Towards a Spin-foam unification of gravity, Yang-Mills interactions and matter fields

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(Dated: May 19, 2011)

We propose a new method of unifying gravity and the Standard Model by introducing a spin-foam model. We realize a unification between an $SU(2)$ Yang-Mills interaction and 3D general relativity by considering a $Spin(4) \sim SO(4)$ Plebanski action. The theory is quantized *à la* spin-foam by implementing the analogue of the simplicial constraints for the broken phase of the $Spin(4)$ symmetry. A natural 4D extension of the theory is shown. We also present a way to recover 2-point correlation functions between the connections as a first way to implement scattering amplitudes between particle states, aiming to connect Loop Quantum Gravity to new physical predictions.

Introduction. One of the hardest challenges of high energy physics in the last decades is to provide a viable quantum theory of gravity. While there are a few theoretically consistent realizations, the main goal of any physical theory is to make contact with experiment. In this spirit, we propose, following the perspective discussed in [1], a unified theory that includes quantum gravity and Yang-Mills (YM) interactions as subgroups of an overall gauge unified theory. Our approach relies on the non-perturbative quantization *à la* Loop Quantum Gravity (LQG) of the theory in its initial phase. Then the theory is broken down to the general relativity (GR) and the Standard Model (SM) parts.

The theory is introduced as a spin-foam model, where the fundamental spin-networks (see subsection below) have quantum number representations of the entire gauge group, which will contain, as subgroups, the $SO(3,1)_{GR}$ and the $SU(3)_C \times SU(2)_W \times U(1)_Y$ of the SM. The spin-foam is defined as living in a *4D* manifold and the spin-network in a foliation of the manifold, as usual in LQG. After a brief discussion of the Coleman-Mandula theorem [2] and the main properties of this unified theory, we propose a method to compute the expectation value of Wilson loops involving the YM and the GR fields. This is an equivalent of the *n-point* function defined in [3] and the method relies on using the boundary formalism [4–6].

So as to provide the underlying structure and logic of our approach and avoid mathematical complexities, we will show calculations in a non trivial, Euclidean *3D* case. Remarkably, this simplified case provides an exactly soluble toy model which shows the emergence of a quantum theory of GR and YM interactions from the spin-foam quantization of the overall theory. We establish exactly how the simplicity constraints, which in *4D* are realized from Thiemann's procedure of the master constraint [7], is connected to the emergence of the YM kinetic term. We then provide the reader with the holonomy representation [8] of the boundary propagator W, encoding spin-foam dynamic, propose an extension of spin-network coherent states for both the GR and YM sectors and discuss the expectation value of the Wilson loops of the connections in the holomorphic representation [8].

A spin-foam proposal towards unification. The theory is defined following this procedure:

i) the action is a modified Plebanski BF theory that lives over a *4D* oriented smooth manifold;

ii) the theory is invariant under some unified Lie group, *i.e.* $SO(N,M)$, with $(N+M) > 4$, which defines a principal $SO(N,M)$-bundle $\mathcal{P}_{SO(N,M)}$;

iii) the basic fields of the theory are a connection A on $\mathcal{P}_{SO(N,M)}$, an ad–$\mathcal{P}_{SO(N,M)}$-valued 2-form B on $\mathcal{M}_4$ and a multiplet of scalar fields Φ on $\mathcal{M}_4$, which will be responsible of the symmetry breaking of the theory;

iv) we overcome the limitations of the Coleman-Mandula theorem for a curve spacetime, due to an initial phase completely background independent, and only a following "broken" phase with an emergent metric, as explained in detail in [9] for a general class of models. In the broken phase all the standard implications of the theorem are recovered in the low energy limit;

v) we use the spin-foam implementation of the LQG dynamics [6]. The details of the spin-foam quantization are based on the discretization of the path integral for the BF theory and on the consequent imposition on the quantized kinematical Hilbert space (constructed from the discretized phase-space of the canonical theory) of the "Plebanski-like" constraints to the BF theory. This produces a generalization of $\mathcal{H}_\Gamma^{\rm grav}$, the Hilbert space of the GR sector for a fixed 1-complex Γ (Here Γ belongs to the dual triangulation Δ^* of $\mathcal{M}_4$ [10] and represents the section of a spin-foam 2-complex.);

vi) the generalized Hilbert space contains as subsets the GR Hilbert space $\mathcal{H}_\Gamma^{\rm grav}$, the YM Hilbert space $\mathcal{H}_\Gamma^{\rm YM}$ and non trivial sectors related to the cosets generated by the symmetry breaking mechanism;

vii) the asymptotic states expanded on spin-network basis elements do not necessarily carry a simplicial interpretation [11]. The spin-foam dynamics interpolating the 1-complexes on which asymptotic states are supported [12, 13], provides the proposal for a LQG predictive scattering process.

We believe that this proposal represents a robust and novel approach that implements LQG techniques in developing a unified theory. There are many peculiar subtleties in the model, both conceptual and technical which may cloud fruitful progress. For instance, the issue of dealing with a *4D* spin-foam with $\{15j\}_G$ re-coupling elements derived by the contraction of the intertwiners of the unification group G, makes the explicit calculations particularly laborious. At the same time, the presence of the sectors associated to the GR and SM cosets makes the interpretation of the results rather obscure. In addition, despite recent successes in the derivation of asymptotics for pure gravity in *4D* [14], persistent challenges to find a formulation of LQG where matter degrees of freedom emerge naturally has created some level of ambiguity as to the expectations for phenomenology. Finally, these aspects are so intertwined with each other that it is a particularly hard task to disentangle the consequences of each single element of the proposal.

In this first work we explore a model that obviates, in a natural way, the non diagonal cosets of the unification Lie group and the intrinsic difficulties of large spin-network state calculations. Nonetheless we are still able to retain the richness of the enlarged spin-network Hilbert space and its proposed phenomenological interpretation. In order to achieve this goal, we study a Plebanski theory over a *3D* oriented smooth manifold $\mathcal{M}_3$, over which we choose to consider a principal $Spin(4)$-bundle $\mathcal{P}_{Spin(4)}$. The basic fields of the theory are in this case a connection A on $\mathcal{P}_{Spin(4)}$, an $ad(\mathcal{P}_{Spin(4)})$-valued 1-form B on $\mathcal{M}_3$ and a multiplet of scalar field Φ_{ABC} on $\mathcal{M}_3$, with capital latin letters labeling indices in the adjoint representation of the algebra $\mathfrak{Spin}(4)=\mathfrak{su}(2)\times\mathfrak{su}(2)$.
The group $SO(4) \sim SU(2) \times SU(2)$ on a *3D* manifold provides us with some evident simplifications:

i) the two $SU(2)$ groups are naturally diagonal, making our theory simpler, although non-trivial;

ii) one $SU(2)$ will be interpreted as the GR sector, and is expected to be similar (at least as a limit) to the standard *3D* LQG, a theory extensively studied; the other sector will be identified with an $SU(2)$ YM, which is the easiest non-abelian gauge theory we can write;

iii) a $Spin(4) \sim SO(4)$ model is expected to share similarities with the standard *4D* LQG (although the manifold dimensionality and the constraints are different);

iv) the absence of fermions and multiplets (at least at this initial stage) will make more evident which properties are intrinsic to the structure of the theory.

An explicit 3-dimensional model. We claim that both an $SU(2)$ YM and GR can be unified in *3D* by a modified BF theory of the form

$$S^{Pleb}=\frac{1}{G}\int_{\mathcal{M}_3} Tr[B\wedge F(A)] + \Phi\cdot\mathcal{B} + g\,\Phi\cdot\mathcal{B}\,(\Phi\cdot\Phi)\,, \quad (1)$$

in which we have defined the 3-form $\mathcal{B}^{IJK}\equiv B^I\wedge B^J\wedge B^K$ and denoted with $\cdot$ contraction of internal indices. By variation of the action, manifestly Spin(4) gauge invariant, Gauß law $\mathcal{D}_A\wedge B^I=0$ is recovered—$\mathcal{D}_A$ is the covariant derivative with respect to A_I. The "field-strength constraint" now reads $F_I=\Phi_{IJK}B^J\wedge B^K(1-g\Phi\cdot\Phi)$, while the generalization to the unified theory of those that are the simplicity constraints in the *4D* BF-theory formulation of pure gravity

$$\mathcal{B}^{IJK}\,(1-g\,\Phi\cdot\Phi)-2g\,\left(\Phi\cdot\mathcal{B}\right)\Phi^{IJK}=0. \quad (2)$$

The Spin(4) symmetry of the theory is here broken by considering the *ansatz* on the decomposition of the multiplet of fields in $\Phi^{IJK}=\Phi^{ijk}\oplus\Phi^{abc}$, where the indices ijk and abc belong each one to a different $SU(2)\in\mathrm{Spin}(4)$ subgroup, which is identified with the GR and YM theory, respectively. We assume that the auxiliary field Φ^{ijk} is order $\sqrt{g}^{\,-1}$ and Φ^{abc} is order $\sqrt{g}^{\,0}$ [29]. We make an *ansatz* on the YM components of the multiplets, $\Phi^{abc}=\chi\epsilon^{abc}$ with χ constant. This latter and the constraint (2) provide (see [15]) the relation between the $\mathfrak{su}(2)$-valued components of B^I, namely

$$B_{GR}=\gamma\,B_{YM}\,, \quad (3)$$

that represents a second class constraint [16] in the phase-space of the theory and in which $\gamma^3=3\lambda\sqrt{2\,g}$. Equation (3) implements the breakdown of the Spin(4) symmetry down to the product $SU(2)\times SU(2)$ in which symmetry between the two subgroups is lost, and in this limit it gives the action for 3D gravity coupled to YM

$$S^{Pleb}_{no\,\Phi}[e,\omega,A,B]=\tfrac{1}{G}\int_{\mathcal{M}_3} e^i\wedge F_i(\omega)\;+\;B^a\wedge F_a(A) + \tfrac{2\,\theta}{G\sqrt{3\,g}}\int_{\mathcal{M}_3}\mathrm{dVol}\,\cdot\,\sqrt{\epsilon^{\mu\nu\rho}\,\epsilon^{\alpha\beta\gamma}\,e^i_\mu e^i_\alpha\,e^j_\nu e^j_\beta\,B^a_\rho B^a_\gamma}\,. \quad (4)$$

In (4) we have split the two subgroups component of the connection in ω^i (whose field strength is denoted as $R(\omega)$) for the GR sector and A^c ($F(A)$ being the field strength) for the YM sector, and denoted the GR $\mathfrak{su}(2)$-valued 1-form as $B^i_{GR}=e^i_\mu dx^\mu$, namely the triad, and the YM ones simply by B^a. The coupling constant θ is related to g by $\theta^2=(1+(g\lambda^2)^{-\frac{1}{3}})^2\,(1+(g\lambda^2)^{\frac{1}{3}})$. After evaluating the action subject to the B-field relation (3), we recover 3D GR coupled to YM:

$$S^{\mathrm{Pleb}}_{no\,\Phi}=\frac{1}{G}\int_{\mathcal{M}_3} e^i\wedge F_i(\omega)+\frac{\sqrt{3\,g}}{2\,\theta}\int_{\mathcal{M}_3}\mathrm{Tr}[F(A)\wedge\star F(A)]\,. \quad (5)$$

Quantization *à la* spin-foam can be easily implemented in this context, following a standard recipe:

i) the manifold is discretized by means of the introduction of an oriented triangulation Δ over $\mathcal{M}_3$, that is an abstract cellular complex constituted of points p, segments s and triangles t. In the dual triangulation Δ^*, constituted by vertices v, edges e and faces f, n-dimensional objects belonging to Δ are mapped in $(3-n)$-dimensional ones.

ii) It follows that each $SU(2)$ subgroup of the B fields are smeared as algebra elements $B_s\equiv l_P^{-1}B^i_\mu l^\mu_s\tau_i\sim l_P^{-1}\tau_i\int_s B^i_\mu(\tilde{x})dx^\mu$, $\tilde{x}\in s$ denoting a weighted point (with respect to the averaging procedure) along the segment s, l_P standing for the Planck length and l_s^μ being an oriented averaged vector whose length is that of s.

iii) Connection A are smeared on the dual triangulation by associating to the discretization procedure group variables representing holonomies over edges $e \in \Delta^*$, namely $U_e \equiv e^{A_\mu^{ij} l_e^\mu} \sim e^{\int_e A}$. These are conjugated variables to B_s obeying canonical Poisson brackets.

iv) Loop quantization of the $SU(2)$-cotangent space over the spatial hypersurfaces of $\mathcal{M}_3$ proceeds constructing the Hilbert space of cylindrical functionals $\mathcal{H}_{\rm Cyl}$ [17], over which holonomies are represented in a multiplicative way and fluxes are represented as left invariant derivative operators with respect to the connections [6].

v) In $3D$ a basis in this space is given by the eigenstates [18] of the area (volume in $4D$) and the length (area in $4D$) operators, *i.e.* the spin-network state basis $\psi_{\Gamma,j,\iota}$. Elements of this basis are supported on a graph $\Gamma \in \Delta^*$ and are labelled by spin j of the irreducible representations (irreps) of each $SU(2)$ subgroup and by the intertwiner quantum number ι. By construction, the elements $\psi_{\Gamma,j,\iota}$ are $SU(2)$ gauge invariant. Invariance under diffeomorphisms is implemented by considering topologically equivalent classes of graph Γ over which $\psi_{\Gamma,j,\iota}$ are supported. The physical Hilbert space $\mathcal{H}_{\rm Phys}$ of the theory, implementing gauge and diffeomorphisms invariance [19], is then easily achieved by considering closure of $\mathcal{H}_{\rm Cyl}$ under the Ashtekar-Lewandowski (A-L) measure [20].

vi) Realization of time re-parametrization encoded in the *field strength* constraint (*scalar constraint* for pure gravity in $4D$) is implemented in a spin-foam setting by considering the discretization of the path integral of the theory. An amplitude between the boundary graph Γ of a 2-complex (over which spin-foam is supported) yields the evolution of states over Γ. Consisting of two topological-BF-theories and $SU(2)$-symmetric sectors constrained by the additional symmetry breaking (3), the theory results in a constrained sum over the two $SU(2)$ subgroups irreps, whose relation, derived by (3), reads $j_{YM} = \gamma j_{GR}$. Denoting hence the SU(2) subgroups irreps as $j_{GR} = j$ and $j_{YM} = \gamma(g) j$, the partition function of the theory (1)

$$\mathcal{Z}_\Delta^{\rm Pleb} = \sum_{j_s,\,\gamma j_s} \prod_s \dim j_s \, \dim(\gamma j)_s \prod_\tau \{6\,j\} \prod_{\tau'} \{6\,\gamma j\}, \quad (6)$$

in which $\dim j$ stands for the dimension of the j $SU(2)$ irreps and $\{6\,j\}$ denotes the 6-j symbol of $SU(2)$ recoupling theory. Notice that switching off g, and hence γ, accounts to obtain the sum from the Ponzano-Regge model, namely for $SU(2)$ topological BF theory.

Boundary propagator for one-vertex amplitude. From (6) we can extract the vertex amplitude and reformulate it in the holonomy representation [8]. As a result, the vertex amplitude is achieved by performing an integration at each node over the gauge-group-elements $\tilde{G} \in$ *Spin*(4). If we are considering a one-vertex-amplitude, the integration over the bulk group element $G^{\rm bulk}$ of the two-complex is not necessary, as each $G^{\rm bulk}$ already represent *Spin*(4) holonomies associated to the link l of the boundary graph Γ_4. Then, assigning to any link l a group-element $G_l \in$ *Spin*(4),

$$W_v^{\rm Pleb}(G_l) = \int_{{\rm Spin}(4)^4} \prod_{n=1}^4 d\tilde{G}_n \prod_l \mathcal{K}_0\left(\tilde{G}_{n_l}\, G_l\, \tilde{G}_{n_l'}^{-1}\right), \quad (7)$$

where $\mathcal{K}_0 = \mathcal{K}_t|_{t=0}$ and $\mathcal{K}_t$ denotes the propagation heat-kernel, whose heat-time is t and that is expressed as a sum over the irreps of each $SU(2)$ subgroup of *Spin*(4):

$$\mathcal{K}_t(G) = \sum_{j,\,\gamma j} \dim j \, \dim(\gamma j)\, e^{-j(j+1)\frac{t}{2}} \,{\rm Tr}\left[\Pi^{(j,\gamma j)}(\tilde{G}_n G \tilde{G}_{n'}^{-1})\right].$$

The vertex amplitude (7) provides the restriction of the boundary propagator to the tetrahedral graph $\Gamma_4 \in \Delta^*$. This restriction can be thought to originate (see *e. g.* [21]) from the perturbative expansion in the coupling constant λ of an appropriate Group Field Theory (GFT) for the unified Plebanski theory here studied. In GFT the sum over two-complexes provides a triangulation independent partition function $\mathcal{Z}^{\rm Pleb}$ [22].

Coherent spin-network states for the broken theory. Spin-network states for the broken phase of the full theory can be constructed generalizing [23], [13] and their related holomorphic spin-foam formulation [12]. Instead of considering only one $\mathrm{SL}(2,\mathbb{C})$ group element for labeling coherent states (such as [12]), we must consider an element $\mathbb{H} = H \times H'$ of $\mathrm{SL}(2,\mathbb{C}) \otimes \mathrm{SL}(2,\mathbb{C})$. We assume that the two group elements H and H' carry the same information about the normals to the 1-cells of the triangulation, *i.e.* to the segments s bounding triangles. The $\mathrm{SL}(2,\mathbb{C})$ elements H_l decompose as a complexification of $SU(2)$ elements by $H_l = n_{s(l)} e^{-iz_l \frac{\sigma_3}{2}} n_{t(l)}^{-1}$. In *3D* each H_l is hence labelled by two normals to the segment s, namely $n_{s(l)}$ and $n_{t(l)}$, whose relative rotation is achieved by a $U(1)$ subgroup of $SU(2)$. For the element H labeling the GR subgroup of the coherent states, the complex parameter $z_l = \xi + i\eta$ has the same meaning as in *4D*: ξ expresses the dihedral angle of a semiclassical Regge geometry [24], while η the length of the 1-simplices, *i.e.* the segments s. The H_l' element labeling the YM subgroup can be thought as the necessary quantities to define a YM copy of the Regge geometry (as a YM lattice [15]).

The complex parameters z_l' of H_l' are associated with the length of the YM lattice spacing, and is related to the flux though s of the electric field B_{YM}. As a consequence of (3), the flux of B_{YM} is the $1/\gamma$ rescaling of the GR electric field flux. In a similar way, the GR dihedral angle is mapped, by multiplication by $1/\gamma^2$, in the equivalent dihedral angle of the YM lattice. This follows from (3) and the expression of the extrinsic curvature in terms of ξ (see *e.g.* [25]). As on the YM lattice $\xi_\gamma = \xi/\gamma^2$ represents the conjugated variable to the flux of the electric field, we can argue that ξ_γ represents the index contraction of the gauge invariant field strength $F(A_{YM})$ (see *e.g.* [26]). Finally, coherent spin-network states read

$$\Psi_{\Gamma,\mathbb{H}_l}(G_l) = \int_{\mathrm{Spin}(4)^4} \left(\prod_n d\tilde{G}_n\right) \prod_l \mathcal{K}_{t_l}\left(G_l, \tilde{G}_n \mathbb{H}_l \tilde{G}_n^{-1}\right) ,$$

in which the heat kernel $\mathcal{K}_t$ has been specified above.

Expectation value of product of holonomies. The reconstruction theorem [27] ensures, at least for a certain class of topologies of the (connected) manifold $\mathcal{M}_3$, that gauge-invariant information about the principle fiber bundle $\mathcal{P}_{\mathrm{Spin}(4)}$ can be recovered from Wilson loops. Therefore the boundary formalism, developed in [3] and [21], paves a way to compute the expectation value of the product of two holonomies, each belonging to a different $SU(2)$ subgroup of the theory. In the most straightforward setting, this expectation value will be calculated on the connected graph Γ_4, the tetrahedral spin-network. We evaluate Wilson loops $U_{\beta_{\mathbf{x}}}(h_l)$ and $U_{\beta'_{\mathbf{y}}}(h'_l)$, where h_l and h'_l are $SU(2)$ group-elements for each subgroup of $Spin(4)$, and $\beta_{\mathbf{x}}$ and $\beta'_{\mathbf{y}}$ are loops with base points $\mathbf{x}$ and $\mathbf{y}$. For convenience, say that the two base points correspond to two nodes of Γ_4, and that the two loops bound two triangles sharing a segment. Within the Euclidean space $\mathcal{M}_3$ taken into account, we can think this graph to be embedded on the Regge submanifold that is the discretization of the boundary of a 3-ball, namely of S^2. The boundary propagator is described by $W_v(G_l)$, while the coherent states, representing the state over which the expectation value is computed, are given by $\Psi_{\Gamma,\mathbb{H}_l}(G_l)$. Both of them are supported on Γ_4. At the first order in the GFT parameter λ we can calculate

$$\mathcal{A} = \langle W_v(G_l)| U_{\beta_{\mathbf{x}}}(h_l) U_{\beta'_{\mathbf{y}}}(h'_l) |\Psi_{\Gamma,\mathbb{H}_l}(G_l)\rangle , \qquad (8)$$

in which we use the inner product of the A-L measure [20] for each $SU(2)$ subgroup. This ensures gauge invariance and space-diffeoinvariance for (8). The result is the sum over $SU(2)$ spin j of the product of the expectation value of $U_{\beta_{\mathbf{x}}}(h_l)$ on the GR subgroup of $W_v(G_l)$ and $\Psi_{\Gamma,\mathbb{H}_l}(G_l)$, say $\tilde{\mathcal{A}}(j_i, s)$ its "spin and intertwiner representation", and of $U_{\beta'_{\mathbf{y}}}(h'_l)$ on the YM subgroup, say it $\tilde{\mathcal{A}}(\gamma j_i, s)$:

$$\mathcal{A} = \sum_{j_l, \gamma j_l} \tilde{\mathcal{A}}(j_m, s_1) \prod_l \mathrm{dim} j_l \, e^{-\frac{(j_l - j_l^0)^2}{2\sigma_l^2}} e^{-i\xi_l j_l} \prod_n \Phi_\iota(n_l) \times$$
$$\tilde{\mathcal{A}}(\gamma j_n, s_1) \prod_{l'} \mathrm{dim} \gamma j_{l'} \, e^{-\frac{(\gamma j_{l'} - \gamma j_{l'}^0)^2}{2\sigma_{l'}^2}} e^{-i\xi_\gamma^{l'} \gamma j_{l'}} \prod_n \Phi_{\iota_\gamma}(n_{l'}) . \qquad (9)$$

In (9) ι denotes a trivalent intertwiner between irreps j, ι_γ denotes a trivalent intertwiner between irreps γj, the coefficients $\Phi_\iota(n_l)$ and $\Phi_{\iota_\gamma}(n_l)$ are the coherent intertwiner defined in [13], and finally $\mathrm{dim}\, j_l^0 = \eta_l / t_{ab}$, $\mathrm{dim}\, \gamma j_l^0 = \gamma \eta_l / t_{ab}$ and $\sigma_l^2 = 1/(2t_l)$. Each $\mathcal{A}$ is the the contraction of twelve Wigner $3j$ symbols involving the six GR $SU(2)$ irreps j (or YM γj) labelling Γ_4 on the boundary of the interaction region. Eq. (8) is the first step to implement the scattering of particle states in this research program, which aims to connect LQG to physical predictions.

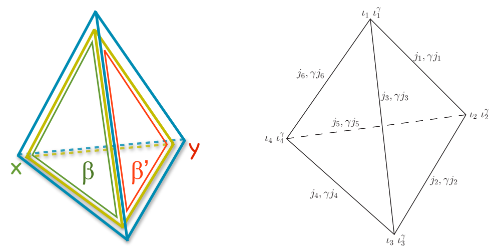

FIG. 1: Two Γ_4 and two loops β_x and β'_y on S^2, as in (8) (left). Γ_4, colored with GR irreps j and YM irreps γj (right).

Conclusions. We present a proposal for unifying gravity and Yang-Mills theory in LQG. The richness and mathematical complexity of the model offer exciting prospects for both theoretical and phenomenological development. In the past there has been interest in gravity and YM in *3D* [28] and it would be important to develop the model to compare it with well established results. Also, the choice of our constraint was dictated by minimality, but it is not the only possible choice. Further work will need to explore more of the *3D* set up.

The procedure is naturally implemented in *4D* and has no obvious or apparent obstacles if not for a more complex manipulability. We believe, though, that much can be already understood in the dimensionally reduced case.

Finally, the proposal for the scattering amplitude provides large room for phenomenological predictions, and could be an important milestone for pushing LQG beyond its present limitations.

Acknowledgements. We thank G. Amelino-Camelia, C. Rovelli and L. Smolin for very stimulating discussions.

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