefore take negative values. Since L_R is linear in q, it cannot modify the leading behavior $\sim q^2$ for large q^2. In the infrared limit of small q^2 it can play an important role, however. One expects that it dominates the behavior near massless propagator poles at $q^2 = 0$.

We need to expand

$$eL_R = -\frac{M^2}{2} e e^{m\mu} e^{n\nu} F_{\mu\nu mn} \tag{113}$$

in quadratic order in H and A around $e_\mu^m = \delta_\mu^m$, $A_{\mu mn} = 0$. Since $F_{\mu\nu mn}$ is a least linear in $A_{\mu mn}$ according to eq. (5), we need the vierbein in linear order in H,

$$\begin{aligned} ee^{m\mu}e^{n\nu} &= \delta^{m\mu}\delta^{n\nu} \\ &+ \frac{1}{2}H_{\rho\sigma}\big(\delta^{m\mu}\delta^{n\nu}\delta^{\rho\sigma} - \delta^{m\mu}\delta^{n\rho}\delta^{\nu\sigma} - \delta^{m\rho}\delta^{n\nu}\delta^{\mu\sigma}\big) . \end{aligned} \tag{114}$$

The linear term $\partial_\mu A_\nu{}^{\mu\nu} - \partial_\nu A_\mu{}^{\mu\nu}$ is a total derivative and therefore vanishes.

In quadratic order one finds two contributions, $L_R = L_R^{(1)} + L_R^{(2)}$,

$$\begin{aligned} eL_R^{(1)} = -\frac{M^2}{4}H_{\rho\sigma}\Big\{&\big(\partial_\mu A_\nu{}^{\mu\nu} - \partial_\nu A_\mu{}^{\mu\nu}\big)\delta^{\rho\sigma} \\ &- \big(\partial_\mu A_\nu{}^{\mu\rho} - \partial_\nu A_\mu{}^{\mu\rho}\big)\delta^{\nu\sigma} - \big(\partial_\mu A_\nu{}^{\rho\nu} - \partial_\nu A_\mu{}^{\rho\nu}\big)\delta^{\mu\sigma}\Big\} , \end{aligned} \tag{115}$$

and

$$eL_R^{(2)} = -\frac{M^2}{2}\Big\{A_\mu{}^{\mu\rho}A_{\nu\rho}{}^{\nu} - A_\nu{}^{\mu\rho}A_{\mu\rho}{}^{\nu}\Big\} . \tag{116}$$

The first term involves the term linear in A_μ in F and the linear term (114), while the second corresponds to the term quadratic in A_μ in F.

For the first term we observe that only the transversal gauge bosons contribute to $F_{\mu\nu mn}$ in linear order,

$$\begin{aligned} eL_R^{(1)} &= \frac{M^2}{2}H_{\rho\sigma}\Big\{\delta^{\rho\sigma}\partial_\mu B_\nu{}^{\nu\mu} - \partial_\mu B^{\sigma\rho\mu} - \partial^\sigma B_\mu{}^{\mu\rho}\Big\} \\ &= \frac{M^2}{2}H_{\rho\sigma}\Big\{\Big(\frac{2}{3}\delta^{\rho\sigma} + \frac{1}{3}\frac{\partial^\rho\partial^\sigma}{\partial^2}\Big)\tilde{b} - \tilde{B}^{(S)\sigma\rho} - \tilde{B}^{(A)\sigma\rho} \\ &\quad - \frac{2}{3}\partial^\sigma w^{(t)\rho} - \partial^\sigma\partial^\rho\tilde{w}\Big\} . \end{aligned} \tag{117}$$

The decomposition yields

$$\begin{aligned} eL_R^{(1)} = \frac{M^2}{2}\Big\{&\frac{2}{3}\tilde{w}\partial^2\sigma + \frac{1}{2}E^{\rho\sigma}\partial^2 t_{\sigma\rho} \\ &+ \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}v_\mu^{(t)}\partial_\nu b_{\rho\sigma} + \frac{2}{3}w^{(t)\mu}\partial^2(\kappa_\mu - \gamma_\mu)\Big\} . \end{aligned} \tag{118}$$

This contributes to the mixing between transversal gauge bosons and vierbein fluctuations, similar to eq. (56). As it should be, the gauge modes $\kappa_\mu + \gamma_\mu$ and u do not appear.

The second term decomposes as

$$\begin{aligned} eL_R^{(2)} = \frac{M^2}{2}\Big\{&\frac{1}{4}E^{\mu\nu}\partial^2 E_{\mu\nu} + C_{\mu\nu\rho}C^{\nu\rho\mu} \\ &+ \frac{2}{9}w^{(t)\mu}w_\mu^{(t)} - \frac{2}{3}\tilde{w}\partial^2\tilde{w} + \frac{1}{2}v^{(t)\mu}v_\mu^{(t)} - \frac{3}{2}\tilde{v}\partial^2\tilde{v} \\ &+ \frac{4}{3}w^{(t)\mu}\partial^2 l_\mu - \varepsilon^{\mu\nu\rho\sigma}v_\mu^{(t)}\partial_\nu M_{\rho\sigma}\Big\} . \end{aligned} \tag{119}$$

The different contributions of eL_R can be listed as follows

$$eL_R = \frac{M^2}{4}\big(\Delta L_{tE} + \Delta L_{\sigma\tilde{w}} + L_{vM} + L_{wL} + L_C + L_{\tilde{v}}\big) . \tag{120}$$

In the sector of transversal traceless tensors we find a further contribution to the t - E mixing and the term quadratic in E,

$$\Delta L_{tE} = E^{\mu\nu}\partial^2 t_{\mu\nu} + \frac{1}{2}E^{\mu\nu}\partial^2 E_{\mu\nu} . \tag{121}$$

It has the opposite sign as compared to the contributions from eq. (60). In the scalar sector a term

$$\Delta L_{\sigma\tilde{w}} = \frac{4}{3}(\tilde{w}\partial^2\sigma - \tilde{w}\partial^2\tilde{w}) , \tag{122}$$

adds to eqs.(60)(61) further contributions. The term

$$L_{vM} = \varepsilon^{\mu\nu\rho\sigma}v_\mu^{(t)}\partial_\nu(b_{\rho\sigma} - 2M_{\rho\sigma}) + v^{(t)\mu}v_\mu^{(t)} \tag{123}$$

can be combined with L_{bM} in eq. (62), involving the same physical combination $b_{\mu\nu} - 2M_{\mu\nu} = 2r_{\mu\nu}$. It induces a new momentum-dependent mixing between $r_{\mu\nu}$ and $v_\mu^{(t)}$. Similarly

$$L_{wl} = \frac{4}{3}w^{(t)\mu}\partial^2(\kappa_\mu - \gamma_\mu + 2l_\mu) + \frac{4}{9}w^{(t)\mu}w_\mu^{(t)\mu} \tag{124}$$

combines with $L_{\kappa\gamma l}$ in eq. (63). It mixes the physical transversal vector fluctuations s_μ and $w_\mu^{(t)}$. The remaining parts,

$$L_C = 2C_{\mu\nu\rho}C^{\nu\rho\mu} , \quad L_{\tilde{v}} = -3\tilde{v}\partial^2\tilde{v} , \tag{125}$$

add mass terms to the gauge boson fluctuations C and $\tilde{v}$.

Adding the contributions of L_R to the ones from $L_U + L_F$ the stability discussion can again be performed separately for different blocks. We need to compute the propagators from a diagonalization of the individual blocks. Stability depends on the behavior of these propagators. In the next section we will compute the propagator in the transverse traceless tensor sector which describes the massless graviton and a massive spin-two particle. In the following sect. VII we turn to the scalar sector. Some of the other blocks are rather simple, but we leave a detailed description to future work.

VI. Graviton propagator

We are now in a position to discuss the propagator of the graviton. In the low momentum limit we expect an inverse propagator $\sim q^2$ according to the emergence of general relativity discussed in the preceding section. Also for high q^2 one has a propagator $\sim q^2$ according to the findings of sect. VI. This momentum region is not affected by the addition of L_R, which is a subleading correction for $q^2 \gg M^2$. One expects some smooth interpolation between the two limits. In the view of later analytic continuation we discuss the graviton propagator for arbitrarily complex q^2.

1. Inverse propagator for transversal traceless tensors

Our model has two fields transforming as transverse traceless tensors, namely $t_{\mu\nu}$ and $E_{\mu\nu}$. They mix, and the graviton has to be identified with one of the eigenmodes of the inverse propagator matrix. In the presence of L_R the renormalized inverse propagator matrix (75) in the $t-E$ - sector is replaced by

$$P_R^{(tE)} = \begin{pmatrix} Zq^2 + m^2 - M^2 & , & \dfrac{m^2 - M^2}{m} q \\ \dfrac{m^2 - M^2}{m} q & , & q^2 \end{pmatrix} . \qquad (126)$$

This matrix is the central quantity for the discussion of the graviton propagator. In addition to the massless graviton it will describe a massive particle with mass $\sim M$. For $M \to 0$ both excitations are massless, as found in sect. IV.

The poles of the propagators or zero eigenvalues of P_R occur for

$$q^2 = 0 \, , \quad q^2 = -\mu^2 \, , \qquad (127)$$

with

$$\mu^2 = \frac{m^2}{Z} y(1-y) \, , \quad y = \frac{M^2}{m^2} \, . \qquad (128)$$

This is seen easily from the condition $\det(P_R) = 0$. No other poles of the propagator are possible. For $M^2 \to 0$ the second pole moves to zero,

$$\mu^2 = \frac{M^2}{Z}\Big(1 - \frac{M^2}{m^2}\Big) \, . \qquad (129)$$

A stable particle requires $\mu^2 \geqslant 0$, or

$$0 \leqslant y \leqslant 1 \, , \quad 0 \leqslant M^2 \leqslant m^2 \, . \qquad (130)$$

Otherwise one encounters a tachyonic instability. The constraint (130) is a first condition for stable gravity.

The inverse particle propagators are given by the eigenvalues of P_R,

$$\lambda_\pm = \frac{1}{2}\Bigg\{(Z+1)q^2 + m^2 - M^2 \pm \sqrt{\big[(Z-1)q^2 + m^2 - M^2\big]^2 + 4\frac{q^2}{m^2}(m^2 - M^2)^2}\Bigg\} . \qquad (131)$$

All properties of the propagation of transverse traceless tensor modes in flat space are encoded in these two eigenvalues. For suitable ranges of parameters we will find that the eigenvector to λ_- describes the massless graviton, while the eigenvector to λ_+ accounts for a stable massive particle. At long distances only the graviton will matter.

2. Graviton

Indeed, for $q^2 \to 0$ the eigenvalue $\lambda_-(q^2)$ is the inverse propagator for the massless graviton, corresponding to a Taylor expansion

$$\begin{aligned} \lambda_- =& \frac{M^2 q^2}{m^2} + \big(Z - \frac{M^2}{m^2}\big)\frac{q^4}{m^2} + \ldots \\ =& \frac{M^2 q^2 + Zq^4 + \ldots}{m^2 + q^2} \end{aligned} \qquad (132)$$

The factor $(m^2 + q^2)^{-1}$ may be absorbed by a normalization which makes the graviton fluctuation dimensionless. The expansion (132) reflects the low-momentum effective action (112). For the other eigenvalue one has

$$\lambda_+(q^2 = 0) = m^2 - M^2 \, . \qquad (133)$$

For $M^2 > 0$ and $M^2 < m^2$ only a single pole remains at $q^2 = 0$. The degeneracy of the double pole in eq. (80) is lifted.

Near the second pole at $q_p^2 = -\mu^2$ one finds for

$$-(Z+1)\mu^2 + m^2 - M^2 < 0 \, , \qquad (134)$$

or

$$Z < Z_c \, , \quad Z_c = \frac{y}{1-y} \, , \qquad (135)$$

a negative λ_- for q^2 near $-\mu^2$,

$$\lambda_- = -m_t^2 - \frac{1}{2}\Big(Z + 1 - \frac{A_t m^2}{m_t^2}\Big)(q^2 + \mu^2) \, . \qquad (136)$$

Indeed, for $Z < Z_c$, the combination m_t^2 is positive,

$$m_t^2 = (Z+1)\mu^2 - m^2 + M^2 = m^2 y(1-y)\Big(\frac{1}{Z} - \frac{1}{Z_c}\Big) \, . \qquad (137)$$

The coefficient A_t is given by

$$A_t = (1-y)\bigg(Z + 1 - \Big(Z + \frac{1}{Z}\Big)y\bigg) \, . \qquad (138)$$

Here we have always assumed the range $0 \leqslant y \leqslant 1$ required by stability according to eq. (130). With the conditions (130) and (135) the eigenvalue $\lambda_-(q^2)$ therefore has a single zero, located at $q^2 = 0$. There is no tachyonic or ghost pole in the graviton propagator.

For large $|q^2|$ one obtains

$$\begin{aligned} \lim_{q^2 \to \infty} \lambda_- &= \begin{cases} q^2 & \text{for } Z > 1 \\ Zq^2 & \text{for } Z < 1 \, , \end{cases} \\ \lim_{q^2 \to -\infty} \lambda_- &= \begin{cases} Zq^2 & \text{for } Z > 1 \\ q^2 & \text{for } Z < 1 \, . \end{cases} \end{aligned} \qquad (139)$$

For $Z < Z_c$ the function $\lambda_-(q^2)$ remains negative and real for the range of real q^2 with $-\mu^2 < q^2 < 0$.

In contrast, for $Z > Z_c$ the graviton propagator $\lambda_-^{-1}(q^2)$ has a second pole at $q^2 = -\mu^2$, with a behavior for q^2 near $-\mu^2$ given by

$$\lambda_-(q^2) = B_t(q^2 + \mu^2) \; , \tag{140}$$

where

$$B_t = \frac{1}{2}\Big(Z + 1 + \frac{A_t m^2}{m_t^2}\Big) = \frac{Zy}{y - Z(1-y)} \; . \tag{141}$$

For $Z > Z_c$ one has $m_t^2 < 0$ and $B_t < 0$. The negative prefactor of $q^2 + \mu^2$ corresponds to a negative residuum at the pole at $q^2 = -\mu^2$. This indicates a ghost instability. A stable theory with a well behaved graviton propagator for all momenta therefore requires the "stability condition" (135), $Z < Z_c$.

With this condition the graviton propagator has only a single pole in the complex q^2- plane at $q^2 = 0$. In the standard normalization with dimensionless metric or vierbein fields the graviton propagator is given by

$$G_{\rm grav}(q^2) = \frac{4}{(m^2 + q^2)\lambda_-(q^2)} \; . \tag{142}$$

This is the graviton propagator (1) mentioned in the introduction. For q^2 near zero it reads, cf. eq. (132),

$$G_{\rm grav}(q^2) = \frac{4}{M^2 q^2}\Big(1 + Z\frac{q^2}{M^2}\Big)^{-1} \; . \tag{143}$$

While the truncated propagator has a ghost pole at $q^2 = -M^2/Z$, this pole is not present in the propagator (142). It is therefore an artefact of the truncation [12].

3. Massive spin-two particle

In the stable range for $Z < Z_c$ an approximate form for $\lambda_+(q^2)$ in a range of momenta in the vicinity of the pole at $q^2 = -\mu^2$ is given by

$$\lambda_+ = B_t(q^2 + \mu^2) \; . \tag{144}$$

With $m_t^2 > 0$ one infers $B_t > 0$. The eigenmode $\lambda_+(q^2)$ corresponds to a stable massive spin-two particle, with mass μ and positive residuum at the pole $B_t^{-1} > 0$.

In the stable range both eigenvalues of the inverse propagator matrix (126) correspond to stable modes. There is no instability in this sector. At $q^2 = 0$ one finds a positive value of λ_+,

$$\lambda_+(q^2 = 0) = m^2 - M^2 \; . \tag{145}$$

For the boundary case $M^2 = m^2$ one observes a second stable massless particle, with

$$\lambda_+ = Zq^2 \; , \quad \lambda_- = q^2 \; . \tag{146}$$

In this limit Z_c diverges.

The condition of stability $Z < Z_c$ corresponds to a lower bound for μ^2,

$$\mu^2 > m^2(1-y)^2 \; . \tag{147}$$

For fixed m^2 and Z, this amounts to a lower bound for M^2 or y

$$\frac{y}{1-y} > Z \; . \tag{148}$$

If for a given m^2 and Z one starts at $M^2 = 0$ and switches on M^2 continuously, the double pole at $q^2 = 0$ for $M^2 = 0$ first turns to a ghost instability since the condition (148) will be violated for small enough y. Once y reaches the critical value saturating eq. (148) the pole jumps from λ_- to λ_+ and the sign of the residuum changes correspondingly. This explains why the massive spin-two particle cannot be found as a small deformation in the vicinity of a massless graviton. The massive particle pole is outside the range of validity of a polynomial expansion in q^2 for the inverse propagator.

4. Analyticity

Analytic continuation can be discussed on different levels. On an overall level one has to specify how all fields are analytically continued. This includes the gauge fields, such that the gauge group $SO(4)$ is analytically continued to the non-compact gauge group $SO(1,3)$. We will describe this procedure in a separate paper. In the present paper we discuss analytic continuation on the level of propagators in flat space. This amounts to a discussion of the behavior of propagators in the plane of complex q^2 or complex q_0. Analytic continuation interpolates from the euclidean relation $q^2 = q^\mu q_\mu = q_0^2 + \vec{q}^{\,2}$ to the Minkowski relation $q^2 = -q_0^2 + \vec{q}^{\,2}$. It requires that there exists a continuous path between the two limits in the complex plane that is not obstructed by non-analyticities as poles or branch cuts.

For an overall picture of the q^2-dependence of the two eigenvalues $\lambda_\pm(q^2)$ we further note that at a critical value $q_c^2 < 0$ both eigenvalue can coincide

$$\lambda_+(q_c^2) = \lambda_-(q_c^2) \; . \tag{149}$$

This occurs when the square root in eq. (131) vanishes, determining

$$\frac{q_{c\pm}^2}{m^2} = -\frac{1-y}{(1-Z)^2}\Big\{Z + 1 - 2y \mp \sqrt{1-y}\,\sqrt{Z-y}\Big\} \; . \tag{150}$$

The condition for the existence of intersection points is $Z > y$. For $Z = y$, which belongs to the stable region, $\lambda_+(q^2)$ and $\lambda_-(q^2)$ touch each other at $q_c^2 = -m^2$. For $Z > y$ one has a finite region $q_{c-}^2 < q^2 < q_{c+}^2$ for which the argument in the square root of eq. (131) becomes negative. In this region $\lambda_+(q^2)$ and $\lambda_-(q^2)$ have a non-vanishing imaginary part for real q^2. (For $Z \to 1$ one has $q_{c-}^2 \to -\infty$, $q_{c+}^2 \to -m^2/4$, while for $Z \neq 1$, $Z > y$ both q_{c-}^2 and q_{c+}^2 are finite.)

The region with an imaginary part of $\lambda_\pm(q^2)$ occurs always beyond the location of the second pole of the propagator, $q_{c\pm} < -\mu^2$. This is visible from

$$\frac{q_c^2}{m^2} = -\frac{\mu^2}{m^2} - x_t \ , \tag{151}$$

with

$$x_\pm = \frac{1}{(Z-1)^2}\Big(b \mp \sqrt{b^2-(Z-1)^2v^2}\Big) \ , \quad v = \frac{m_t^2}{m^2} \ , \tag{152}$$

where

$$\begin{aligned} b &= 2(1-y)^2 + (Z-1)\Big(1-y-\frac{\mu^2}{m^2}\Big) \\ &= (1-y)\Big[(Z+1)(1-y)+\frac{y}{Z}(Z-1)^2\Big] \ . \end{aligned} \tag{153}$$

The argument of the square root being smaller than b^2, both x_+ and x_- are positive since $b > 0$.

In the complex q^2- plane the graviton propagator has a cut for real negative q^2, extending from q_{c-}^2 to q_{c+}^2. Except for the pole at $q^2 = 0$ this cut, the graviton propagator is analytic, decaying $\sim |q|^{-2}$ for large $|q|$. Nothing obstructs analytic continuation from euclidean space to Minkowski space. We can therefore extend our analysis to a Minkowski metric, $g_{\mu\nu} = \eta_{\mu\nu}$ and $q^2 = -q_0^2 + \vec{q}^{\,2}$. In the complex q_0 - plane the graviton propagator has two poles at $q_0 = \pm\sqrt{\vec{q}^{\,2}}$. It inherits the cuts on the real q_0 - axis, extending from $q_0^2 = |q_{c-}^2| + \vec{q}^{\,2}$ to $|q_{c+}^2| + \vec{q}^{\,2}$. The prescription for the usual infinitesimal $i\varepsilon$ - terms is dictated by analytic continuation. This graviton propagator is well behaved in Minkowski space, without any instability.

In summary of this section both stability conditions (130) and (135) are necessary for a stable graviton sector. If realized, the massless graviton is accompanied by a massive spin two particle which is stable as well. This is an example how a pair of massless and massive spin two particles can be obtained in a rather simple setting. No problem of consistency or stability is visible in this sector.

VII. Metastability of flat space

In the presence of L_R flat space cannot be the minumum of the euclidean effective action. For flat space one has $\Gamma = 0$. The curvature scalar R can take positive and negative values, however. Since the term L_R is dominant for small momenta, positive R and therefore negative L_R lead to $\Gamma < 0$. This issue is well known in Einstein gravity. In Einstein gravity it leads to an unbounded Einstein-Hilbert action such that the euclidean functional integral is not well defined. For our model of pregeometry the functional integral based on the classical action is well defined. This extends to the high momentum behavior of the effective action, and therefore the behavior for large $|R|$, which are well behaved. The presence of L_R in the effective action indicates, however, that flat space is a saddle point of the euclidean effective action, but not a minimum.

1. Scalar propagators

The metastability of flat space is visible in the propagators for scalar fields. In the scalar sector one has three physical scalar modes, namely σ, $\tilde{w}$, $\tilde{v}$. The scalar u is a gauge mode. The scalar $\tilde{v}$ does not mix with the other two physical scalars. It describes a massive particle with a stable propagator

$$G_{\tilde{v}} \sim (Zq^2 + m^2 + 2M^2)^{-1} \ , \tag{154}$$

as obtained from the action

$$S_{\tilde{v}} = \int_q v(-q)\frac{3q^2}{8}(Zq^2+m^2+2M^2)v(q) \ . \tag{155}$$

The squared mass $m^2 + 2M^2$ is increased by L_R.

In the $\sigma - \tilde{w}$ sector one finds the inverse propagator matrix

$$P^{(\sigma\tilde{w})}(q^2) = \frac{1}{3}\begin{pmatrix} Zq^4+(m^2+2M^2)q^2 & -\Big(\frac{1}{2}m^2+M^2\Big)q^2 \\ -\Big(\frac{1}{2}m^2+M^2\Big)q^2 & \frac{m^2}{4}q^2 \end{pmatrix} . \tag{156}$$

Restoring the normalization to scalar fields with dimension mass the corresponding renormalized propagator matrix becomes

$$P_R^{(\sigma\tilde{w})}(q^2) = \frac{4}{3}\begin{pmatrix} Zq^2+m^2+2M^2 \ , & -(m^2+2M^2)\frac{q}{m} \\ -(m^2+2M^2)\frac{q}{m} \ , & q^2 \end{pmatrix} . \tag{157}$$

Metastability is expected to become visible in the behavior of one of the eigenfunctions for $q^2 \to 0$. According to the effective low energy theory being general relativity, the physical scalar fluctuation in the metric should have the "wrong" sign of the inverse propagator.

The zero eigenvalues of P_R occur for

$$q^2 = 0 \ , \quad q^2 = \nu^2 = \frac{2M^2}{Z}(1+2y) \ . \tag{158}$$

For the range of stability of the graviton propagator, $y > 0$, the second pole of the scalar propagator occurs for positive q^2, in contrast to $q^2 = -\mu^2$ for the graviton sector. For general q^2 the two eigenvalues of P_R are given by

$$\begin{aligned} \lambda_\pm(q)^2 = \frac{1}{2}\Big\{ &(Z+1)q^2 + m^2(1+2y) \\ &\pm\sqrt{\Big[(Z-1)q^2+m^2(1+2y)\Big]^2 + 4(1+2y)^2m^2q^2}\Big\} \ . \end{aligned} \tag{159}$$

For q^2 near zero one obtains

$$\lambda_-(q^2) = -4yq^2 + \dots \quad , \tag{160}$$

in contrast to the opposite sign of $\lambda_- = yq^2$ for the graviton sector. This is the expected behavior for the Einstein-Hilbert action. The second zero at $q^2 = \nu^2$ also occurs for λ_-, while λ_+ never vanishes. The mode corresponding to

λ_+ does not describe a propagating particle. For $\lambda_-(q^2)$ one finds for q^2 near ν^2

$$\lambda_-(q^2) = B_\sigma(q^2 - \nu^2)\ , \tag{161}$$

with positive B_σ,

$$B_\sigma = \frac{2yZ}{Z + 2y(1+Z)}\ . \tag{162}$$

The function $\lambda_-(q^2)$ has for positive q^2 a minumum in the range between $q^2 = 0$ and $q^2 = \nu^2$. This reflects the observation that the configuration with $e_\mu^m = \delta_\mu^m$, $A_{\mu mn} = 0$ cannot be the minimum of the euclidean action. The term L_R linear in F is not positive definite and can take negative values. The flat space configuration with vanishing gauge fields is therefore a saddle point.

In contrast to Einstein gravity, where $-(M^2/2)\int_x R$ can take arbitrary negative values, the behavior for large F and U is governed in our case by quadratic terms with positive coefficients such that the euclidean action has a minimum.

2. Metastability of flat space

The euclidean action based on $L_U + L_F$ involves two squares of real tensors. Since we restrict the vierbein to values for which the determinant e is positive, $e \geqslant 0$, the effective action is positive definite, $\Gamma \geqslant 0$. The minimum occurs for $\Gamma = 0$. This is realized by flat space and zero gauge fields. Adding the invariant L_R, the flat space solution with vanishing gauge fields becomes a saddle point. As argued before, this follows from the simple fact that $F = F_{\mu\nu}{}^{\mu\nu}$ can take positive and negative values. For every $M^2 \neq 0$ there are therefore field configurations for which the action becomes negative. Flat space remains a solution, but is no longer a minimum of the euclidean action. It is turned to a saddle point. For $M^2 > 0$ the graviton sector is stable, i.e. the action increases to positive values $\Gamma > 0$ for nonzero inhomogeneous small $t_{\mu\nu}$ or $E_{\mu\nu}$. In contrast, the scalar direction has partially the opposite property. The action can decrease for nonzero small values of the scalar fields σ and $\tilde{v}$. Since the action for large field values and large momenta is dominated by $L_U + L_F$, it remains bounded from below. The minimum occurs, however, for vierbein configurations and gauge fields different from the solution $e_\mu^m = \delta_\mu^m$, $A_{\mu mn} = 0$.

We may decompose the squared field strength as

$$\frac{Z}{8} F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma} = \alpha F^2 + \beta \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \gamma W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma}\ , \tag{163}$$

with

$$\tilde{F}_{\mu\nu} = F_{\mu\nu} - \frac{1}{4} F g_{\mu\nu}\,, \qquad \tilde{F}^{\mu\nu} g^{\mu\nu} = 0\,, \tag{164}$$

and

$$\begin{aligned} W_{\mu\nu\rho\sigma} &= F_{\mu\nu\rho\sigma} - \frac{1}{2}(g_{\mu\rho}F_{\nu\sigma} + g_{\nu\sigma}F_{\mu\rho} - g_{\mu\sigma}F_{\nu\rho} - g_{\nu\rho}F_{\mu\sigma}) \\ &\qquad + \frac{1}{6} F(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}), \\ W_{\mu\nu\rho\sigma} g^{\nu\sigma} &= W_{\mu\nu\rho\sigma} g^{\mu\rho} = 0\ . \end{aligned} \tag{165}$$

The coefficients are given by

$$\alpha = \frac{Z}{48}, \quad \beta = \frac{Z}{4}, \quad \gamma = \frac{Z}{8}\ . \tag{166}$$

Combining

$$L_F + L_R = \alpha\left(F - \frac{M^2}{4\alpha}\right)^2 - \frac{M^4}{16\alpha} + \beta \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} + \gamma W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma}\ , \tag{167}$$

the minumum of $L_F + L_R$ occurs for

$$F = \frac{M^2}{4\alpha}\,, \quad \tilde{F}_{\mu\nu} = 0\,, \quad W_{\mu\nu\rho\sigma} = 0\ . \tag{168}$$

For configurations obeying eq. (168) the sum $L_F + L_R$ is negative. Since we can also achieve $L_U = 0$ by taking $A_{\mu mn} = \omega_{\mu mn}$, this demonstrates that the euclidean action can become negative.

3. Open points

The interpretation of this saddle-point behavior for flat space remains an open issue. We first note that in the low-momentum effective theory the issue is the same as for Einstein gravity. The action for the euclidean vierbein of Einstein gravity is also a saddle point, with scalar fluctuations around flat space leading to a lowering of the euclidean action. At least for the analytically continued model in Minkowski space this is actually not a sign of instability. Due to the positive energy condition Minkowski space is stable in Einstein gravity. The scalar mode is actually not a propagating mode. Combined with the necessary projectors the scalar propagator turns out to be given by $(\vec{q}^{\,2})^{-1}$ instead of $(q^2)^{-1}$ [41]. There is therefore no pole for $\vec{q}^{\,2} \neq 0$. The scalar propagator accounts for Newton's potential, rather than being a physical propagating particle. We expect a similar behavior for our model, at least for the region of momenta $q^2 \ll m^2$. While in euclidean Einstein gravity the action is unbounded from below, this disease is cured in our settings of pregeometry.

For the discussion of stability this points to an important difference between the classical action and the effective action. For the classical action all field configurations are included in the functional integral. A lack of boundedness from below obstructs a well defined functional integral. For the effective action we are concerned with the stability of solutions of the field equations derived from it. Not all arbitrary field configurations are reached by such solutions. Einstein's equation imply constraints that imply that the scalar part in the metric cannot be chosen arbitrarily. It is a generalization of Newton's potential which is determined as a fixed functional of the other fluctuations. For Minkowski signature this is the basic reason why the apparent metastability of flat space does not induce any instability of the solutions of field equations. One concludes that on the level of the effective action not every instability in the space of all arbitrary field configuration leads to an instability of the field configurations that can be reached by solutions of field equations.

It seems rather likely that these features also play a role for the euclidean setting. The field configurations that can be reached by solutions of the field equations include the presence of conserved sources and arbitrary boundary conditions. Nevertheless, the metastability for arbitrary configurations in the scalar sector may not lead to metastable solutions of field equations.

We may also ask what happens if one includes L_R in the classical action. Now all field configurations are present in the functional integral. The configuration minimizing the euclidean action is not known in our setting. Naively, one could minimize $L_U+L_F+L_R$ by $A_{\mu mn} = \omega_{\mu mn}$, and achieving $F = R = M^2/4\alpha$ by a sphere with radius $\sim M^{-1}$. This leads to $L_U + L_F + L_R \sim M^4$, and $\int_x e \sim M^{-4}$. The action would be negative and of the order one. Much smaller values of the action can be achieved for flat space with gauge fields leading to a traced field strength $F \sim M^2$. In this case the action can be negative $\sim M^4$ even in presence of $L_U \sim M^4 > 0$. The volume factor $\int_x e$ diverges for flat space, such that the euclidean action can be much more negative than for the sphere discussed above. It is a further interesting question if gauge field configurations in flat space with $F \sim M^2$ could be phenomenologically acceptable. This would require that these configurations are invariant under the combined $SO(4)$ - rotations which leave the vierbein $e_\mu{}^m = \delta_\mu^m$ invariant, and that macroscopic translation symmetry is preserved.

VIII. Cosmological constant

The issue of metastability of the flat space solution concerns partly the physics at low momenta and geometries with large volumes. If we add a cosmological constant U in eq. (87), it plays a central role for the behavior for $q^2 \to 0$. The cosmological constant U does not vanish for $q^2 \to 0$, in contrast to all other terms that involve derivatives. In this section we include in the effective action a small positive cosmological constant $U > 0$. We will see its profound implications an the question of boundedness of the effective action for euclidean gravity.

1. Spheres with vanishing covariant derivative of vierbein

Let us consider a particular family of field configurations for which the vierbein is covariantly conserved, while the metric describes a sphere with radius L,

$$\begin{aligned} U_{\mu\nu}{}^m &= 0 \,, \qquad F_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} \,, \\ R_{\mu\nu\rho\sigma} &= \frac{1}{L^2}(g_{\mu\rho}g_{\nu\sigma} - g_{\nu\rho}g_{\mu\sigma}) \,, \\ F_{\mu\nu} = R_{\mu\nu} &= \frac{3}{L^2}g_{\mu\nu} \,, \quad F = R = \frac{12}{L^2} \,. \end{aligned} \tag{169}$$

The effective action (87) for this configuration is given for $L_G = 0$ by

$$\begin{aligned} \Gamma(L) &= cL^4\left(U - \frac{6M^2}{L^2} + \frac{3Z}{L^4}\right) \\ &= c(UL^4 - 6M^2L^2 + 3Z) \,. \end{aligned} \tag{170}$$

Here we have employed

$$\int_x e = cL^4 \,. \tag{171}$$

Adding L_G will only modify the constant term in $\Gamma(L)$, replacing Z by a similar constant.

The minimum of $\Gamma(L)$ obeys

$$UL_0^2 - 3M^2 = 0 \,, \tag{172}$$

or

$$L_0^2 = \frac{3M^2}{U} \,. \tag{173}$$

This corresponds to the solution of the Einstein equation

$$M^2(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) = -Ug_{\mu\nu} \,, \quad R = \frac{4U}{M^2} \,. \tag{174}$$

At the minimum, the effective action is negative,

$$\Gamma_0 = -c(UL_0^4 - 3Z) = -c\left(\frac{9M^4}{U} - 3Z\right) \,. \tag{175}$$

For a small ratio $U/M^4 \ll 1$ the effective action Γ_0 takes large negative values, and the contribution $\sim Z$ can be neglected.

Nevertheless, the function $\Gamma(L)$ remains bounded from below even for very small nonzero U/M^4. For $U > 0$ flat space is no longer a solution of the field equations. Instead, we find a new solution of the field equations corresponding to the sphere (169) with L^2 given by eq. (173).

2. Minimum of euclidean effective action

It is an interesting question if this solution corresponds to the minumum of the euclidean action (87), or at least to a minimum on the space of possible solutions of the field equations. For this question we have to analyze fluctuations around this solution. We present several arguments suggesting that the solution (169), (173) could indeed be the minumum of the euclidean effective action on the space of possible solutions of field equations.

We start by discussing the subspace of fluctuations for which the vierbein is covariantly conserved, $U_{\mu\nu}{}^m = 0$. On this subspace the effective action becomes

$$\Gamma = \int_x e\left\{U - \frac{M^2}{2}R + \alpha R^2 + \beta\tilde{R}_{\mu\nu}\tilde{R}^{\mu\nu} + \gamma C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right\} \,, \tag{176}$$

with $\tilde{R}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ and $C_{\mu\nu\rho\sigma}$ the Weyl tensor. We assume a form of L_G for which the three coefficients α, β, γ remain positive. With $e = \sqrt{g}$ eq. (176) is a particular form of metric gravity. We may discuss the question of the minumum of Γ for arbitrary couplings $U > 0$, $M^2 > 0$, $\alpha, \beta, \gamma > 0$.

Consider first the limit $\beta \to \infty, \gamma \to \infty$. The minimum of Γ has to obey in this limit $\tilde{R}_{\mu\nu} = 0$, $C_{\mu\nu\rho\sigma} = 0$. Spaces with these properties are maximally symmetric spaces. On this subspace a minimum requires $R > 0$, and we end with spheres. We have already found the minumum for the space of spheres. We conclude that for $\beta \to \infty, \gamma \to \infty$ the minimum of the effective action is indeed given by a sphere with radius determined by eq. (173).

We can consider an alternative limit with finite β and γ, taking now U, M^2 and α to zero. The effective action is positive semidefinite and its minimum occurs for maximally symmetric spaces. For spheres there is a flat direction corresponding to L. Allowing for $\alpha > 0$, the flat direction remains present, c.f. eq. (170). Adding M^2 and U the degeneracy is lifted, and $\Gamma(L)$ develops a rather deep minimum for small U/M^4. The large negative value of Γ_0 is due to the large volume of the sphere. For $U/M^4 \ll 1$ the geometry of space is almost flat, as observed for the almost de Sitter geometry of our present Universe for Minkoswsi signature.

In the limit $U/M^4 \to 0$ the geometry approaches flat space. In this limit the graviton fluctuations around the configuration (169), (173) remain stable. They still correspond to massless excitations with a positive kinetic term. This differs from graviton fluctuations around flat space which develop a tachyonic instability for $U > 0$. Besides the graviton, the only physical fluctuation that remains is the scalar fluctuation of the metric. For flat space the effective action contains a term linear in the scalar fluctuation, indicating a different minimum with $\Gamma_0 < 0$. The configuration (169), (173) corresponds precisely to this required new minimum for the scalar fluctuation.

In view of these arguments it seems rather likely that the sphere could be on acceptable minimum for the effective action (176) for metric gravity, at least within the space of configurations that can be reached by solutions of field equations. For a pure metric theory the propagators derived from this polynomial effective action show tachyonic or ghost instabilities for q^2 around M^2. This is of no worry in our case since we have already found stability in this momentum region for the full effective action (87).

Relaxing the restriction on configurations with vanishing covariant derivative of the vierbein, it seems useful to consider $e_\mu{}^m$ and $U_{\mu\nu}{}^m$ as variables, instead of $e_\mu{}^m$ and $A_{\mu mn}$. The term L_U provides for a positive quadratic term for $U_{\mu\nu}{}^m$. Expanding around the sphere there is no term linear in $U_{\mu\nu}{}^m$. The only terms linear in $U_{\mu\nu}{}^m$ are from L_F via eq. (109), and similarly from L_G. They vanish for a covariantly constant curvature tensor. In principle, mixed terms linear in $U_{\mu\nu}{}^m$ and linear in fluctuations of the vierbein could turn the extremum characterizing the sphere to a saddle point. This seems not very likely in the low momentum range, given that the vierbein fluctuations have to generate covariant derivatives of the curvature tensor.

3. Realistic euclidean quantum gravity

All these arguments do not constitute a proof that the sphere (169), (173) is the minimum of the euclidean effective action (87). If this sphere is the minimum for the possible solutions of field equations, our formulation of pregeometry leads to a rather satisfactory state of euclidean quantum gravity. The euclidean effective action is bounded from below for $U > 0$, and the minimum corresponds for tiny U/M^4 almost to flat space. Its analytic continuation to Minkowski signature would correspond to de Sitter space with a tiny cosmological constant, close to the present Universe for $U/M^4 \approx 10^{-120}$.

With $U/M^4 = 10^{-120}$, the overall view of the ground state of euclidean pregeometry is a very deep minimum with $\Gamma_0 \approx -10^{120}$, assumed by a sphere with huge radius in units of the Planck mass, $L_0 M \approx 10^{60}$. The four dimensional volume of the sphere is extremely large, $L_0^4 M^4 \approx 10^{240}$, corresponding to the size of our observable Universe in Planck units, $M^4/H^4 \approx 10^{240}$. Field configurations with even larger volume L^4 will lead to a huge positive action, due to the positive term $\int_x eU \approx (L/L_0)^4 (M^4/U) \approx 10^{120}(L/L_0)^4$, which overwhelms all other contributions to Γ. Fluctuations with effective volume L^4 substantially smaller than L_0^4 change the effective action only by a small relative amount $\Delta\Gamma/\Gamma_0$, suppressed by a volume factor L^4/L_0^4.

4. Metastability

We may turn back to the issue of metastability. The minimum of the effective action for scalar fluctuations, corresponding to the minimum of $\lambda_-(q^2)$ in eq. (159), occurs for $U = 0$ in the range $q^2 \approx M^2$. This range will not be affected by a very small positive U/M^4. Since the minimum occurs for negative $\lambda_-(q^2)$, this type of scalar fluctuation leads to an effective action that is lower than the one of the sphere. The sphere is therefore not the absolute minimum in the space of arbitrary field configurations. In the momentum range $q^2 \gg L_0^{-2}$ a tiny cosmological constant does not change the situation as compared to flat space. It seems likely that the scalar fluctuations that indicate an apparent metastability of the sphere solution are constrained fluctuations. The generalized Newton potential to which they correspond has to be created by other "propagating fluctuations". Together with the contribution of these fluctuations this may lead to an increase of the effective action, similar to the positive energy theorem for Einstein gravity in Minkowski space. The sphere may the minimum in the space of possible solutions of field equations.

IX. Conclusions and discussion

In this paper we have proposed an euclidean functional integral for a model of pregeometry that contains quantum gravity. The classical action is bounded from below and all two-point correlation functions or propagators have a simple short distance behavior without any instabilities. The inverse propagators in flat space are for all physical excitations proportional to the squared momentum q^2 in the limit $q^2 \to \infty$. Analytic continuation to Minkowski signature does not encounter any problems, and there are neither ghost nor tachyon instabilities in the high momentum region.

The functional integral still needs a regularisation. Since our model of pregeometry is a Yang-Mills gauge theory, the situation is in many respects similar to other gauge theories. The particularity is the diffeomorphism invariance of the classical action that should be preserved by the regularization. Dimensional regularization may be one of the possibilities, needing as all other continuum regularizations the introduction of some type of gauge fixing. It will be interesting to find out if a suitable lattice regularization exists, with A_μ and e_μ replaced by variables on the links of a lattice. This may require lattice diffeomorphism invariance [17, 42] for the discrete action, which should extend to the usual diffeomorphism symmetry in the continuum limit.

For low momenta it seems reasonable to expect that the invariants consistent with the symmetries and involving a low number of derivatives are present in the quantum effective action. It is sufficient that the term linear in the derivatives of the gauge field is generated by the fluctuations, with a coefficient $M^2 > 0$. In this case general relativity with the Einstein-Hilbert action for the composite metric emerges as the effective low momentum theory. By analytic continuation our model of pregeometry can then describe quantum gravity.

It is straightforward to add fermions in our model. The corresponding Grassmann variables ψ are scalars with respect to general coordinate transformations and transform as Weyl or Dirac spinors with respect to the $SO(4)$ - gauge group. The covariant derivative contains the gauge field in the usual way. The gauge invariant kinetic term for the fermions reads

$$S_\psi = i\int_x e\overline{\psi}\gamma^m e_m{}^\mu D_\mu\psi\ ,\quad \overline{\psi} = \psi^\dagger\gamma^0\ ,$$
$$D_\mu\psi = \partial_\mu\psi - \frac{1}{2}A_{\mu mn}\Sigma^{mn}\psi\ , \tag{177}$$

with γ^m the euclidean Dirac matrices and Σ^{mn} the $SO(4)$ - generators in the spinor representation,

$$\Sigma^{mn} = -\frac{1}{4}\big[\gamma^m,\gamma^n\big]\ ,\quad \big\{\gamma^m,\gamma^n\big\} = 2\delta^{mn}\ . \tag{178}$$

With $ee_m{}^\mu$ being a polynomial in the vierbein,

$$ee_m{}^\mu \sim \varepsilon^{\mu\nu\rho\sigma}\varepsilon_{mnpq}\, e_\nu{}^n e_\rho{}^p e_\sigma{}^q\ , \tag{179}$$

this action does actually not involve the inverse vierbein. Weyl spinors obtain by suitable projections.

It is possible that the vierbein arises from an even more fundamental pregeometry as a composite fermion bilinear

$$e_\mu{}^m = i\overline{\psi}\gamma^m D_\mu\psi\ . \tag{180}$$

Insertion of this expression into eqs. (177), (179) yields an invariant term involving eight powers of the fermion fields ψ. This is a gauge theory based only on fermions and gauge fields. Finally, the gauge fields may also be expressed as composites of fermions similar to ref. [43]. This results in a type of spinor gravity with local "Lorentz" symmetry $SO(4)$ and diffeomorphism invariance [17, 18]. While conceptually rather interesting, it is clear that suitable technologies have to be developed for dealing with actions that involve only multi-fermion invariants.

In the direction of a more realistic model for particle physics coupled to gravity, one needs to include additional gauge symmetries and corresponding gauge fields, as for the standard model gauge symmetry $SU(3)\times SU(2)\times U(1)$ or some grand unified extension. Furthermore, scalar fields are needed for a description of spontaneous symmetry breaking. Interesting cosmologies can be obtained if the parameters Z, m^2, M^2 become functions of a scalar singlet field χ, similar to variable gravity [44].

The central remaining issue with which we have not dealt in this paper concerns the renormalizability of our model of pregeometry. This involves an investigation of the dependence of the couplings Z, m^2, M^2 on some renormalization scale k. More generally, beyond the three couplings mentioned explicitly, one needs to understand the dependence of the whole scale-dependent effective action Γ_k on k. This question is a typical issue for functional renormalization which can be addressed by suitable truncations of the exact flow equation for the effective average action [4]. The scale-dependent effective action should admit a fixed point or scaling solution for $k\to\infty$, which remains sufficiently simple and close to the proposed classical action. Only in this case our model of pregeometry can be considered as a fully satisfactory description of emerging quantum gravity.

[1] K. S. Stelle, "Renormalization of higher-derivative quantum gravity," Phys. Rev. D **16**, 953 (1977).

[2] E. S. Fradkin and A. A. Tseytlin, "Renormalizable asymptotically free quantum theory of gravity," Nucl. Phys. B **201**, 469 (1982).

[3] I. G. Avramidy and A. O. Barvinsky, "Asymptotic freedom in higher-derivative quantum gravity," Phys. Lett. B **159**, 269 (1985).

[4] Christof Wetterich, "Exact evolution equation for the effective potential," Phys. Lett. B **301**, 90–94 (1993), arXiv:1710.05815 [hep-th].

[5] S. Weinberg, "Ultraviolet divergences in quantum theories of gravitation," in *General Relativity: an Einstein Centenary Survey* (Cambridge University Press, 1980) p. 790.

[6] M. Reuter, "Nonperturbative evolution equation for quantum gravity," Phys. Rev. D **57**, 971 (1998), arXiv:hep-th/9605030 [hep-th].

[7] D. Dou and R. Percacci, "The running gravitational couplings," Class. Quant. Grav. **15**, 3449 (1998), arXiv:hep-th/9707239 [hep-th].

[8] W. Souma, "Non-Trivial Ultraviolet Fixed Point in Quantum Gravity," Prog. Theor. Phys. **102**, 181 (1999), https://academic.oup.com/ptp/article-pdf/102/1/181/19571317/102-1-1

[9] M. Reuter and F. Saueressig, "Renormalization group flow of quantum gravity in the Einstein-Hilbert truncation," Phys. Rev. D **65**, 065016 (2002).

[10] O. Lauscher and M. Reuter, "Ultraviolet fixed point and generalized flow equation of quan-

tum gravity," Phys. Rev. D **65**, 025013 (2001), arXiv:hep-th/0108040 [hep-th].

[11] Alfio Bonanno, Astrid Eichhorn, Holger Gies, Jan M. Pawlowski, Roberto Percacci, Martin Reuter, Frank Saueressig, and Gian Paolo Vacca, "Critical reflections on asymptotically safe gravity," Front. in Phys. **8**, 269 (2020), arXiv:2004.06810 [gr-qc].

[12] A. Platania and C. Wetterich, "Non-perturbative unitarity and fictitious ghosts in quantum gravity," (2020), arXiv:2009.06637 [hep-th].

[13] K. Akama, "An Attempt at Pregeometry: – Gravity with Composite Metric –," Progr. Theor. Phys. **60**, 1900 (1978), https://academic.oup.com/ptp/article-pdf/60/6/1900/5224288/60-6-1900.pdf.

[14] D. Amati and G. Veneziano, "A unified gauge and gravity theory with only matter fields," Nucl. Phys. B **204**, 451 (1982).

[15] G. Denardo and E. Spallucci, "Curvature and torsion from matter," Class. Quant. Grav. **4**, 89 (1987).

[16] C. Wetterich, "Spontaneous Symmetry Breaking Origin for the Difference Between Time and Space," Phys. Rev. Lett. **94**, 011602 (2005), arXiv:hep-th/0405223 [hep-th].

[17] C. Wetterich, "Spinor Gravity and Diffeomorphism Invariance on the Lattice," Lect. Notes Phys. , 67 (2013), arXiv:1201.2871 [gr-qc].

[18] C. Wetterich, "Geometry and symmetries in lattice spinor gravity," Ann. Phys. **327**, 2184 (2012), arXiv:1201.6505 [hep-th].

[19] C. Wetterich, "Gravity from spinors," Phys. Rev. D **70**, 105004 (2004), arXiv:hep-th/0307145 [hep-th].

[20] A. Hebecker and C. Wetterich, "Spinor gravity," Phys. Lett. B **574**, 269 (2003), arXiv:hep-th/0307109 [hep-th].

[21] C. Wetterich, "Dimensional Reduction of Fermions in Generalized Gravity," Nucl. Phys. B **242**, 473 (1984).

[22] C. Wetterich, "Fundamental Scale Invariance," (2020), arXiv:2007.08805 [hep-th].

[23] Roberto Percacci, "Spontaneous soldering," Physics Letters B **144**, 37 – 40 (1984).

[24] E. Cartan, "Sur une généralisation de la notion de courbure de Riemann et les espaces á torsion," C. R. Acad. Sci. **174**, 593 (1922).

[25] R. Utiyama, "Invariant Theoretical Interpretation of Interaction," Phys. Rev. **101**, 1597 (1956).

[26] T. W. B. Kibble, "Lorentz Invariance and the Gravitational Field," J. Mathm. Phys. **2**, 212 (1961).

[27] D. W. Sciama, "The Physical Structure of General Relativity," Rev. Mod. Phys. **36**, 463 (1964).

[28] F. W. Hehl, P. v. d. Heyde, G. D. Kerlick, and J. M. Nester, "General relativity with spin and torsion: Foundations and prospects," Rev. Mod. Phys. **48**, 393 (1976).

[29] I. L. Shapiro, "Physical aspects of the space–time torsion," Phys. Rep. **357**, 113 (2002).

[30] F. W. Hehl, J. D. McCrea, E. W. Mielke, and Y. Ne'eman, "Metric-affine gauge theory of gravity: field equations, Noether identities, world spinors, and breaking of dilation invariance," Phys. Rep. **258**, 1 (1995).

[31] L. Heisenberg, "A systematic approach to generalisations of General Relativity and their cosmological implications," Phys. Rep. **796**, 1 (2019), arXiv:1807.01725 [gr-qc].

[32] R. Percacci, "Towards Metric-Affine Quantum Gravity," (2020), arXiv:2003.09486 [gr-qc].

[33] R. Percacci and E. Sezgin, "New class of ghost- and tachyon-free metric affine gravities," Phys. Rev. D **101**, 084040 (2020), arXiv:1912.01023 [hep-th].

[34] M. Shaposhnikov, A. Shkerin, I. Timiryasov, and S. Zell, "Einstein-Cartan gravity, matter, and scale-invariant generalization," (2020), arXiv:2007.16158 [hep-th].

[35] S. Matsuzaki, S. Miyawaki, K. y. Oda, and M. Yamada, "Dynamically emergent gravity from hidden local Lorentz symmetry," (2020), arXiv:2003.07126 [hep-th].

[36] J.-E. Daum and M. Reuter, "Einstein–Cartan gravity, Asymptotic Safety, and the running Immirzi parameter," Ann. Phys. **334**, 351 (2013), arXiv:1301.5135 [hep-th].

[37] U. Harst and M. Reuter, "A new functional flow equation for Einstein–Cartan quantum gravity," Ann. Phys. **354**, 637 (2015), arXiv:1410.7003 [hep-th].

[38] J.-E. Daum and M. Reuter, "Renormalization group flow of the Holst action," Phys. Lett. B **710**, 215 (2012), arXiv:1012.4280 [hep-th].

[39] K. Krasnov, "Gravity as a diffeomorphism-invariant gauge theory," Phys. Rev.D **84**, 024034 (2011), arXiv:1101.4788 [hep-th].

[40] K. Krasnov, "Pure Connection Action Principle for General Relativity," Phys. Rev. Lett. **106**, 251103 (2011), arXiv:1103.4498 [gr-qc].

[41] C. Wetterich, "Quantum correlations for the metric," Phys. Rev. D **95**, 123525 (2017), arXiv:1603.06504 [gr-qc].

[42] C. Wetterich, "Lattice diffeomorphism invariance," Phys. Rev. D **85**, 104017 (2012), arXiv:1110.1539 [hep-lat].

[43] C. Wetterich, "Scalar lattice gauge theory," Nucl. Phys. B **876**, 147–186 (2013), arXiv:1212.3507 [hep-lat].

[44] C. Wetterich, "Variable gravity Universe," Phys. Rev. D **89**, 024005 (2014), arXiv:1308.1019 [astro-ph.CO].