

Quantum gravitational signatures in next-generation gravitational wave detectors

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Abstract

A recent study established a correspondence between the Generalized Uncertainty Principle (GUP) and Modified theories of gravity, particularly Stelle gravity. We investigate the consequences of this correspondence for inflation and cosmological observables by evaluating the power spectrum of the scalar and tensor perturbations using two distinct methods. First, we employ PLANCK observations to determine the GUP parameter γ_0 . Then, we use the value of γ_0 to investigate the implications of quantum gravity on the power spectrum of primordial gravitational waves and their possible detectability in the next-generation detectors, like Einstein Telescope and Cosmic explorer.

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I. INTRODUCTION

Since its discovery, the cosmic microwave background (CMB) has been an indispensable tool for understanding the very early universe. Consequently, observational cosmology has made incredible progress over the last three decades and thanks to the observation of temperature fluctuations at the last scattering surface, one has been able to identify the physical origins of the primordial density perturbations in the very early universe [1–7]. The inflationary paradigm provides a causal mechanism for the origin of these perturbations and the inflationary epoch magnifies the tiny fluctuations in the quantum fields present at the beginning of the epoch into classical perturbations that leave an imprint as anisotropies in the CMB [8]. A key prediction of inflation is the generation of primordial gravitational waves (PGWs) [9, 10].

Therefore, significant effort is currently underway to detect PGWs via B-mode polarization measurements of the CMB [5, 11–16]. This is because B-modes of CMB are *only* sourced by the differential stretching of spacetime associated with the PGWs [10, 17]. However, the primordial B-mode polarization signal is weak and could be swamped inside dust emission in our galaxy [5, 14, 18]. Efforts are also underway to directly detect PGWs [19–23]. In this work, we explicitly show that PGWs carry quantum gravitational signatures that can potentially be observed in the next-generation gravitational wave detectors such as the Einstein Telescope and Cosmic Explorer.

Theories of Quantum Gravity (QG) predict the existence of a minimum measurable length and/or a maximum measurable momentum of particles [24, 25]. The experimental implications of this scale are explored in Quantum Gravity Phenomenology (QGP) [26]. This QGP/QG scale is most easily realized by deforming the standard Heisenberg uncertainty relation to the so-called Generalized Uncertainty Principle (GUP) [27–36]. Recently, we proposed a Lorentz invariant implementation of GUP, wherein we modified the canonical commutation relation to [37]:

$$[x^\mu, p^\nu] = i\hbar\eta^{\mu\nu}(1 - \gamma p^\sigma p_\sigma) - 2i\hbar\gamma p^\mu p^\nu, \quad (1)$$

where in natural units ($\hbar = c = 1$),

$$\gamma = \gamma_0 M_{\text{Planck}}^{-2} = \gamma_0 l_{\text{Planck}}^2, \quad (2)$$

is the Lorentz invariant scale, and γ_0 is a numerical parameter used to fix the scale. Ex-

perimental data and QG theories suggest that the intermediate scale γ should be found somewhere between the electroweak length scale $\sim 10^{17}l_{\text{Planck}}$ and the Planck length scale.

In a recent study [38], we employed the GUP model with maximum momentum uncertainty in Eq. (1) to establish a relationship between the GUP-modified dynamics of a massless spin-2 field and Stelle gravity with mass degeneracy [39]. We also showed that the mass-degenerate Stelle gravity leads to inflation with a natural exit and can be *mapped* to Starobinsky gravity in the sense that the dynamics predicted by both models are the same. Using this, we obtained some of the best-known bounds on GUP parameters [38].

Recently, it was shown that the massive spin-2 modes in the Stelle gravity carry more energy than the scalar modes in Starobinsky model [40]. It is also known that the Starobinsky model is in perfect agreement with the Planck observations as it predicts a low value of the scalar-to-tensor ratio (r) [5–7, 41]. This leads to the following questions: Are there signatures distinguishing mass-degenerate Stelle gravity and Starobinsky model? What are the observable signatures of the GUP modified gravity? In this work, we address these questions by investigating the evolution of the scalar and tensor perturbations in these two scenarios.

In order for the inflationary model of the mass-degenerate Stelle gravity to be successful, it must lead to suitable primordial density perturbations that are consistent with the CMB observations [3, 5]. In this work, we evaluate the spectral tilt of the scalar perturbations $n_{\mathcal{R}}$, and the scalar to tensor ratio r . By comparing the primordial scalar power spectrum with the PLANCK [5, 6], we obtain the bounds on the GUP parameter. Interestingly, the values for γ_0 we obtain here are consistent with the bounds obtained in Refs. [38, 42] and bounds from quantum mechanical considerations [37, 43, 44].

Using the bounds of γ_0 , we evaluate the power spectrum of PGWs generated during inflation. We show that these can be observed in the upcoming gravitational wave detectors, such as Einstein telescope [19] and Cosmic explorer [22, 23]. Additionally, such detection can provide a direct constrain on the number of e-foldings of inflation.

II. FROM STELLE TO $f(R)$ + WEYL-SQUARED

The GUP modified gravity action is [38]:

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} [R - 2\gamma R^{\mu\nu} R_{\mu\nu} + \gamma R^2] , \quad (3)$$

where $\kappa = 8\pi G$, γ is defined in Eq. (2). The above action is identical to Stelle gravity [39]. Interestingly, the masses of the two additional Yukawa Bosons *coincide* ($m_{\text{Yukawa}} = 1/\sqrt{2\gamma}$) [38]. The new quadratic terms in the action can be written as the following linear combination

$$-2\gamma R^{\mu\nu} R_{\mu\nu} + \gamma R^2 = -\gamma [-G^2 + C^2 - R^2/3] , \quad (4)$$

where $C^2 = C_{\alpha\beta\rho\sigma} C^{\alpha\beta\rho\sigma} \equiv \mathcal{L}_{\text{Weyl}}$ is the Weyl tensor¹. squared, G^2 is the Gauss-Bonnet invariant defined as [47]:

$$G^2 \equiv R^2 - 4R^{\mu\nu} R_{\mu\nu} + R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} , \quad (5)$$

and R^2 is the Ricci scalar squared. Since the Gauss-Bonnet invariant is a boundary term in (3 + 1)–dimensions, it does not contribute to the dynamics [48]. The remaining parts of the action (3) is:

$$S_f = \frac{1}{2\kappa} \int d^4x \sqrt{-g} [f(R) - \gamma \mathcal{L}_{\text{Weyl}}] , \quad (6)$$

where, $f(R) = R + \gamma R^2/3$. (7)

Using the fact that $\mathcal{L}_{\text{Weyl}}$ is invariant under conformal transformations, we do a series of transformations [49]. First, we define the inflaton field and its potential as

$$\phi \equiv \frac{df}{dR} , \quad V(\phi) \equiv \frac{1}{2\kappa} \left(R \frac{df}{dR} - f \right) , \quad (8)$$

Next, we perform the following transformation [49]:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} (df/dR) = \phi g_{\mu\nu} , \quad (9)$$

under which the potential transforms as

$$U \equiv \frac{1}{2\kappa\phi^2} \left(R \frac{df}{dR} - f \right) = \frac{1}{2\kappa} \frac{V}{\phi^2} , \quad (10)$$

¹ We use the following definition of the Weyl tensor in 4-D: $C_{\alpha\beta\rho\sigma} = R_{\alpha\beta\rho\sigma} - (g_{\alpha[\rho} R_{\sigma]\beta} - g_{\beta[\rho} R_{\sigma]\alpha}) + \frac{1}{3} R g_{\alpha[\rho} g_{\sigma]\beta}$ [45, 46].

where $f(R)$ is given in Eq. (7). Lastly, we redefine the scalar field ϕ as:

$$\chi \equiv \sqrt{\frac{1}{3\kappa}} \ln[\phi], \quad (11)$$

for which the potential $U(\chi)$ takes the form

$$U(\chi) = \frac{3}{4} \frac{M_{\text{Planck}}^2}{8\gamma} \left(1 - e^{\sqrt{2/3}\chi/M_{\text{Planck}}}\right)^2. \quad (12)$$

Thus, the action (3) is transformed to:

$$\tilde{S} = \int d^4x \sqrt{-\tilde{g}} \left[\frac{\tilde{R}}{2\kappa} - \frac{\tilde{g}^{\mu\nu}}{2} \partial_\mu \chi \partial_\nu \chi - U(\chi) - \frac{\gamma}{2\kappa} \mathcal{L}_{\text{Weyl}} \right], \quad (13)$$

in which the scalar field has been separated from the curvature term. The equations of motion reduce to

$$G_{\mu\nu} - 2\gamma B_{\mu\nu} = \kappa^2 T_{\mu\nu}^\chi, \quad (14)$$

where $G_{\mu\nu}$ is the Einstein tensor, Bach tensor ($B_{\mu\nu}$) is

$$B_{\mu\nu} \equiv \nabla^\rho \nabla^\sigma C_{\mu\rho\nu\sigma} - \frac{1}{2} R^{\rho\sigma} C_{\mu\rho\nu\sigma}, \quad (15)$$

and $T_{\mu\nu}^\chi$ is the stress-energy tensor of the χ field which has the same form as a scalar field stress-energy tensor

$$T_{\mu\nu}^\chi \equiv \partial_\mu \chi \partial_\nu \chi - \delta_{\mu\nu} \left(\frac{1}{2} \tilde{g}^{\rho\sigma} \partial_\rho \chi \partial_\sigma \chi + U(\chi) \right). \quad (16)$$

From Eq. (14) we see that the scalar field (χ) and the geometrical part can be quantized independently.

As mentioned earlier, in Ref. [38], we analytically showed that Stelle gravity, when applied to a homogeneous, isotropic background, leads to inflation with exit. In this work, to connect to observables, we consider the following perturbed FRW line-element [8]:

$$ds^2 = a^2(\eta) \left[-(1 + 2\varphi)d\eta^2 + 2\partial_i B dx^i d\eta + [(1 + 2\psi)\delta_{ij} + 2\partial_i \partial_j E + h_{ij}] dx^i dx^j \right], \quad (17)$$

where the functions φ , B , ψ and E represent the scalar sector whereas the tensor h_{ij} , satisfying $h_i^i = \partial^i h_{ij} = 0$, represent gravitational waves. $a(\eta)$ is the expansion factor in conformal time η . Note that all these first-order perturbations are functions of (η, \mathbf{x}) . For

convenience, we do not write the dependence explicitly. Interestingly, the GUP modified action (3), like Einstein gravity, leads to scalar and tensor fluctuations [50]. In other words, GUP modified action does not contain any growing vector fluctuations, or additional scalar and tensor fluctuations.

Substituting the above line-element (17) in Eq. (14), scalar perturbation equation (in Fourier 3-space) is [51]:

$$\gamma \left[W^{(4)} - \frac{\ddot{H}}{\dot{H}} W^{(3)} \right] + C_2 \ddot{W} + C_1 \dot{W} + C_0 W = 0, \quad (18)$$

where $W = \psi - \varphi$, H is the background Hubble parameter, and $\{C_0, C_1, C_2\}$ are coefficients determined by the wavenumber of the scalar perturbations k , the QG scale γ , the scale factor a , the Hubble parameter H , and their derivatives. The EoMs of the tensor perturbations are given by:

$$\square \bar{h}_{ij} - 2\mathcal{H} \bar{h}'_{ij} - \frac{\gamma}{a^2} \square^2 \bar{h}_{ij} = 0. \quad (19)$$

In the rest of this work, we use two distinct methods to connect to observables. First, we use the Fakeon procedure [50] to constraint the GUP parameter from the CMB observations [5]. Later, using the constraint on γ_0 from CMB observations, we derive the power spectrum of PGWs [51], and compare with the design sensitivity of Einstein telescope [19] and Cosmic explorer [22, 23].

III. OBSERVABLES

Recently, imposing constraints of locality, unitarity, and renormalizability for quantum gravity, and using the fakeon procedure [52], it was shown that quantum gravity could contain only two more independent parameters than Einstein gravity. This was identified as the inflaton χ and the spin-2 fakeon $\chi_{\mu\nu}$ [50]. Interestingly, the extra degrees of freedom are turned into fake ones and projected away. Note that the Stelle action and the $f(R) + C^2$ action are renormalizable, although they contain Fadeev-Popov (non-malicious) ghosts.

Repeating the analysis [50, 53], yields the following expressions for the power spectra of the scalar and tensor perturbations, respectively:

$$\mathcal{P}(k)_{\mathcal{R}} = A_{\mathcal{R}} \left(\frac{k}{k^*} \right)^{n_{\mathcal{R}}-1}; \mathcal{P}(k)_T = A_T \left(\frac{k}{k^*} \right)^{n_T}, \quad (20)$$

where k_* is the wave-number of the fluctuations at the horizon crossing. $P(k)_{\mathcal{R}}$, $A_{\mathcal{R}}$ and $n_{\mathcal{R}}$ are the power spectrum, the amplitude, and the spectral tilt of the scalar perturbations respectively. Analogously, $P(k)_T$, A_T , and n_T are the power spectrum, the amplitude, and the spectral tilt of the tensor perturbations. We will obtain these quantities during slow-roll inflation [8].

During the slow-roll inflation, the number of e-folds of inflation N can be rewritten in terms of the slow-roll parameter (ε):

$$N = \int_{\varepsilon}^1 \frac{H(t(\varepsilon))}{\dot{\varepsilon}(t(\varepsilon))} d\varepsilon \simeq \frac{1}{2\varepsilon} - \frac{1}{12} \ln \varepsilon + \mathcal{O}(\varepsilon^0). \quad (21)$$

Thus, in the leading order, ε and N are inversely proportional, i. e., $\varepsilon \approx 1/2N$. Up to the leading order, the amplitudes and the spectral tilts are given by

$$n_{\mathcal{R}} - 1 = -4\sqrt{\frac{\varepsilon}{3}}, \quad A_{\mathcal{R}} = \frac{Gm_{\phi}^2}{12\pi\varepsilon^2} \left(1 - \sqrt{3\varepsilon} - (n_{\mathcal{R}} - 1)(2 - \gamma_E - \ln 2) \right), \quad (22a)$$

$$A_T = \frac{8Gm_{\chi}^2 m_{\phi}^2}{\pi(m_{\phi}^2 + 2m_{\chi}^2)} \left(1 - \frac{2\sqrt{3\varepsilon}m_{\chi}^2}{m_{\phi}^2 + 2m_{\chi}^2} - \frac{\varepsilon m_{\chi}^2(2m_{\chi}^2 + 37m_{\phi}^2)}{6(m_{\phi}^2 + 2m_{\chi}^2)^2} - n_T(2 - \gamma_E - \ln 2) \right) \quad (22b)$$

$$n_T = -\frac{4\varepsilon m_{\chi}^2}{m_{\phi}^2 + 2m_{\chi}^2}, \quad r = \frac{A_T}{A_{\mathcal{R}}} = \frac{32\varepsilon m_{\chi}^2}{m_{\phi}^2 + 2m_{\chi}^2} \left(1 + \frac{\sqrt{3\varepsilon}m_{\phi}^2}{m_{\phi}^2 + 2m_{\chi}^2} + (n_{\mathcal{R}} - 1 - n_T)(2 - \gamma_E - \ln 2) \right) \quad (22c)$$

where, $m_{\phi}(= m_{\chi}) = 1/(2\sqrt{\gamma})$ correspond to the mass of the spin-2 (spin-0) mode, and γ_E is the Euler-Mascheroni constant. Since Bach tensor is conformally invariant, it vanishes in the conformal flat FRW geometry.

The variables describing the power spectra of the scalar and tensor excitations $P(k)_{\mathcal{R}}$ and $P(k)_T$ can be written to the leading order as

$$A_{\mathcal{R}} = \frac{m_{\phi}^2 N^2}{3\pi M_{\text{Pl}}^2} = \frac{N^2}{18\pi\gamma_0}; \quad A_T = \frac{8m_{\chi}^2 m_{\phi}^2}{\pi(m_{\phi}^2 + 2m_{\chi}^2)M_{\text{Pl}}^2} = \frac{1}{\pi\gamma_0} \quad (23)$$

This is the first key result regarding which we would like to discuss the following points: First, the spectral tilt $n_{\mathcal{R}}$ and n_T , and the scalar-to-tensor ratio r , depend on γ_0 . However, it does not contain additional deviations from the Bach tensor. Second, from the above expressions on $A_{\mathcal{R}}$ and A_T , we can derive bounds on the magnitude of γ_0 . We obtain this by comparing Eq. (23) with PLANCK observations [5, 41, 54]. Additionally, we compare the QG modified observables with Starobinsky model [8]. This is because the Starobinsky model perfectly agrees with the Planck observations as it predicts a low value of the scalar-to-tensor ratio and mass-degenerate Stelle gravity leads to inflation with a natural exit and can be

mapped to Starobinsky gravity. Third, Fig. 1 contains the comparison of the spectral tilt of the scalar modes $n_{\mathcal{R}}$ and the scalar-tensor ratio r in the Starobinsky and $f(R) + C^2$ gravity model. While the spectral tilt of the scalar power spectrum is the same in both cases, the introduction of $R_{\mu\nu}R^{\mu\nu}$ decreases the ratio between the scalar contribution and the tensor modes. This is because Stelle gravity contains extra massive spin-2 modes and they carry more energy compared to the scalar modes. This is consistent with the recent analysis where it is shown that QG effects suppress the tensor modes [40]. Finally, the bounds on the GUP parameter γ_0 can be obtained from the PLANCK observations [5, 41, 54]:

$$A_{\mathcal{R}} = 2.474 \pm 0.116 \times 10^{-9}. \quad (24)$$

Comparing the theoretical results of Eq. (23) with the above expression leads to the following value for γ_0 for two different e-folds ($N = 40$ and $N = 60$) of inflation:

$$\gamma_0^{N=40} \approx 3.430 \times 10^{10}, \quad \gamma_0^{N=60} \approx 7.719 \times 10^{10}. \quad (25)$$

Thus, the values for the intermediate scale are consistent with the bounds obtained in Refs. [37, 38, 43, 44].

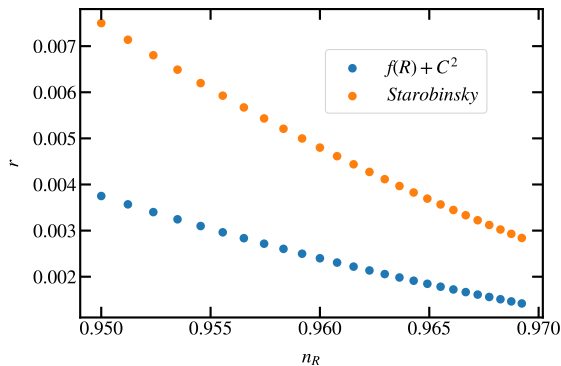


FIG. 1. The graph represents the spectral tilt of the scalar perturbations $n_{\mathcal{R}}$ and the scalar to tensor ratio $r = n_{\mathcal{R}}/n_{\mathcal{T}}$ from $N = 40$ to $N = 70$ number of e-folds. One can see the QG suppression of the tensor perturbations.

IV. POWER SPECTRUM OF PGWS

The next generation of ground-based GW detectors [19, 22, 23] and LISA [21] are expected to deliver data that will help to answer some of the deep questions in fundamental physics,

astrophysics, and cosmology [55]. Here, we show that the quantum gravitational signatures in PGWs can potentially be observed in the next-generation gravitational wave detectors.

Rewriting Eq. (19) as two second-order differential equations, in the Fourier domain, we have [51]:

$$\begin{aligned} \frac{d^2 \mu_k^{(1)}}{dz^2} + \left(1 - \frac{2}{z^2}\right) \mu_k^{(1)} &= 0, \\ \frac{d^2 \mu_k^{(2)}}{dz^2} + \left(1 + \frac{1}{\gamma H^2 z^2}\right) \mu_k^{(2)} &= 0, \end{aligned} \quad (26)$$

where $\mu^{(1)}$ ($\mu^{(2)}$) corresponds to the standard gravitational wave (massive spin-2 mode from the modified theory), and $z \equiv -k\eta$. For pure de Sitter, the power spectrum can be evaluated exactly [56] and is given by:

$$\mathcal{P}_{\text{PGW}}(k) = \mathcal{P}_0(\gamma_0, H) \left[1 + \left[\frac{k}{k_*}\right]^2 - \frac{\pi}{2} \left[\frac{k}{k_*}\right]^3 |e^{i\pi\nu/2} H_\nu^{(1)}\left[\frac{k}{k_*}\right]|^2 \right], \quad (27)$$

where $\mathcal{P}_0(\gamma_0, H) = (2\kappa H^2/\pi^2)(1 + 2\gamma H^2)$, $k_* = aH$ is the wavenumber at horizon crossing, $H_\nu^{(1)}$ is the Henkel function of the first kind, H is the Hubble scale during inflation and $\nu = \frac{1}{4}\sqrt{1 - \frac{4}{\gamma H^2}}$. The power spectrum given in Eq. (27) is valid for $\gamma_0 > 0$. For $\gamma_0 = 0$, the unmodified power spectrum for the PGWs is:

$$\mathcal{P}_{\text{PGW}}(k) = \mathcal{P}_0(\gamma_0, H) \left(\frac{k}{k_*}\right)^{n_T}. \quad (28)$$

Evaluating the power spectrum at the horizon crossing (i.e., $\eta \rightarrow 0$), for three values of γ_0 — $\gamma_0 = 0$, $\gamma_0^{N=40}$ and $\gamma_0^{N=60}$ (25) — we obtain the following values for the normalization constant $\mathcal{P}_0(\gamma_0, H)$:

$$\mathcal{P}_0(\gamma_0 = 0, H = 10^{15} \text{ GeV}) = 1.329 \times 10^{-25} \quad (29a)$$

$$\mathcal{P}_0(\gamma_0, N=40, H = 10^{15} \text{ GeV}) = 4.841 \times 10^{-29} \quad (29b)$$

$$\mathcal{P}_0(\gamma_0, N=60, H = 10^{15} \text{ GeV}) = 2.152 \times 10^{-29}, \quad (29c)$$

Fig. 2 contains the plot of the power spectrum of PGWs (for three different values of γ_0) as a function of frequency. From the figure, we infer that the power spectrum is almost constant for a large range of frequencies and is consistent with standard inflationary scenario [57]. Also, the degenerate Stelle gravity model suppresses the amplitude of PGWs. This implies that the longer the inflation, the larger the suppression of the amplitude of the PGWs. This is because the massive spin-2 modes carry more energy than the scalar modes [40].

Further, to quantify the detectability of PGWs in the upcoming GW detectors, we evaluate the energy density of PGWs [58]:

$$\Omega_{\text{GW}}(k) = \mathcal{P}_{\text{PGW}}(k) (k^2/(12H_0^2)). \quad (30)$$

where we have set the scale factor at the present time to unity, and $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$. Figure 2 contains the plot of differential energy density is $d(\Omega_{\text{GW}}(k))/d\ln(k)$ as a function of f . From the figure, we infer that the differential energy density is 8-orders of magnitude larger in kHz compared to $1 Hz$ range for all three values of γ_0 . To confirm this, we now compare the projected characteristic strain for PGWs and the characteristic strain of the detectors. The characteristic strain of the PGWs is given by [59]:

$$h_{\text{PGW}} = \int \mathcal{P}_{\text{PGW}}(f) df. \quad (31)$$

where $\mathcal{P}_{\text{PGW}}(f)$ is given by Eq. (27). Since $\mathcal{P}_{\text{PGW}}(f)$ is approximately constant with frequency (cf. Figure 2), we approximate $\mathcal{P}_{\text{PGW}}(f) \approx \mathcal{P}_0(H, \gamma_0)$. Thus, we get:

$$h_{\text{PGW}} \approx f \mathcal{P}_0(H, \gamma_0). \quad (32)$$

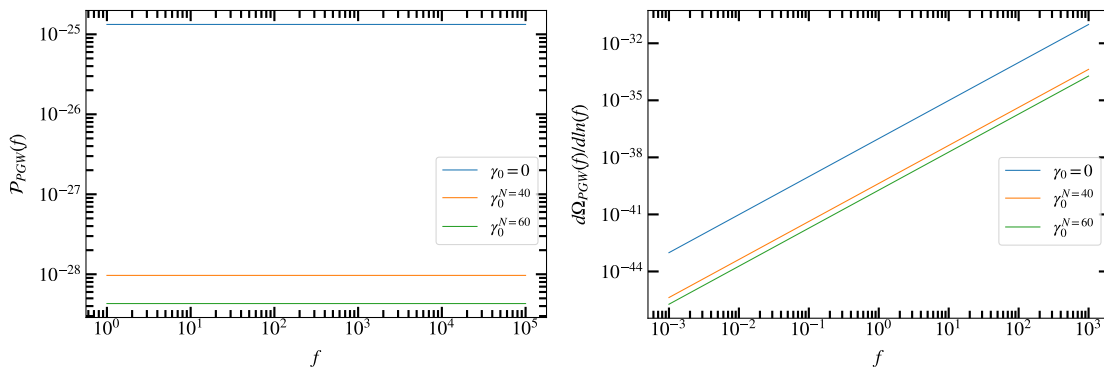


FIG. 2. The top graph shows a logarithmic plot of the power spectrum $\mathcal{P}_{\text{PGW}}(f)$ as a function of the frequency f and the GUP parameter γ_0 . The bottom graph shows the differential energy density of PGWs as a function of frequency.

Figure 3 contains the characteristic strain of PGWs for three different values of γ_0 (corresponding to different e-foldings of inflation) along with the design sensitivity of the Einstein Telescope [19] and the Cosmic explorer mission [22, 23]. From the figure, we infer the following: First, the characteristic strain of PGWs for the standard inflation (blue curve) is

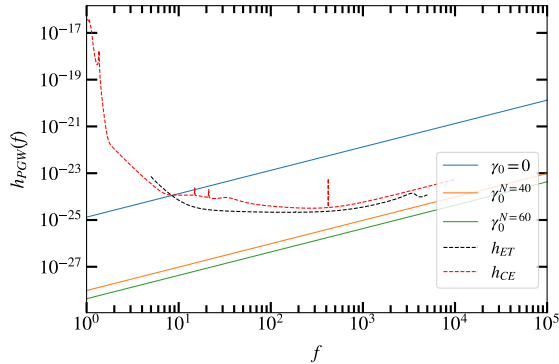


FIG. 3. Logarithmic graph of the characteristic strain of the gravitational waves produced during inflation h_{GW} as a function of frequency f and the GUP parameter γ_0 , compared to the characteristic strain of the Einstein Telescope h_{ET} [19] and the Cosmic explorer mission h_{CE} [22, 23].

well within the observational capability of both the Einstein telescope and Cosmic explorer. Second, for degenerate Stelle gravity, as the number of e-foldings increases, the suppression of the amplitude of the PGWs is larger (orange and green curves). Hence, it is not possible to confirm their detection with confidence. However, the non-detectability of the PGWs will directly constrain the value of γ_0 . Thus, the Einstein Telescope [19] and the Cosmic explorer mission [22, 23] can put a severe constraints on the value of the GUP parameter. Third, our analysis shows that LISA will not be able to detect PGWs as the characteristic strain is much larger than PGWs in low-frequency (~ 1 Hz). Lastly and most importantly, our analysis explicitly shows that we can strongly constrain the number of e-foldings of inflation from the Einstein telescope and cosmic explorer.

V. CONCLUSIONS

In conclusion, we demonstrated that our model is described by a linear combination of $f(R)$ gravity, particularly the Starobinsky model and Weyl-squared gravity. This allowed us to use the fakeon procedure [50] to obtain the scalar spectral tilt $n_{\mathcal{R}}$ and the tensor-scalar ratio r . Although our model and Starobinsky model are identical in the background FRW universe, they lead to different observable consequences in the linear order in perturbations. More specifically, we showed that the introduction of $R_{\mu\nu}R^{\mu\nu}$ decreases the ratio between the scalar contribution and the tensor modes. This is consistent with the recent analysis [40]. Using PLANCK observations, we determined the bounds on the GUP parameter γ_0 . These

bounds are consistent with bounds derived from other physical considerations [37, 38, 42–44].

Later, using the constraint on γ_0 from the CMB observations, we explicitly showed that PWGs carry quantum gravitational signatures in their power spectrum and energy density. Furthermore, from Figure 3 one can conclude that these quantum gravitational effects will potentially be observable in the next-generation ground-based gravitational wave detectors such as the Einstein Telescope and Cosmic Explorer. Additionally, depending on the energy density of detected PGWs, one can strongly constrain the number of e-foldings of inflation using these detectors.

After the historical detection of GWs in the frequency range $100 - 10^4$ Hz, there is a surge in activity for the possibility of detection of GWs in the MHz-GHz frequency range [60]. As mentioned earlier, for a large frequency range, \mathcal{P}_{PGW} is approximately constant. This means that high-frequency GW detectors with sensitivity around 10^{-20} can potentially detect PGWs. Such detectors will reveal a disparate view of the early universe compared to their electromagnetic counterparts.

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