

Dimensionless physics: continuation

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Several approaches to quantum gravity (including the model of superplastic vacuum; Diakonov tetrads emerging as the bilinear combinations of the fermion fields; and effective acoustic metric) suggest that in general relativity the metric must have dimension 2, i.e. $[g_{\mu\nu}] = 1/[L]^2$, irrespective of the dimension of spacetime. This leads to the "dimensionless physics" discussed in the review paper¹. Here we continue to exploit this unusual dimension of the metric.

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I. INTRODUCTION: ELASTICITY TETRADS, QUANTUM MECHANICS AND GENERAL RELATIVITY

Here we discuss some issues related to the dimensions of physical variables, which follow from the peculiar dimensions of the tetrads and metric. The particular example is represented by the elasticity tetrads – the translational gauge fields, which have dimension $1/[L]$, The corresponding metric field, which is the bilinear combination of the tetrad fields, has dimension $1/[L]^2$. There are the other approaches to gravity, which also result in the same dimensions. All this leads to the "dimensionless physics".¹ Here we continue to exploit the dimensionful tetrads. In particular we consider how such tetrads make connection between quantum mechanics and general relativity.

II. DIMENSIONS OF TETRADS AND METRIC

A. Dimensional tetrads

The 3 + 1-dimensional vacuum crystal is the plastic (malleable) medium,² which is described in terms of the so-called elasticity tetrads:³⁻⁶

$$E_{\mu}^a = \frac{\partial X^a}{\partial x^{\mu}}, \quad (1)$$

where equations $X^a(x) = 2\pi n_a$ are equations of the (deformed) crystal planes. Since the functions X^a play the role of the geometric $U(1)$ phases and thus are dimensionless, the elasticity tetrads play the role of the gauge fields (translation gauge fields). That is why tetrads have the same dimension 1 as the dimension of gauge fields:

$$[E_{\mu}^a] = \frac{1}{[L]}. \quad (2)$$

The dimension n of quantity A means $[A] = [L]^{-n}$, where $[L]$ is dimension of length. Tetrads with dimension 1 lead to dimensionless physics,¹ which we discuss here.

Note that the $U(1)$ gauge fields, which form the dimensional tetrads, are not necessarily related to translations and the crystalline structure of the vacuum. The latter is only one of the possible sources of the dimensional tetrads. In particular, the dimension 1 tetrads appear in the Diakonov theory,⁷⁻¹⁰ see Sec. III C.

Note also, that in principle, the matrix E_{μ}^a is not necessarily quadratic. The extension of tetrads to the rectangular vilebein is considered in Ref.¹¹.

B. Dimensional metric

Elasticity tetrads in Eq.(1) give rise to the metric with lower indices, which is the following bilinear combination of tetrads:

$$g_{\mu\nu} = \eta_{ab} E_{\mu}^a E_{\nu}^b. \quad (3)$$

This covariant metric tensor has dimension $n = 2$, while the contravariant metric $g^{\mu\nu}$ has dimension $n = -2$:

$$[g_{\mu\nu}] = \frac{1}{[L]^2}, \quad [g^{\mu\nu}] = [L]^2. \quad (4)$$

Another example of the metric with the dimension 2 is the effective metric describing the propagation of Goldstone modes (phonons) in moving superfluids, the so-called acoustic metric,^{12,13} see Sec. III E.

The tetrad determinant has dimension 4 in the 4-dimensional spacetime:

$$[e] = [\sqrt{-g}] = \frac{1}{[L]^4}. \quad (5)$$

In the N -dimensional spacetime, the dimensions of the metric elements are the same as in Eq.(4), while the tetrad determinant has dimension N :

$$[e] = [\sqrt{-g}] = \frac{1}{[L]^N}. \quad (6)$$

C. Action and Lagrangian

Eq.(6) makes the spacetime integration dimensionless:

$$\left[\int d^N x \sqrt{-g} \right] = [1] = 0. \quad (7)$$

Since the action is dimensionless, Eq.(7) leads to the dimensionless Lagrangian, $[\mathcal{L}] = [1] = 0$:

$$[S] = \left[\int d^N x \sqrt{-g} \mathcal{L} \right] = \left[\int d^N x \sqrt{-g} \right] \cdot [\mathcal{L}] = [1] \cdot [1] = [1]. \quad (8)$$

D. Dynamics of classical point particle

Eq.(3) gives rise to the dimensionless interval:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad [s^2] = \frac{1}{[L]^2} \cdot [L]^2 = [1] = 0. \quad (9)$$

In general relativity, the interval describes the classical dynamics of a point particle with action:

$$S = M \int ds. \quad (10)$$

The variation of action leads to the Hamilton–Jacobi equation expressed in terms of the contravariant metric $g^{\mu\nu}$:

$$g^{\mu\nu} \partial_\mu S \partial_\nu S + M^2 = 0. \quad (11)$$

Since the action (10) is dimensionless, the dimensionless interval makes the particle mass M also dimensionless, $[M] = [1] = 0$. Then both terms in Eq.(11) are dimensionless. This is valid for the arbitrary dimension N of spacetime.

In the spacetime crystal, the interval between the events is counted in terms of the lattice points, and this is the geometric reason why the interval is dimensionless. In case of the other possible origins of the dimensionless interval, this demonstrates that the interval determines the dynamics of the particle, rather than the geometric distance. In the first case the dynamics comes from geometry, while in the second case the geometry follows from dynamics.

III. DIMENSIONS OF QUANTUM FIELDS IN CURVED SPACE

A. Universal dimension of scalar fields

Let us consider the quadratic terms in the action for the scalar field Φ in the N -dimensional spacetime:

$$S = \int d^N x \sqrt{-g} (g^{\mu\nu} \nabla_\mu \Phi^* \nabla_\nu \Phi + M^2 |\Phi|^2) , \quad (12)$$

Taking into account equations (4) and (7), one obtains that the scalar field is dimensionless, $[\Phi] = [1] = 0$, for arbitrary spacetime dimension N . This universal zero dimension differs from the N -dependent dimension of scalar fields in the conventional approach, where the dimension is $n = (N - 2)/2$.

B. Dimension of wave function in quantum mechanics

Expanding the Klein-Gordon equation for scalar Φ in Eq.(12) over $1/M$ one obtains the non-relativistic Schrödinger action in curved space-time. Let us for simplicity consider the space-time independent metric and assume that $g^{0i} = 0$. Then introducing the Schrödinger wave function ψ :

$$\Phi(\mathbf{r}, t) = \frac{1}{\sqrt{M}} \exp\left(iMt/\sqrt{-g^{00}}\right) \psi(\mathbf{r}, t) , \quad (13)$$

one obtains the Schrödinger-type action in the form

$$S_{\text{Sch}} = \int d^3 x dt \sqrt{-g} \mathcal{L} , \quad (14)$$

$$2\mathcal{L} = i\sqrt{-g^{00}} (\psi \partial_t \psi^* - \psi^* \partial_t \psi) + \frac{g^{ik}}{M} \nabla_i \psi^* \nabla_k \psi . \quad (15)$$

The normalization condition for the wave function ψ is:

$$\int d^3r \sqrt{\gamma} |\psi|^2 = 1, \quad (16)$$

where $\sqrt{\gamma} = \sqrt{-g} \sqrt{-g^{00}}$ is the determinant of the space part of the metric. This corresponds to the particle number conservation, see e.g. Eq.(13) in Ref.¹⁴.

Since the dimension of this determinant is $[\sqrt{\gamma}] = \frac{1}{[L]^3}$, the wave function ψ is dimensionless. The dimension $[\psi] = [1]$ can be also obtained directly from Eqs. (14) and (15), since the Lagrangian is dimensionless, $[\mathcal{L}] = [1]$. This is distinct from the conventional Schrödinger equation without gravity, where the dimension of the wave function is $[\psi] = [L]^{-3/2}$ (or $[\psi] = [L]^{-(N-1)/2}$ for the N dimensional spacetime). Inclusion of gravity in the form of the vacuum crystal tetrads provides the natural zero dimension for the probability amplitude in quantum mechanics, $[\psi] = [1]$, for any spacetime dimension N .

The same can be obtained using the Dirac bra-ket notations. The overlap of the quantum states is naturally dimensionless:

$$\langle \mathbf{r} | \mathbf{r}' \rangle = \frac{1}{\sqrt{\gamma}} \delta(\mathbf{r} - \mathbf{r}'). \quad (17)$$

Then for the wave function

$$\psi(\mathbf{r}) = \langle \mathbf{r} | \psi \rangle, \quad |\psi\rangle = \int d^{N-1}r \sqrt{\gamma} \psi(\mathbf{r}) |\mathbf{r}\rangle, \quad (18)$$

one obtains Eq.(16) for normalization:

$$1 = \langle \psi | \psi \rangle = \int d^{N-1}r \sqrt{\gamma} |\psi|^2. \quad (19)$$

This confirms that the wave function is dimensionless, $[\psi] = [1] = 0$, and this is the consequence of the presence of the tetrad field in Eq.(19). This demonstrates the close connection between quantum mechanics and general relativity.

Note, that the action (14) and Lagrangian (15) do not contain \hbar . The role of \hbar in equation $E_m - E_n = \hbar \omega_{mn}$ is now played by $\sqrt{g^{00}}$ in the red shift equation,¹⁵ $M_m - M_n = \sqrt{g^{00}} \omega_{mn}$. The corresponding dimensions are: $[M] = [\sqrt{g^{00}}][\omega] = [L] \cdot 1/[L] = [1]$.

C. Weyl and Dirac fermions

On the other hand, the natural dimension of tetrads, $[E_\mu^a] = 1/[L]$, can be obtained without consideration of the model of superplastic vacuum. It is enough to use the natural

zero dimension of wave functions, $[\psi] = [1]$, which gives rise to the dimensionless Weyl and Dirac fields, $[\Psi] = [1] = 0$. For example, from the dimensionless action for massless Dirac fermions

$$S = \int d^4x e e_a^\mu \bar{\Psi} \gamma^a \nabla_\mu \Psi, \quad (20)$$

assuming that the quantum field operators Ψ are dimensionless, $[\Psi] = [1] = 0$, one obtains $[e e_a^\mu] = 1/[L]^3$, which gives

$$[e_a^\mu] = [L], [E_\mu^a] = \frac{1}{[L]}, [e] = \frac{1}{[L]^4}. \quad (21)$$

The Hamiltonian for massless Dirac fermions has dimension 1, i.e. the same as the dimension of frequency:

$$H = \int_{x_0=\text{const}} d^3r e e_a^i \bar{\Psi} \gamma^a \nabla_i \Psi, [H] = [\omega] = \frac{1}{[L]}. \quad (22)$$

However, the dimension of the Hamiltonian does not coincide with the dimension of mass M , which is zero.

Although the fermionic fields are dimensionless, the current density operator of these fields has the conventional dimension 3. This is due to the dimension of tetrads:

$$j^\mu = e e_a^\mu \bar{\Psi} \gamma^a \Psi, [j^\mu] = [e][e_a^\mu] = \frac{1}{[L]^3}. \quad (23)$$

The dimensionful tetrads also appear in the Diakonov theory,⁷⁻¹⁰ where tetrads emerge as the bilinear combinations of the fermionic fields:

$$E_\mu^a \propto \langle \bar{\Psi} \gamma^a \nabla_\mu \Psi \rangle, [E_\mu^a] = \frac{1}{[L]}. \quad (24)$$

In this approach the metric emerges as the bilinear combination of tetrads, and thus as the quadrilinear combination of the fermionic fields.

D. Gauge fields

The action for the $U(1)$ gauge field in the N -dimensional spacetime is:

$$S \sim \int d^N x \sqrt{-g} g^{\mu\nu} g^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta}. \quad (25)$$

In case of the conventional dimensionless tetrads, the action in Eq.(25) is dimensionless only for $N = 4$. But with dimensionful tetrads the action is dimensionless for arbitrary N , since

$$[g^{\mu\nu}] = [L]^2, [F_{\mu\nu}] = \frac{1}{[L]^2}, [\sqrt{-g}] = \frac{1}{[L]^N}. \quad (26)$$

The dimensionless prefactor $1/\alpha$ in Eq.(25) will be discussed in Sec. IV D.

E. Dimension of acoustic metric

Acoustic metric describes propagation of sound in a non-homogeneous flowing fluid,^{12,13} and also the propagation of Goldstone modes in moving superfluids. The action for the propagating Goldstone mode (the phase ϕ of the Bose condensate) is similar to the action (12) for the massless scalar field:

$$S = \int d^4x \sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi. \quad (27)$$

There are some differences between Eq.(27) and Eq.(12): (i) the phase ϕ of the superfluid order parameter is dimensionless in any approach; (ii) $\tilde{g}^{\mu\nu}$ is the emergent (effective) acoustic metric, which is determined by the velocity and density of the superfluid liquid; (iii) the real gravity is not taken into account and thus the conventional dimensions can be used.

From the action (27) it follows that $\tilde{g}^{\mu\nu}$ has dimension -2 , i.e. $[\tilde{g}^{\mu\nu}] = [l]^2$. This can be also seen from the dependence of the effective interval on the hydrodynamic variables:^{13,16}

$$d\tilde{s}^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu = \frac{n}{ms} [-s^2 dt^2 + (dx^i - v^i dt) \delta_{ij} (dx^j - v^j dt)]. \quad (28)$$

Here n is the density of atoms in the liquid; m is the mass of the atom; s is the speed of sound; and v^i is the velocity of the superfluid liquid, which coincides with the shift vector N^i in the Arnowitt-Deser-Misner formalism in Sec. IV D.

Using the conventional dimensions of hydrodynamic quantities in Eq.(28) one obtains the dimension 2 for the covariant metric $\tilde{g}_{\mu\nu}$:

$$[\tilde{g}_{\mu\nu}] = [n] \cdot \frac{1}{[m]} = \frac{1}{[l]^3} \cdot [l] = \frac{1}{[l]^2}. \quad (29)$$

Such dimension of the effective metric follows solely from the dynamics of the superfluid. This provides another example when the geometry comes from dynamics.

The effective metric in Eq.(28) depends on two "Planck" energy scales. One of them is related to the mass of the atom of the liquid and another one is determined by the average distance between the atoms in the liquid. Together they produce the effective Planck scale:¹⁶

$$E_{\text{Planck}} = E_{\text{Planck}1}^{-1/2} E_{\text{Planck}2}^{3/2}, \quad E_{\text{Planck}1} = ms^2, \quad E_{\text{Planck}2} = n^{1/3}. \quad (30)$$

IV. GENERAL RELATIVITY

A. Einstein action

Let us consider the GR action on example of q -theory. This is the class of theories which avoid the cosmological constant problem: the huge contributions of zero point energy to the cosmological constant is cancelled in the equilibrium state of the vacuum due to thermodynamics.¹⁷⁻¹⁹ For the particular q -theory on the four-dimensional (4D) “brane” the action is:^{20,21}

$$S = - \int d^4x \sqrt{-g} \left[\epsilon(q) + \frac{R}{16\pi G_N(q)} + \Lambda_0 + \mathcal{L}^M[\psi, q] \right] + \mu \int d^4x n, \quad (31)$$

$$q = \frac{n}{\sqrt{-g}}. \quad (32)$$

Here n is the 4D analog of the particle density in the quantum vacuum (density of the “spacetime atoms”), which has the same dimension 4 as the tetrad determinant

$$[n] = [\sqrt{-g}] = \frac{1}{[L]^4}, \quad (33)$$

q is the vacuum variable, and μ plays the role of the chemical potential in the vacuum thermodynamics.

For illustration, we consider the metric of the expanding Friedmann-Robertson-Walker universe:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -d\tau^2 + a^2(\tau) d\mathbf{r}^2, \quad (34)$$

$$H(\tau) = \frac{da/d\tau}{a(\tau)}, \quad (35)$$

where τ is the conformal time; $a(\tau)$ is the scale factor; and $H(\tau)$ is time-dependent Hubble parameter.

The scale factor $a(\tau)$ has dimensions 1:

$$[a(\tau)] = \frac{1}{[L]}, \quad (36)$$

while the following quantities have zero dimension ($1/[L]^0 = [1] = 0$):

$$\begin{aligned} [q] &= [\mu] = [\epsilon] = [R] = [G_N] = [\Lambda_0] = \\ &= [H] = [\tau] = [\psi] = [M] = [1] = 0. \end{aligned} \quad (37)$$

Some of the dimensionless quantities can be fundamental, or correspond to some integer valued topological invariants. For example, the "chemical potential" μ may correspond to the topological invariant, $\mu = \pm 1$, which changes sign at the Big-Bang quantum phase transition.¹⁹ Since masses of particles are dimensionless, and there is no fundamental mass scale, one can choose any convenient mass as a unit mass.

Note also that the dimensionless interval in Eq.(9) does not mean the existence of the fundamental length, such as Planck length. First, because the gravitational coupling $1/G_N$ is not fundamental (see, however, Sec.IV C). Second, in the model of the superplastic (malleable) vacuum there is no equilibrium value of the distance between the neighbouring lattice points. As distinct from the solid state crystals, arbitrary deformations of the vacuum crystal are possible. Also, in the Diakonov model,⁷ the metric is the emergent phenomenon, while on the fundamental level the distance between the spacetime points is not determined.

B. Black hole entropy

In terms of the dimensionful metric, the acceleration is dimensionless:¹

$$a^2 = g_{\mu\nu} \frac{d^2 x^\mu}{ds^2} \frac{d^2 x^\nu}{ds^2}, \quad (38)$$

$$[a^2] = [g_{\mu\nu}][x^\mu][x^\nu] = \frac{1}{[l]^2} \cdot [l]^2 = [1] = 0. \quad (39)$$

This leads to the dimensionless Unruh temperature:

$$T_U = \frac{a}{2\pi}, \quad [T_U] = [1] = 0. \quad (40)$$

The same is valid for the Gibbons-Hawking temperature of the cosmological horizon. As follows from Eq.(37), it is also dimensionless:

$$T_H = \frac{H}{2\pi}, \quad [T_H] = [H] = [1] = 0, \quad (41)$$

Both equations, (40) and (41), look fundamental: they do not contain parameters.

However, for the temperature of the Hawking radiation from the black hole horizon, $T_{\text{BH}} = 1/8\pi G_N M$, situation is different. Although the Hawking temperature is dimensionless ($[T_{\text{BH}}] = [1]$, since $[G_N] = [M] = [1]$), it is not fundamental, because it depends on the parameter G_N , which is not fundamental (see, however, Sec. IV C). The same concerns the

conventional Bekenstein-Hawking entropy,

$$S_{\text{BH}} = \frac{A}{4G_N}. \quad (42)$$

The entropy in Eq.(42) is dimensionless, since both G_N and the area A are dimensionless.¹ That the horizon area is dimensionless can be seen from the general equation for area:

$$dA = \sqrt{dS^{ik}dS_{ik}}, \quad [A] = [1] = 0, \quad (43)$$

and from the simple spherical case:

$$A = \int_0^\pi d\theta \int_0^{2\pi} d\phi \sqrt{g_{\phi\phi}g_{\theta\theta}}, \quad [g_{\phi\phi}] = [g_{\theta\theta}] = [A] = 0. \quad (44)$$

The Bekenstein-Hawking entropy (42) determines the black hole thermodynamics, but similar to the Hawking temperature it does not look as fundamental, since it contains the gravitational coupling $1/G_N$. Also it is not clear why the microscopic degrees of freedom responsible for the black hole entropy should be characterized by the Planck length.²² In the superplastic vacuum approach,² the Planck length scale is simply absent, since there is no equilibrium value of the distance between the lattice points: the superplastic vacuum can be arbitrarily deformed.

On the other hand, since the area is dimensionless, one may suggest that the entropy of the black hole horizon can be expressed in terms of the area only:

$$S_{\text{BH}} = \eta A, \quad [\eta] = [S_{\text{BH}}] = [A] = [1] = 0. \quad (45)$$

Here η is some fundamental dimensionless parameter, like the topological invariant. In this case one may take the point of view that Einstein's gravity equations can be derived solely from thermodynamics.²³ Then the constant of proportionality η between the entropy and the area determines the gravitational coupling, $1/G_N = 4\eta$. In this thermodynamic approach, $1/G_N$ may become fundamental due to the fundamentality of the parameter η .

However, in the thermodynamic approach to gravity there is the ‘‘species problem’’,²⁴ i.e. the gravitational coupling G_N may depend on the number of fermionic and bosonic fluctuating quantum fields.²⁵⁻²⁷ In principle, with some relations between the numbers of quantum fields, $1/G_N$ can be negative, or even zero. This destroys many conjectures, which are based on positivity of the gravitational coupling.²⁸ If so, $1/G_N$ cannot be the fundamental parameter.

In principle, this “no-go theorem” can be avoided, if $1/G_N$ is the quantum number, which is related to symmetry and/or topology. In this case the parameter $1/G_N$ does not depend on the interaction between gravity and quantum field, though it may experience jumps during the topological quantum phase transitions. The latter takes place in topological materials when one varies the parameters of interaction,^{29,30} and may take place when the Universe crosses the Big Bang.¹⁹

C. Einstein-Cartan, Barbero-Immirzi, Nieh-Yan and topology

The topological invariants, which are relevant for the quantum vacuum, are known in the crystalline matter,^{4,5,31} and thus they can be extended to the superplastic quantum vacuum. The topology in the crystalline quantum vacua is enriched due to the dimensional elasticity tetrads in Eq.(1), which come from the geometric $U(1)$ phases. This topological approach is not applicable to the conventional Einstein gravity, where tetrads are absent. But it may take place in the Einstein-Cartan gravity (Einstein–Cartan–Sciama–Kibble theory), which is expressed in terms of tetrads, and thus is more fundamental than the conventional Einstein gravity, which is based on metric. Such type of gravity emerging in superplastic crystals has been discussed in Ref.³².

The gravitational action in the Einstein-Cartan gravity can be expressed in terms of the differential forms, which contain the elasticity tetrads as the translational gauge fields:

$$S_{\text{EC}} \sim \epsilon_{abcd} \int d^4x E^a \wedge E^b \wedge R^{cd}. \quad (46)$$

This action is dimensionless because the one-form tetrad has dimension 1, $[E_\mu^a] = \frac{1}{[L]}$, while the curvature two-form R^{ab} has dimension 2:

$$[R_{\mu\nu}^{ab}] = \frac{1}{[L]^2}. \quad (47)$$

That is why with the dimensional elasticity tetrads, the topology of the 3 + 1 the crystalline phases,^{4,5,31} may provide the fundamental topological prefactor in Eq.(46), with $1/G_N$ being integer or fractional topological number.

The same can be valid for the dimensionless parameter in the Barbero-Immirzi action:

$$S_{\text{BI}} \sim \int d^4x E^a \wedge E^b \wedge R_{ab}. \quad (48)$$

Eq.(48) looks similar to the Nieh-Yan term in the action, see e.g. Ref.³³. Due to dimensional tetrads, $[E_\mu^a] = \frac{1}{[L]}$, the prefactors in the Nieh-Yan and in the Barbero-Immirzi actions are dimensionless, and thus can be fundamental.¹ It is not excluded that these parameters are the topological invariants similar to that in topological insulators, semimetals and superconductors.⁴

D. ADM formalism and fine structure constant

The Arnowitt-Deser-Misner (ADM) formalism³⁴ is used for the Hamiltonian formulation of general relativity. Let us consider this formalism and its application using the dimensional metric. One has the following metric elements and their dimensions:

$$g_{ik} = \gamma_{ik} , [\gamma_{ik}] = \frac{1}{[L]^2} , \quad (49)$$

$$g_{0i} = N_i = \gamma_{ik} N^k , [N_i] = \frac{1}{[L]^2} , [N^i] = 0 , \quad (50)$$

$$g_{00} = \gamma_{ik} N^i N^k - N^2 = N^i N_i - N^2 , [N] = \frac{1}{[L]} , \quad (51)$$

$$g^{00} = -\frac{1}{N^2} , [g^{00}] = [L]^2 , \quad (52)$$

$$g^{0i} = \frac{N^i}{N^2} , [g^{0i}] = [L]^2 , \quad (53)$$

$$g^{ik} = \gamma^{ik} - \frac{N^i N^j}{N^2} , [\gamma^{ik}] = [L]^2 , \quad (54)$$

$$\sqrt{-g} = N \sqrt{\gamma} , [\sqrt{\gamma}] = \frac{1}{[L]^3} , \quad (55)$$

$$\gamma^{ik} \gamma_{kl} = \delta_l^i . \quad (56)$$

Here N and N^i are lapse and shift functions correspondingly, and γ_{ik} are the space components of the metric. Actually the ADM metric is similar to the acoustic metric in Sec.III E, where the role of the shift vector N^i is played by the velocity v^i of the liquid.

The ADM formalism allows to consider dynamics in curved space in terms of the Poisson brackets. Let us consider this on example of Poisson brackets for the classical 3 + 1 electrodynamics in curved space:

$$\{A_i(\mathbf{r}), D^k(\mathbf{r}')\} = \delta_i^k \delta(\mathbf{r} - \mathbf{r}') , \quad (57)$$

which in terms of the gauge invariant fields is:

$$\{B^i(\mathbf{r}), D^k(\mathbf{r}')\} = e^{ikl} \nabla_l \delta(\mathbf{r} - \mathbf{r}') . \quad (58)$$

Here \mathbf{B} is magnetic field, and the vector \mathbf{D} is the electric induction of the quantum vacuum (electric displacement field). The electric induction \mathbf{D} is expressed in terms of the electric field $E_i = F_{0i}$:

$$D^k = \frac{1}{\alpha} \frac{\sqrt{\gamma}}{N} \gamma^{ik} E_i. \quad (59)$$

Here α is the dimensionless fine structure constant, which determines the dielectric constant (electric permittivity) of the relativistic quantum vacuum, ϵ_{vac} , and the magnetic permeability μ_{vac} :

$$\epsilon_{\text{vac}} = \frac{1}{\mu_{\text{vac}}} = \frac{1}{\alpha}, \quad (60)$$

and we do not include the artificial factor 4π . In the $3 + 1$ spacetime, \mathbf{D} has the same dimension 2 as the electric field in spite of the dimensional metric:

$$[D^i] = [E_i] = \frac{1}{[L]^2}. \quad (61)$$

The Poisson brackets in Eqs. (57) and (58) look as fundamental. They do not depend on the metric and do not contain physical parameters of the quantum vacuum. However, the function \mathbf{D} in Eq.(59) breaks this fundamentality. It is the phenomenological variable, which describes the response of the vacuum to electric field. This response contains the electromagnetic coupling $1/\alpha$, which is not fundamental because of the corresponding ‘‘species problem’’: it depends on the fluctuating bosonic and fermionic fields in the quantum vacuum. While according to Sec. IV C, the gravitational coupling $1/G_N$ can be fundamental due to topology, there are no topological invariants which could support the fundamentality of the electromagnetic coupling $1/\alpha$. This is in favour of the scenario in which the quantum electrodynamics is the effective low-energy theory, where for example the gauge fields emerge as the bilinear combinations of the fermionic fields, or/and the gauge fields emerge in the vicinity of the topologically stable Weyl points in the fermionic spectrum.^{16,35–37}

V. CONCLUSION

Several approaches to quantum gravity (including the model of superplastic vacuum; Diakonov tetrads emerging as the bilinear combinations of the fermionis fields; and effective acoustic metric) suggest that in general relativity the metric has dimension 2, i.e. $[g_{\mu\nu}] = 1/[L]^2$, irrespective of the dimension of spacetime. One consequence of such dimension of the

metric is that the wave function in quantum mechanics is dimensionless, $[\psi(x)] = [1] = 0$. This also leads to the dimensionless quantum fields.

On the other hand, if one starts with the conjecture that in quantum mechanics the wave function is naturally dimensionless, one obtains that the metric acquires the dimension 2. This suggests the close connection between quantum mechanics and general relativity.

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