Gravity based on internal symmetry of quantum fields

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The standard model of particle physics describes electromagnetic, weak and strong interactions, which are three of the four known fundamental forces of nature[1–3]. The fourth interaction, gravity, describes how the fields and matter curve the space-time[4,5]. The unification of gravity with the standard model has been one of the most challenging problems of modern physics due to incompatibilities of the underlying theories – general relativity and quantum field theory[1,6]. While quantum field theory utilizes symmetries associated with internal vector spaces of quantum fields[1–3], general relativity is based on external space-time symmetries[7–12]. Here we employ the internal special unitary symmetry of quantum fields in the revised formulation of quantum electrodynamics[13] to couple the electromagnetic and Dirac electron-positron fields to a tensor gauge field. The dynamical equations of the tensor gauge field are shown to describe gravity. General relativity is obtained in the limit of small coupling, while the general case of nonzero coupling enables beyond-general-relativity studies of strong gravitational fields encountered in black holes and at the possible beginning of time. Our work provides a viable way to a theory of all fundamental interactions within a single coherent mathematical framework – the theory of everything.

Quantum field theory is a theoretical framework, which synthesizes classical field theory, quantum mechanics, and special relativity. The standard model of particle physics arises on the basis of this framework through unitary symmetries related to invariances of a physical system[3,14]. The gauge invariance of electrodynamics, related to the Abelian phase rotation transformations, is the most trivial example of such a symmetry. The Yang–Mills theory extends the gauge theory to non-Abelian special unitary symmetries[3,14,15], which enable mutually interacting force carriers. It describes the behavior of other fundamental interactions of the standard model being at the core of the unification of electrodynamics to weak and strong interactions. A similar special-unitary-symmetry-based approach to the description of gravity has remained unknown. Therefore, alternative approaches, such as string theory[16,17] and loop quantum gravity[18–20], are being developed. Many authors have approached the problem by attempting to reformulate space-time symmetries in a way compatible with the Yang-Mills theory[21–31]. The difference between external space-time symmetries and internal vector space symmetries of quantum fields, however, represents a challenge for this gauge gravitation theory approach[12].

In this work, we employ an internal special unitary symmetry of quantum fields in the revised eight-spinor formulation of quantum electrodynamics[13]. We utilize the Lie algebra associated with this symmetry to couple the electromagnetic and Dirac electron-positron fields to a new tensor gauge field in a way that is analogous to the gauge couplings of the fields in the electromagnetic, weak, and strong interactions of the standard model. Once the new gauge field is introduced, the powerful machinery of the Yang-Mills theory leads to dynamical equations, which are generalizations of Einstein's field equations of general relativity. The ensuing Yang-Mills gauge theory of gravity is subject to quantization using the approach of the Yang-Mills theory[3,14]. The quanta of the gauge field, the gravitons, are spin-2 tensor bosons. These quanta are to be added in the spectrum of the known elementary particles extending the existing standard model to describe gravity. The detailed study of this quantization is left to future works.

Gravity couples to all fields and matter, regardless of whether they are massive or massless. Therefore, one cannot exclude any field or matter from the complete dynamical description of gravity. However, it is possible to limit our study to the coupling between gravity and electrodynamics since we consider the system of the electromagnetic field, Dirac electron-positron field, and the gravitational field to be such an elementary system that can provide all the insight needed for obtaining a unified description of gravity on equal footing with the other known fundamental forces of nature. The extension of the theory to the coupling of gravity to the other massive and massless fields can be approached once the present gravity-electrodynamics coupling problem is solved.

We start with Maxwell's theory of electrodynamics[32] and the revised eight-spinor formulation of quantum electrodynamics[13]. The eight-spinor formulation of quantum electrodynamics reveals a profound connection between the internal special unitary symmetry of quantum fields and the symmetric stress-energy tensor. Since the symmetric stress-energy tensor is the source of the gravitational field in general relativity, it becomes obvious that the gauge theory based on the special unitary symmetry should describe gravitational interaction. This is the basis for the development of the Yang-Mills gauge theory of gravity in the present work.

Generating Lagrangian density of gravity

The electromagnetic field is described by an eight-component spinor, given in terms of the conventional three-component real-valued electric and magnetic fields $\mathbf{E}_{\Re}$ and $\mathbf{B}_{\Re}$ by $\Psi_{\Re}=\sqrt{\varepsilon_0/2}\,[0,\mathbf{E}_{\Re},0,ic\mathbf{B}_{\Re}]^T$. Here the superscript T denotes the transpose, ε_0 is the vacuum permittivity, and c is the speed of light in vacuum. The four-component Dirac spinor field ψ is used in its conventional form. The Dirac gamma matrices $\boldsymbol{\gamma}_{\mathrm{F}}^{\mu}$ and the electromagnetic-gauge-covariant derivatives $\vec{D}_{\mu}$, where the index $\mu\in\{0,x,y,z\}$ ranges over the four Minkowski space-time dimensions, form eight-component spinors $\boldsymbol{\gamma}_{\mathrm{F}}$ and $\vec{D}$, see Methods.

The eight-spinor theory is formulated in terms of four 8×8 bosonic gamma matrices $\boldsymbol{\gamma}_{\mathrm{B}}^{\mu}$ and $\boldsymbol{\gamma}_{\mathrm{B}}^{5}=i\boldsymbol{\gamma}_{\mathrm{B}}^{0}\boldsymbol{\gamma}_{\mathrm{B}}^{x}\boldsymbol{\gamma}_{\mathrm{B}}^{y}\boldsymbol{\gamma}_{\mathrm{B}}^{z}$. These matrices are explicitly presented in Ref.[13]. They satisfy the Dirac algebra, i.e., the Clifford algebra $\mathcal{C}\ell_{1,3}(\mathbb{C})$. The defining property of the Dirac algebra of $\boldsymbol{\gamma}_{\mathrm{B}}^{\mu}$ is the anticommutation relation $\{\boldsymbol{\gamma}_{\mathrm{B}}^{\mu},\boldsymbol{\gamma}_{\mathrm{B}}^{\nu}\}=2\eta^{\mu\nu}\mathbf{I}_8$, where $\mathbf{I}_8$ is the 8×8 identity matrix and $\eta^{\mu\nu}$ is the Minkowski metric tensor with $\eta^{00}=1$ and $\eta^{xx}=\eta^{yy}=\eta^{zz}=-1$. Using $\boldsymbol{\gamma}_{\mathrm{B}}^{\mu}$ and $\boldsymbol{\gamma}_{\mathrm{B}}^{5}$, the generating Lagrangian density of gravity for the electromagnetic and Dirac fields is given by[13]

$$\begin{aligned}\mathcal{L}_0 &= \frac{\hbar c}{4g_{\mathrm{g}}}\bar{\psi}(\overleftarrow{\bar{D}}\bar{\mathbf{I}}_8\boldsymbol{\gamma}_{\mathrm{B}}^{5}\boldsymbol{\gamma}_{\mathrm{B}}^{\nu}\vec{\partial}_{\nu}\mathbf{I}_8\boldsymbol{\gamma}_{\mathrm{F}}-\bar{\boldsymbol{\gamma}}_{\mathrm{F}}\bar{\mathbf{I}}_8\boldsymbol{\gamma}_{\mathrm{B}}^{5}\boldsymbol{\gamma}_{\mathrm{B}}^{\nu}\vec{\partial}_{\nu}\mathbf{I}_8\vec{D})\psi\\ &\quad+\frac{i}{g_{\mathrm{g}}}\bar{\Psi}_{\Re}\mathbf{I}_8^{\dagger}\boldsymbol{\gamma}_{\mathrm{B}}^{5}\boldsymbol{\gamma}_{\mathrm{B}}^{\nu}\vec{\partial}_{\nu}\bar{\mathbf{I}}_8^{\dagger}\Psi_{\Re}-m_{\mathrm{e}}c^2\bar{\psi}\psi+\bar{\Psi}_{\Re}\Psi_{\Re}.\end{aligned}\tag{1}$$

Here $\hbar$ is the reduced Planck constant and g_{g} is the gravitational coupling constant of the present theory, given in units of inverse distance. Through the first two terms of Eq. (1), the Dirac and electromagnetic fields will be coupled to the gravitational field as an outcome of the present theory. The third term is the well-known mass term of the Dirac field, where m_{e} is the electron rest mass. The fourth term is the eight-spinor representation of the conventional electromagnetic Lagrangian density. The partial derivatives $\vec{\partial}_{\nu}$ in Eq. (1) act on $\mathbf{I}_8$ and do not extend to the spinors $\Psi_{\Re}$ and ψ. See Methods for technical definitions of quantities.

Special unitary symmetry

We follow the conventional Yang-Mills theory to seek for global symmetries with respect to which the generating Lagrangian density of gravity in Eq. (1) is invariant. Then, we introduce gauge fields to make these symmetries local. The generating Lagrangian density trivially satisfies the U(1) symmetry of quantum electrodynamics. This symmetry is satisfied locally since the electromagnetic gauge field is included and the electromagnetic-gauge-covariant derivative $\vec{D}$ is used. We next apply the special unitary symmetry transformation, given by[13]

$$\mathbf{I}_8\to\mathbf{U}\mathbf{I}_8,\qquad\text{where }\mathbf{U}=e^{i\phi_{\mu}\mathbf{t}^{\mu}}.\tag{2}$$

Here ϕ_{μ} is a real-valued four-vector describing the symmetry transformation parameters, the symmetry transformation matrix $\mathbf{U}$ has determinant 1. The transformation generators $\mathbf{t}^{\mu}$ are constant traceless Hermitian matrices given in terms of the complex-conjugated gamma matrices as $\mathbf{t}^{\mu}=(\boldsymbol{\gamma}_{\mathrm{B}}^{0}\boldsymbol{\gamma}_{\mathrm{B}}^{5}\boldsymbol{\gamma}_{\mathrm{B}}^{\mu})^*$. This set of matrices generates the Clifford algebra $\mathcal{C}\ell_{4,0}(\mathbb{C})$ with the anticommutation relation $\{\mathbf{t}^{\mu},\mathbf{t}^{\nu}\}=2\delta^{\mu\nu}\mathbf{I}_8$, where $\delta^{\mu\nu}$ is the Kronecker delta. The commutation relation is given by $[\mathbf{t}^{\mu},\mathbf{t}^{\nu}]=if_{\rho}{}^{\mu\nu}\mathbf{t}^{\rho}$, where $f_{\rho}{}^{\mu\nu}=2\varepsilon^{0\rho\mu\nu}$ are real-valued constants in which $\varepsilon^{\alpha\beta\gamma\delta}$ is the Levi-Civita symbol. The traces satisfy $\mathrm{Tr}(\mathbf{t}^{\mu}\mathbf{t}^{\nu})=8\delta^{\mu\nu}$. The matrices $\mathbf{t}^{\mu}$ are also generators of a Lie algebra, where the commutator has the role of the Lie bracket and $f_{\rho}{}^{\mu\nu}$ are the totally antisymmetric structure constants. This fact allows us to use the powerful machinery of the Yang-Mills gauge theory[3]. All that follows is, thus, a direct consequence of the gauge invariance with respect to the symmetry transformation in Eq. (2) following the conventional approach of quantum field theory.

While the standard model of particles physics is based on the unitary group U(1) and the special unitary groups SU(2) and SU(3), the symmetry group corresponding to the transformation in Eq. (2) is a four-dimensional subgroup of SU(8), which we denote SU(8)$_{\mathrm{4D}}$. The transformation in Eq. (2) commutes with $\boldsymbol{\gamma}_{\mathrm{B}}^{5}$ and pairs of gamma matrices as $[\mathbf{U},\boldsymbol{\gamma}_{\mathrm{B}}^{5}]=\mathbf{0}$ and $[\mathbf{U},\boldsymbol{\gamma}_{\mathrm{B}}^{\rho}\boldsymbol{\gamma}_{\mathrm{B}}^{\sigma}]=\mathbf{0}$. These relations are needed for the invariance of the Lagrangian density in the gauge theory that follows. Furthermore, $\mathbf{U}$ commutes with the tensor-field Lorentz transformation $\boldsymbol{\Lambda}_{\mathrm{J}}=e^{\frac{1}{8}\Omega_{\rho\sigma}[\boldsymbol{\gamma}_{\mathrm{B}}^{\rho},\boldsymbol{\gamma}_{\mathrm{B}}^{\sigma}]}$ as $[\mathbf{U},\boldsymbol{\Lambda}_{\mathrm{J}}]=\mathbf{0}$, where $\Omega_{\rho\sigma}$ is a matrix parametrizing the Lorentz transformation[13]. A more detailed study of the symmetry properties of $\mathbf{U}$ is left as a topic of further work.

Gauge-covariant derivative

The symmetry transformation in Eq. (2) is global for constant values of ϕ_{ν}. To promote this global symmetry to a local symmetry, we allow $\phi_{\nu}(t,x,y,z)$ to be space-time dependent. Since the symmetry transformation matrices $\mathbf{U}$ are noncommuting for different values of ϕ_{ν}, the symmetry transformation represents a noncommuting symmetry, and our field theory is termed a non-Abelian gauge theory. A prototype example of a non-Abelian gauge theory is the original theory of Yang and Mills considering the proton-neutron doublet transformed under isotopic spin[3,14,15,33]. Following the standard approach of the Yang-Mills theory[3,14], it follows that the generating Lagrangian density of gravity in Eq. (1) can be made locally invariant in the symmetry transformation in Eq. (2) when we introduce the gauge-covariant derivatives $\vec{\boldsymbol{\mathcal{D}}}_{\nu}\mathbf{I}_8$ and $\bar{\vec{\boldsymbol{\mathcal{D}}}}_{\nu}^{\dagger}\bar{\mathbf{I}}_8^{\dagger}$ as

$$\begin{aligned}\vec{\boldsymbol{\mathcal{D}}}_{\nu}&=\mathbf{I}_8\vec{\partial}_{\nu}-ig_{\mathrm{g}}\mathbf{h}_{\nu},\\ \bar{\vec{\boldsymbol{\mathcal{D}}}}_{\nu}^{\dagger}&=\mathbf{I}_8\vec{\partial}_{\nu}-ig_{\mathrm{g}}\bar{\mathbf{h}}_{\nu},\end{aligned}\qquad\text{where }\mathbf{h}_{\nu}=h_{\mu\nu}\mathbf{t}^{\mu}.\tag{3}$$

Here the Hermitian gauge field $\mathbf{h}_\nu$ is given in terms of $\mathbf{t}^\mu$ and unitless real-valued components $h_{\mu\nu}$. The gauge field $\mathbf{h}_\nu$ and its components $h_{\mu\nu}$ are invariant in the Lorentz transformation as $\mathbf{\Lambda}_\mathrm{J}\mathbf{h}_\nu\mathbf{\Lambda}_\mathrm{J}^{-1} = \mathbf{h}_\nu$. The Lorentz invariance is the defining property for the components of the metric tensor[3]. That the gauge field introduced is indeed linked to the metric tensor becomes obvious from the field equations derived below. The gauge-covariant derivatives transform by matrices $\mathbf{U}$ and $\bar{\mathbf{U}}^\dagger$ as $\vec{\boldsymbol{\mathcal{D}}}_\nu\mathbf{I}_8 \rightarrow \mathbf{U}\vec{\boldsymbol{\mathcal{D}}}_\nu\mathbf{I}_8$ and $\bar{\vec{\boldsymbol{\mathcal{D}}}}_\nu^\dagger\bar{\mathbf{I}}_8^\dagger \rightarrow \bar{\mathbf{U}}^\dagger\bar{\vec{\boldsymbol{\mathcal{D}}}}_\nu^\dagger\bar{\mathbf{I}}_8^\dagger$. These relations require that the transformation of $\mathbf{h}_\nu$ is given by $\mathbf{h}_\nu \rightarrow (\mathbf{U}\mathbf{h}_\nu - \frac{i}{g_\mathrm{g}}\partial_\nu\mathbf{U})\mathbf{U}^\dagger$. Using the gauge-covariant derivative operators $\vec{\boldsymbol{\mathcal{D}}}_\nu$ and $\bar{\vec{\boldsymbol{\mathcal{D}}}}_\nu^\dagger$ in place of the partial derivatives $\vec{\partial}_\nu$ makes the generating Lagrangian density of gravity in Eq. (1) invariant with respect to the local form of the symmetry transformation in Eq. (2).

Gauge-invariant Lagrangian density

To write the complete gauge-invariant Lagrangian density, we must also include a gauge-invariant term that depends only on the gauge field $h_{\mu\nu}$. Utilizing the Yang-Mills gauge theory[3], this can be obtained from the commutator of the gauge-covariant derivatives[3,14]. The relation $[\vec{\boldsymbol{\mathcal{D}}}_\mu, \vec{\boldsymbol{\mathcal{D}}}_\nu] = -ig_\mathrm{g}\mathbf{H}_{\mu\nu}$ is used to define an antisymmetric field strength tensor $\mathbf{H}_{\mu\nu}$ as

$$\begin{aligned}\mathbf{H}_{\mu\nu} &= \partial_\mu\mathbf{h}_\nu - \partial_\nu\mathbf{h}_\mu - ig_\mathrm{g}[\mathbf{h}_\mu, \mathbf{h}_\nu] = H_{\rho\mu\nu}\mathbf{t}^\rho,\\ H_{\rho\mu\nu} &= \partial_\mu h_{\rho\nu} - \partial_\nu h_{\rho\mu} + g_\mathrm{g}f_\rho{}^{\sigma\lambda}h_{\sigma\mu}h_{\lambda\nu}.\end{aligned} \tag{4}$$

The commutator term of this field strength tensor is one of the prime novelties of the Yang-Mills theory since it leads to direct interaction between the gauge field quanta[3]. The gauge symmetry transformation law for the field strength tensor $\mathbf{H}_{\mu\nu}$ follows from the relations above, and it is given by $\mathbf{H}_{\mu\nu} \rightarrow \mathbf{U}\mathbf{H}_{\mu\nu}\mathbf{U}^\dagger$. Following the Yang-Mills theory procedure[3], we obtain the gauge-invariant Lagrangian density term for the gauge field $h_{\mu\nu}$, given by $\mathcal{L}_\mathrm{g} = -\frac{1}{64\kappa}\mathrm{Tr}(\mathbf{H}_{\mu\nu}\mathbf{H}^{\mu\nu}) = -\frac{1}{8\kappa}H_{\rho\mu\nu}H^{\rho\mu\nu}$. Here $\kappa = 8\pi G/c^4$ is Einstein's constant, where G is the gravitational constant. The prefactor of $\mathcal{L}_\mathrm{g}$ has been determined by comparison of the resulting theory to general relativity as discussed below. The complete gauge-invariant generalization of the generating Lagrangian density of gravity in Eq. (1) is then given by

$$\begin{aligned}\mathcal{L} = {} & \frac{\hbar c}{4g_\mathrm{g}}\bar{\psi}(\bar{\overleftarrow{D}}\bar{\mathbf{I}}_8\boldsymbol{\gamma}_\mathrm{B}^5\boldsymbol{\gamma}_\mathrm{B}^\nu\vec{\boldsymbol{\mathcal{D}}}_\nu\mathbf{I}_8\boldsymbol{\gamma}_\mathrm{F} - \bar{\boldsymbol{\gamma}}_\mathrm{F}\bar{\mathbf{I}}_8\boldsymbol{\gamma}_\mathrm{B}^5\boldsymbol{\gamma}_\mathrm{B}^\nu\vec{\boldsymbol{\mathcal{D}}}_\nu\mathbf{I}_8\vec{D})\psi\\ & + \frac{i}{g_\mathrm{g}}\bar{\Psi}_\Re\mathbf{I}_8^\dagger\boldsymbol{\gamma}_\mathrm{B}^5\boldsymbol{\gamma}_\mathrm{B}^\nu\bar{\vec{\boldsymbol{\mathcal{D}}}}_\nu^\dagger\bar{\mathbf{I}}_8^\dagger\Psi_\Re - m_\mathrm{e}c^2\bar{\psi}\psi + \bar{\Psi}_\Re\Psi_\Re\\ & - \frac{1}{64\kappa}\mathrm{Tr}(\mathbf{H}_{\mu\nu}\mathbf{H}^{\mu\nu}).\end{aligned} \tag{5}$$

As shown in Methods, the Lagrangian density in Eq. (5) reduces to the known Lagrangian density of quantum electrodynamics in the Minkowski metric limit $h_{\mu\nu} \rightarrow \eta_{\mu\nu}$. In the general case, the gauge field $h_{\mu\nu}$ is obtained as a solution of the field equations derived below.

Dynamical equations

Through the well-known Euler-Lagrange equations, one can derive the dynamical equations for all fields appearing in the Lagrangian density in Eq. (5). The resulting generalized Maxwell's and Dirac's equations for the electromagnetic and Dirac electron-positron fields are presented in Methods. The Euler-Lagrange equations for $h_{\mu\nu}$ are given by $\partial\mathcal{L}/\partial h_{\mu\nu} - \partial_\rho[\partial\mathcal{L}/\partial(\partial_\rho h_{\mu\nu})] = 0$. After some algebra, we then obtain the dynamical equation

$$\partial_\rho H^{\mu\rho\nu} + g_\mathrm{g}f_\rho{}^{\mu\lambda}h_{\lambda\sigma}H^{\rho\sigma\nu} = 2\kappa\bar{T}^{\mu\nu}. \tag{6}$$

The source term on the right of Eq. (6) is given by the so-called trace-reversed stress-energy tensor $\bar{T}^{\mu\nu} = T^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}T$, where $T^{\mu\nu}$ is the total symmetric stress-energy tensor of the electromagnetic field and the Dirac field[13]. Tensor contractions here and below are briefly denoted by leaving out the indices as $T = T^\rho{}_\rho$. In terms of the electromagnetic and Dirac field spinors, we obtain

$$\begin{aligned}\bar{T}^{\mu\nu} = {} & \frac{i\hbar c}{4}\bar{\psi}(\bar{\overleftarrow{D}}\boldsymbol{\gamma}_\mathrm{B}^5\boldsymbol{\gamma}_\mathrm{B}^\nu\mathbf{t}^\mu\boldsymbol{\gamma}_\mathrm{F} - \bar{\boldsymbol{\gamma}}_\mathrm{F}\boldsymbol{\gamma}_\mathrm{B}^5\boldsymbol{\gamma}_\mathrm{B}^\nu\mathbf{t}^\mu\vec{D})\psi\\ & + \bar{\Psi}_\Re\mathbf{t}^\mu\boldsymbol{\gamma}_\mathrm{B}^\nu\boldsymbol{\gamma}_\mathrm{B}^5\Psi_\Re.\end{aligned} \tag{7}$$

As shown in Ref.[13], the two terms of $\bar{T}^{\mu\nu}$ agree, respectively, with the well-known trace-reversed stress-energy tensors of the Dirac and electromagnetic fields[3].

Defining $R^{\mu\nu} = \partial_\rho H^{\mu\rho\nu} + g_\mathrm{g}f_\rho{}^{\mu\lambda}h_{\lambda\sigma}H^{\rho\sigma\nu}$ and lowering the tensor indices, Eq. (6) can be written compactly as $R_{\mu\nu} = \kappa\bar{T}_{\mu\nu}$. Furthermore, we can write this equation in a form, where the source term is $T_{\mu\nu}$ instead of its trace-reversed form. Contracting the tensor indices, we obtain $R = \kappa\bar{T} = -\kappa T$. Multiplying this relation by $-\frac{1}{2}\eta_{\mu\nu}$ and adding to $R_{\mu\nu} = \kappa\bar{T}_{\mu\nu}$ side by side, we obtain

$$R_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}R = \kappa T_{\mu\nu}. \tag{8}$$

Equations (6) and (8) indicate that the stress-energy tensor acts as the source term in the dynamics of the gauge field $h_{\mu\nu}$. Furthermore Eq. (8) reminds Einstein's field equations of general relativity in the absence of the cosmological constant. The difference is that no dynamical metric tensor has been defined for the present theory yet, whence the Minkowski metric tensor appears in Eq. (8). This observation is related to the fact that, in general relativity, one can locally regard space-time as the Minkowski space-time. The local Minkowski space-time corresponds to a tangent space of the four-dimensional space-time manifold. Only in the global consideration, it becomes clear that the space-time is curved by the presence of matter. Due to the similarity with Einstein's field equations, we propose that the gauge field $h_{\mu\nu}$ describes the gravitational field.

Discussion

To elaborate the physics of the field equations (6) and (8), we consider the limit of vanishing coupling constant $g_{\mathrm{g}} \to 0$. Then, Eq. (6) becomes $\partial^\rho\partial_\rho h_{\mu\nu} - \partial^\rho\partial_\nu h_{\mu\rho} = 2\kappa\bar{T}_{\mu\nu}$. It is convenient to use the harmonic gauge[34], in which we set $\partial^\rho h_{\mu\rho} = 0$. Then, taking the trace reverse and denoting $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$, we obtain a tensor-field wave equation, given by

$$\partial^\rho\partial_\rho\bar{h}_{\mu\nu} = 2\kappa T_{\mu\nu}. \tag{9}$$

In the case of the stress-energy tensor being zero, this equation describes gravitational waves propagating through vacuum at the speed of light[35]. Equation (9) is equivalent in form to the well-known equation of gravitational waves[4,34].

Next, we consider the classical Newtonian limit. The trace-reversed gauge field in Eq. (9) is approximated by the deviation from the Minkowski metric as $\bar{h}_{\mu\nu} = \bar{\eta}_{\mu\nu} + \Delta\bar{h}_{\mu\nu}$, where $\Delta\bar{h}_{\mu\nu}$ represents the deviation. The stress-energy tensor is assumed to have only a single nonzero time-independent component $T_{00} = \rho c^2$, where ρ is the mass density. Consequently, $\Delta\bar{h}_{\mu\nu}$ has only a single nonzero component $\Delta\bar{h}_{00}$, for which using Eq. (9) gives $\Delta\bar{h}_{00} = -4\phi/c^2$, where ϕ is the classical gravitational potential. Thus, Eq. (9) is rewritten as $\nabla^2\phi = 4\pi G\rho$, which is the well-known Newtonian equation of gravity in the mass density form.

Another interesting consideration is the study of Eq. (8) in a curvilinear coordinate system with a metric tensor $g_{\mu\nu}$. The partial derivatives in Eqs. (4) and (6) are replaced by coordinate-covariant derivatives corresponding to the affine connection and the Minkowski metric tensor in Eqs. (8) is replaced by $g_{\mu\nu}$. Then, searching for vacuum solutions for which the right-hand side of Eqs. (8) is zero, we find that, in the limit $g_{\mathrm{g}} \to 0$, Eq. (8) has a nontrivial exact solution $h_{\mu\nu} = g_{\mu\nu}$. Therefore, any solutions of Einstein's field equations, such as the well-known Schwarzschild[36] and Kerr[37] metrics, are also solutions of the field equations of the present theory in the limit $g_{\mathrm{g}} \to 0$. Thus, in this limit, the present theory reproduces the experimentally verified predictions of Einstein's field equations, such as the precession of the perihelion of Mercury[4], the bending of light by the Sun[38], and the gravitational redshift of light[39]. In this limit, we also recover the principle of equivalence of general relativity. The eventual possibility to generalize the principle of equivalence to the present theory, in which g_{g} is nonzero, is a topic of further work.

Even though, in the limit $g_{\mathrm{g}} \to 0$, the predictions of the present theory agree with those of general relativity, the theories are in general fundamentally different. It is expected that, in the limit of strong gravitational field, the commutator term of the field strength tensor in Eq. (4) becomes substantial when the coupling constant g_{g} is nonzero, even if it is small. Such strong gravitational fields are encountered in black holes and at the possible beginning of time. Therefore, our theory may provide a tool for the investigation of intense gravitational fields beyond the applicability of general relativity.

It is obvious that Eq. (6) is subject to quantization using the methods of quantizing Yang-Mills theories[3,14,40,41]. The resulting quanta of the field should be elementary particles. Since the present gauge field is a tensor field, its quanta must be spin-2 tensor bosons[4]. Thus, the particle associated with the gauge field $h_{\mu\nu}$ must be the graviton, the force carrier for gravity. As characteristic for Yang-Mills theories, the gravitons are mutually interacting through the commutator term of the field strength tensor. After replacing the classical fields with pertinent quantum operators, Eqs. (6) and (8) can be viewed as a quantum field theoretical generalization of Einstein's field equations of general relativity. This quantization is a topic of further work.

Conclusion

The internal $\mathrm{SU}(8)_{\mathrm{4D}}$ symmetry of the eight-spinor formulation of quantum electrodynamics[13] leads to the Yang-Mills gauge theory of gravity. The theory is based on the generating Lagrangian density of gravity, which is related to the conservation of the stress-energy tensor, and which can be made locally invariant in the special unitary transformation by introducing a tensor gauge field. The weak and strong interactions and the associated particles and fields of the standard model are still to be added to the present theory. Due to the analogous forms of the underlying theories, this is assumed a technical exercise. Then, the entire dynamics of the known particles and fields, including gravity, can be described by a single master Lagrangian of the Universe through the Euler-Lagrange equations in a unified way. Therefore, our work opens a new era of quantum science of gravity. After rigorous quantization of the present theory using known methods to quantize Yang-Mills theories, physicists may finally have the long-sought tool for the investigation of intense gravitational fields in black holes and the possible beginning of time. The present theory includes a single undetermined parameter, which is the coupling constant g_{g}. The determination of the value of this parameter is of large interest.

Methods

Technical definitions of quantities

The eight-spinor representation of the conventional electromagnetic Lagrangian density term in Eqs. (1) and (5) can be alternatively written as $\bar{\Psi}_{\Re}\Psi_{\Re} = -\frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu}$, where $F_{\mu\nu} = \partial_\mu A_{\Re\nu} - \partial_\nu A_{\Re\mu}$ is the electromagnetic tensor and μ_0 is the permeability of vacuum[13]. The electric and magnetic fields are related to the electromagnetic four-potential $A_{\Re\mu} = (\phi_{\mathrm{e}\Re}/c, -\mathbf{A}_{\Re})$ by the conventional relations $\mathbf{E}_{\Re} = -\nabla\phi_{\mathrm{e}\Re} - \frac{\partial}{\partial t}\mathbf{A}_{\Re}$ and $\mathbf{B}_{\Re} = \nabla\times\mathbf{A}_{\Re}$, where $\phi_{\mathrm{e}\Re}$ and $\mathbf{A}_{\Re}$

are the scalar and vector potentials, respectively[32]. In the eight-spinor notation, these relation are written as a single equation $\Psi_{\Re} = -\boldsymbol{\gamma}_{\mathrm{B}}^{\nu}\partial_{\nu}\Theta_{\Re}$[13]. The Dirac and electromagnetic adjoint spinors are denoted by $\bar{\psi} = \psi^{\dagger}\boldsymbol{\gamma}_{\mathrm{F}}^{0}$ and $\bar{\Psi}_{\Re} = \Psi_{\Re}^{\dagger}\boldsymbol{\gamma}_{\mathrm{B}}^{0}$, where $\psi^{\dagger}$ and $\Psi_{\Re}^{\dagger}$ are Hermitian adjoints. For a generic matrix $\mathbf{M}$, the corresponding adjoint operation is defined as $\bar{\mathbf{M}} = \boldsymbol{\gamma}_{\mathrm{B}}^{0}\mathbf{M}^{\dagger}\boldsymbol{\gamma}_{\mathrm{B}}^{0}$. The vector arrows in equations indicate the direction in which the differential operators operate. If arrows do not exist, the operators operate to the right as conventional. The electromagnetic-gauge-covariant derivative spinor operator $\vec{D} = [0, \vec{D}_x, \vec{D}_y, \vec{D}_z, -\vec{D}_0, 0, 0, 0]^T$ and its adjoint $\overleftarrow{D} = [0, \overleftarrow{D}_x, \overleftarrow{D}_y, \overleftarrow{D}_z, \overleftarrow{D}_0, 0, 0, 0]$ are defined in terms of the conventional electromagnetic-gauge-covariant derivative operators $\vec{D}_{\mu} = \vec{\partial}_{\mu} + iq_{\mathrm{e}}A_{\Re\mu}/\hbar$ and $\overleftarrow{D}_{\mu} = \overleftarrow{\partial}_{\mu} - iq_{\mathrm{e}}A_{\Re\mu}/\hbar$, where $q_{\mathrm{e}} = \pm e$ is the electric charge in which e is the elementary charge. The quantity $\boldsymbol{\gamma}_{\mathrm{F}} = [\mathbf{0}, \boldsymbol{\gamma}_{\mathrm{F}}^{x}, \boldsymbol{\gamma}_{\mathrm{F}}^{y}, \boldsymbol{\gamma}_{\mathrm{F}}^{z}, \boldsymbol{\gamma}_{\mathrm{F}}^{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}]^T$ is a spinor made of the Dirac gamma matrices. The corresponding adjoint spinor is given by $\bar{\boldsymbol{\gamma}}_{\mathrm{F}} = [\mathbf{0}, \boldsymbol{\gamma}_{\mathrm{F}}^{x}, \boldsymbol{\gamma}_{\mathrm{F}}^{y}, \boldsymbol{\gamma}_{\mathrm{F}}^{z}, -\boldsymbol{\gamma}_{\mathrm{F}}^{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}]$[13].

Minkowski metric limit of the Lagrangian density

In the Minkowski metric limit, $h_{\mu\nu} \to \eta_{\mu\nu}$, we can write $\vec{\boldsymbol{\mathcal{D}}}_{\nu}\mathbf{I}_8 = -ig_{\mathrm{g}}\eta_{\mu\nu}\mathbf{t}^{\mu}$ and $\bar{\vec{\boldsymbol{\mathcal{D}}}}_{\nu}^{\dagger}\bar{\mathbf{I}}_8^{\dagger} = -ig_{\mathrm{g}}\eta_{\mu\nu}\bar{\mathbf{t}}^{\mu}$. Furthermore, using these relations, we obtain $\bar{\mathbf{I}}_8\boldsymbol{\gamma}_{\mathrm{B}}^{5}\boldsymbol{\gamma}_{\mathrm{B}}^{\nu}\vec{\boldsymbol{\mathcal{D}}}_{\nu}\mathbf{I}_8\boldsymbol{\gamma}_{\mathrm{F}} = -2ig_{\mathrm{g}}\boldsymbol{\gamma}_{\mathrm{F}}$, $\bar{\mathbf{I}}_8\boldsymbol{\gamma}_{\mathrm{B}}^{5}\boldsymbol{\gamma}_{\mathrm{B}}^{\nu}\vec{\boldsymbol{\mathcal{D}}}_{\nu}\mathbf{I}_8\vec{D} = -2ig_{\mathrm{g}}\vec{D}$, and $\mathbf{I}_8^{\dagger}\boldsymbol{\gamma}_{\mathrm{B}}^{5}\boldsymbol{\gamma}_{\mathrm{B}}^{\nu}\bar{\vec{\boldsymbol{\mathcal{D}}}}_{\nu}^{\dagger}\bar{\mathbf{I}}_8^{\dagger}\Psi_{\Re} = \mathbf{0}$. Substituting these relations into Eq. (5), we obtain

$$\mathcal{L}_{\mathrm{QED}} = \frac{i\hbar c}{2}\bar{\psi}(\bar{\boldsymbol{\gamma}}_{\mathrm{F}}\vec{D} - \overleftarrow{D}\boldsymbol{\gamma}_{\mathrm{F}})\psi - m_{\mathrm{e}}c^2\bar{\psi}\psi + \bar{\Psi}_{\Re}\Psi_{\Re}. \qquad (10)$$

As shown in detail in Ref.[13], this is the eight-spinor representation of the well-known Lagrangian density of quantum electrodynamics. Note that the generating Lagrangian density of gravity, $\mathcal{L}_0$ in Eq. (1), cannot be used to derive $\mathcal{L}_{\mathrm{QED}}$ in Eq. (10) without first introducing the gauge field $h_{\mu\nu}$ and then taking the Minkowski metric limit. This highlights the fundamental role of the gravitational gauge field for the entire structure of the space-time.

Generalized Maxwell's equations

The Euler-Lagrange equations for the electromagnetic potential spinor field $\bar{\Theta}_{\Re}$ are given by $\partial\mathcal{L}/\partial\bar{\Theta}_{\Re} - \partial_{\rho}[\partial\mathcal{L}/\partial(\partial_{\rho}\bar{\Theta}_{\Re})] = 0$. Using $\Psi_{\Re} = -\boldsymbol{\gamma}_{\mathrm{B}}^{\nu}\partial_{\nu}\Theta_{\Re}$ and $\Phi_{\Re} = q_{\mathrm{e}}(2\varepsilon_0)^{-1/2}\bar{\psi}(\boldsymbol{\gamma}_{\mathrm{F}})\psi$, we then obtain after some algebra

$$\boldsymbol{\gamma}_{\mathrm{B}}^{\rho}(\mathbf{I}_8 + h_{\mu\nu}\boldsymbol{\gamma}_{\mathrm{B}}^{5}\mathbf{t}^{\mu}\boldsymbol{\gamma}_{\mathrm{B}}^{\nu})\partial_{\rho}\Psi_{\Re} = -\frac{1}{2}h_{\mu\nu}\boldsymbol{\gamma}_{\mathrm{B}}^{5}\boldsymbol{\gamma}_{\mathrm{B}}^{\nu}\mathbf{t}^{\mu}\Phi_{\Re}. \qquad (11)$$

This equation is the generalization of all Maxwell's equations in the present Yang-Mills gauge theory of gravity. In the Minkowski metric limit $h_{\mu\nu} \to \eta_{\mu\nu}$, we have $\eta_{\mu\nu}\boldsymbol{\gamma}_{\mathrm{B}}^{\rho}\boldsymbol{\gamma}_{\mathrm{B}}^{5}\mathbf{t}^{\mu}\boldsymbol{\gamma}_{\mathrm{B}}^{\nu}\partial_{\rho}\Psi_{\Re} = \mathbf{0}$ and $\eta_{\mu\nu}\boldsymbol{\gamma}_{\mathrm{B}}^{5}\boldsymbol{\gamma}_{\mathrm{B}}^{\nu}\mathbf{t}^{\mu}\Phi_{\Re} = 2\Phi_{\Re}$. Thus, Eq. (11) becomes $\boldsymbol{\gamma}_{\mathrm{B}}^{\rho}\partial_{\rho}\Psi_{\Re} = -\Phi_{\Re}$. As shown in Ref.[13], this spinorial photon equation is equivalent to the conventional Maxwell's equations.

Generalized Dirac equation

The Euler-Lagrange equations for the Dirac field $\bar{\psi}$ are given by $\partial\mathcal{L}/\partial\bar{\psi} - \partial_{\rho}[\partial\mathcal{L}/\partial(\partial_{\rho}\bar{\psi})] = 0$. After some algebra, we obtain

$$\frac{i\hbar c}{2}h_{\mu\nu}\bar{\boldsymbol{\gamma}}_{\mathrm{F}}\boldsymbol{\gamma}_{\mathrm{B}}^{5}\boldsymbol{\gamma}_{\mathrm{B}}^{\nu}\mathbf{t}^{\mu}\vec{D}\psi - m_{\mathrm{e}}c^2\psi = 0. \qquad (12)$$

This equation is the generalization of the conventional Dirac equation of quantum electrodynamics. In the Minkowski metric limit $h_{\mu\nu} \to \eta_{\mu\nu}$, we have $\eta_{\mu\nu}\boldsymbol{\gamma}_{\mathrm{B}}^{5}\boldsymbol{\gamma}_{\mathrm{B}}^{\nu}\mathbf{t}^{\mu}\vec{D} = 2\vec{D}$. Thus, we obtain $i\hbar c\bar{\boldsymbol{\gamma}}_{\mathrm{F}}\vec{D}\psi - m_{\mathrm{e}}c^2\psi = 0$. This equation is equivalent to the conventional Dirac equation[13].

Conventional electrodynamic gauge theory

To highlight the complete analogy between the present Yang-Mills gauge theory of gravity and the traditional quantum field theories, we next briefly present the derivation of the conventional electrodynamic gauge theory. In analogy to $\mathcal{L}_0$ in Eq. (1), we start from the generating Lagrangian density of quantum electrodynamics, which is the Lagrangian density of the Dirac field in the absence of the electromagnetic field, given by

$$\mathcal{L}_{\mathrm{QED},0} = \frac{i\hbar c}{2}\bar{\psi}(\boldsymbol{\gamma}_{\mathrm{F}}^{\mu}\vec{\partial}_{\mu} - \overleftarrow{\partial}_{\mu}\boldsymbol{\gamma}_{\mathrm{F}}^{\mu})\psi - m_{\mathrm{e}}c^2\bar{\psi}\psi. \qquad (13)$$

This generating Lagrangian density satisfies the global unitary symmetry U(1). The unitary transformation associated with this symmetry is, in analogy to Eq. (2), given by

$$\psi \to U_{\mathrm{e}}\psi, \qquad \text{where } U_{\mathrm{e}} = e^{i\theta}. \qquad (14)$$

Here θ is the single real-valued symmetry transformation parameter. To promote the global symmetry of constant θ to a local symmetry of space-time dependent θ, the partial derivatives in $\mathcal{L}_{\mathrm{QED},0}$ in Eq. (13) are replaced by electromagnetic-gauge-covariant derivative, given in analogy to Eq. (3) by

$$\vec{D}_{\mu} = \vec{\partial}_{\mu} + i\frac{q_{\mathrm{e}}}{\hbar}A_{\Re\mu}. \qquad (15)$$

The electromagnetic four-potential $A_{\Re\mu}$ is the gauge field. The electromagnetic-gauge-covariant derivative transforms as $\vec{D}_{\mu}\psi \to U_{\mathrm{e}}\vec{D}_{\mu}\psi$. This relation requires that the transformation of $A_{\Re\mu}$ is given by $A_{\Re\mu} \to (U_{\mathrm{e}}A_{\Re\mu} + \frac{i\hbar}{q_{\mathrm{e}}}\partial_{\mu}U_{\mathrm{e}})U_{\mathrm{e}}^{*} = A_{\Re\mu} - \frac{\hbar}{q_{\mathrm{e}}}\partial_{\mu}\theta$. Using the electromagnetic-gauge-covariant derivative operator $\vec{D}_{\mu}$ and its adjoint $\overleftarrow{D}_{\mu}$ in place of the partial derivatives $\vec{\partial}_{\mu}$ and $\overleftarrow{\partial}_{\mu}$ in Eq. (13) makes $\mathcal{L}_{\mathrm{QED},0}$ invariant with respect to the local form of the symmetry transformation in Eq. (14). To write the complete electromagnetic-gauge-invariant Lagrangian density, we must also include an electromagnetic-gauge-invariant term that depends only on the gauge field $A_{\Re\mu}$. This can be obtained from the commutator of the electromagnetic-gauge-covariant derivatives. The relation $[\vec{D}_{\mu}, \vec{D}_{\nu}] = \frac{iq_{\mathrm{e}}}{\hbar}F_{\mu\nu}$ can be used to define an antisymmetric field strength tensor $F_{\mu\nu}$, in analogy to Eq. (4), as

$$F_{\mu\nu} = \partial_{\mu}A_{\Re\nu} - \partial_{\nu}A_{\Re\mu}. \qquad (16)$$

The gauge symmetry transformation law for the field strength tensor $F_{\mu\nu}$ follows from the relations above, and it is given by $F_{\mu\nu} \to U_{\mathrm{e}}F_{\mu\nu}U_{\mathrm{e}}^{*} = F_{\mu\nu}$. Following the gauge theory procedure, we obtain an electromagnetic-gauge-invariant Lagrangian density term for the gauge field $A_{\Re\mu}$, given by $\mathcal{L}_{\mathrm{em}} = -\frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu}$. The prefactor of $\mathcal{L}_{\mathrm{em}}$ has been determined by comparison of the resulting dynamical equations of the gauge field to Maxwell's equations. The complete electromagnetic-gauge-invariant generalization of $\mathcal{L}_{\mathrm{QED},0}$ in Eq. (13) is then given, in analogy to Eq. (5), by

$$\mathcal{L}_{\mathrm{QED}} = \frac{i\hbar c}{2}\bar{\psi}(\boldsymbol{\gamma}_{\mathrm{F}}^{\mu}\vec{D}_{\mu} - \overleftarrow{D}_{\mu}\boldsymbol{\gamma}_{\mathrm{F}}^{\mu})\psi - m_{\mathrm{e}}c^2\bar{\psi}\psi - \frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu}. \qquad (17)$$

This is equivalent to Eq. (10). Equations (13)–(17) represent the gauge theory procedure of quantum electrodynamics that is completely analogous to the gauge theory procedure corresponding to Eqs. (1)–(5) of the present work.

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