The Barbero Immirzi parameter: an enigmatic paramater of loop quantum gravity

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Abstract:

The Barbero - Immirzi parameter (γ) is introduced in loop quantum gravity (LQG) whose physical significance is still a biggest open question; because of its profound traits. In some cases, it is real-valued; while, it is complex-valued in other cases. This parameter emerges out in the process of denoting a Lorentz connection with noncompact group $SO(3,1)$ in the form of a complex connection with values in a compact group of rotations, either $SO(3)$ or $SU(2)$. Initially, it was appeared in the Ashtekar variables. Fernando Barbero proposed its possibility to include within formalism. Its present value is fixed by counting of microstates in loop quantum gravity and matching with the semiclassical black hole entropy computed by Stephen Hawking. This parameter is used to count the size of the quantum of area in Planck units. Untill, the the discovery of the spectrum of the area operator in LQG; its significance remains unknown. However, its complete physical significance is yet to be explored. In the present article, an introduction to the Barbero - Immirzi parameter in LQG, timeline of this research area, various proposals regarding its physical significance are given.

Keywords: Loop quantum gravity, Ashtekar variable, Barbero-Immirzi parameter, area operator, black hole entropy,

Contents

1 Introduction

Loop quantum gravity (LQG) is one of the supposed candidate of theory of quantum gravity. It can unify general relativity (GR) with quantum field theory (QFT). It is non - perturbative and backgorund independent approach of quantum gravity theory. LQG begins with GR; thereafter, takes some conceptual base from QFTs to deliver a quantum theory of gravity. LQG is a theory of constraints, in which various constraints such as Hamiltonian, diffeomorphism and Gauss constraints are converted into operators. Due to limited space, basics of LQG is not given. There are many classic texts [1 - 10] and papers [11 -23] that explain LQG lucidly [1-5].

In 1986, Abhay Ashtekar [24] found new kind of variable (Ashtekars variable) in classical and quantum gravity. In Ashtekar's formulations, the constraints are simplified by considering a complex-valued form for the connection and tetrad variables and these are known as Ashtekar's variables [1 - 5].

While dealing with the reality condition of the formalism of Ashtekar's variable, Barbero [25] firstly, introduced a free parameter in the expression of Ashtekar's variable and then in the exression of constraints. Thereafter, Immirzi [28 - 29] used various possibilities of this free parameter in the expression of LQG in his paper. This free parameter is now known as the Barbero - Immirzi parameter (γ). Whether the γ is complex or real; it gives many results in LQG. In some cases, real-valued γ is required; while, in the other cases complex-valued γ is necessary [1 - 5].

The physical significance of area operator in LQG with the complex γ becomes ambiguous. The LQG kinematics i.e., kinematical Hilbert space can only be comprehended; if, the γ is real number. the $SU(2)$ spin network of LQG can only be created with the real value of the γ [1 - 5].

With the complex value of the $\gamma(\gamma = i)$, the spatial connection can be seen as spacetime connection; since, it transforms under diffeomorphism in right way. There are also some cases that shows that the complex valued γ is also crucial in LQG formalism. For instance, the form of Hamiltonian constraints becomes simpler; if, $\gamma = i$ is taken [1 - 5].

In the next section, various proposals regarding the Barbero - Immirzi parameter are briefly explained; in which some proposals advocates the real valued γ, while, the other advocate complex valued γ.

1.1 Historical timeline

In the below table, historical timeline is introduced; in which, title of various papers on the Barbero - Immirzi parameter are given in chronological order [24 - 65].

Sr. No.	Year	**Research on the Barbero Immirzi parameter and its significance**
1	1986	Discovery of Ashtekar variables
2	1995	Real Ashtekar variables for Lorentzian signature space-times
3	1996	Black hole entropy from loop quantum gravity
4	1996	From Euclidean to Lorentzian General Relativity: The Real Way
5	1996	Real and complex connections for canonical gravity
6	1997	Quantum gravity and Regge calculus
7	1997	Counting surface states in LQG
8	1997	Immirzi parameter in quantum general relativity
9	1997	On the constant that fixes the area spectrum in canonical quantum gravity
10	1998	Quantum Geometry and Black Hole Entropy
11	2000	Is Barberos Hamiltonian formulation a Gauge Theory of Lorentzian Gravity?
12	2001	Comment on Immirzi parameter in quantum general relativity
13	2003	Quasinormal Modes, the Area Spectrum, and Black Hole Entropy
14	2004	Black-hole entropy in loop quantum gravity
15	2004	Black-hole entropy from quantum geometry
16	2005	Origin of the Immirzi parameter
17	2005	Physical effects of the Immirzi parameter
18	2005	On choice of connection in loop quantum gravity
19	2007	On a covariant formulation of the BarberoImmirzi connection
20	2007	Renormalization and black hole entropy in Loop Quantum Gravity
21	2008	From the Einstein-Cartan to the Ashtekar-Barbero canonical constraints, passing through the Nieh-Yan functional
22	2008	The Barbero-Immirzi Parameter as a Scalar Field: K-Inflation from Loop Quantum Gravity?
23	2008	Topological Interpretation of Barbero-Immirzi Parameter
24	2009	Peccei-Quinn Mechanism in Gravity and the Nature of the Barbero-Immirzi Parameter

25	2010	A relation between the BarberoImmirzi parameter and the standard model
26	2011	Complex Ashtekar variables, the Kodama state and spinfoam gravity
27	2012	The Quantum Gravity Immirzi ParameterA General Physical and Topological Interpretation
28	2012	Complex Ashtekar Variables and Reality Conditions for Holsts Action
29	2013	Black Hole Entropy from complex Ashtekar variables
30	2014	Geometric temperature and entropy of quantum isolated horizons
31	2014	A Correction to the Immirizi Parameter of SU(2) Spin Networks
32	2014	The Microcanonical Entropy of Quantum Isolated Horizon, 'quantum hair' N and the Barbero-Immirzi parameter fixation
33	2015	The holographic principle and the Immirzi parameter of loop quantum gravity
34	2017	Immirzi parameter without Immirzi ambiguity: Conformal loop quantization of scalar-tensor gravity
35	2018	Horizon entropy with loop quantum gravity methods
36	2018	Generalizing the Kodama State I: Construction
37	2018	Generalizing the Kodama State II: Properties and Physical Interpretation
38	2018	Chiral vacuum fluctuations in quantum gravity
39	2018	Black hole entropy from the SU(2)-invariant formulation of Type I isolated horizons
40	2018	Black hole entropy and SU(2) Chern-Simons theory
41	2020	On the value of the Immirzi parameter and the horizon entropy

Table 1.1: Timline of research on the Barbero Immirzi parameter.

1.2 Ashtekar's formalism

Before the discovery of Ashtekar's variables, the Palatini action i.e., the first order formulation is not complete. But, Ashtekars formalism makes it complete. By converting tetrads into triads i.e. three dimensional hypersurfaces Σ_t; one gets $e^J_\mu \rightarrow e^j_c$ where, $\mu \rightarrow c \in \{1, 2, 3\}$, $J \rightarrow j \in \{1, 2, 3\}$ and the spin connection is also transformed as $\Gamma^j_c = \omega_{ckl}\varepsilon^{klj}$ [1 -5, 23].

The Hamiltonian constraint is a complicated non-polynomial function in Palatini formulation; thus, canonical quantization is not easy within this formalism. In Palatini formulation, the variables of phase space are (e_c^j, Γ_c^j); where, e_c^j is the intrinsic metric of the spacelike manifold Σ and Γ_c^j is a function of its extrinsic curvature [1 -5, 23].

In Ashtekars formalism, complex valued connection Γ_c^j replaces the real connection ω_μ^{JK} with duality (either self (+1) or anti-self (-1)) [1 -5, 23].

$$\tilde{E}_j^c \to \frac{1}{i}\tilde{E}_j^c, K_c^j \to A_c^j = \Gamma_c^j - iK_c^j \tag{1}$$

Where, $\tilde{E}_j^c$ is the scalar density or triad electric field, A_c^j is the Ashtekar-Barbero connection or spatial connection, $K_c^j = k_{cd}e^{dj}$ with k_{cd} the extrinsic curvature of Σ. Thus, there are two phase space variables i.e. A_c^j and $\tilde{E}_j^c$ [1 -5, 23].

Since, the Ashtekars connection formulation variables, i.e. A_c^j and $\tilde{E}_j^c$ follows rotation of $SU(2)$ symmetry with respect to the internal indices; the Ashtekars formalism plays the role of $SU(2)$ gauge theory and this $SU(2)$ group is a subgroup of $SL(2, \mathbb{C})$ [1 -5, 23].

All three constraints are simplified in Ashtekars variables and their expression are [1 -5, 23]

$$\mathcal{G}_j = D_c\tilde{E}_j^c \tag{2}$$

$$\mathcal{C}_c = \tilde{E}_j^d F_{cd}^j - A_c^j\mathcal{G}_j \tag{3}$$

$$\mathcal{H} = \varepsilon_l^{jk}\tilde{E}_j^c\tilde{E}_k^d F_{cd}^l \tag{4}$$

The equations (2), (3), and (4) are Gauss, diffeomorphism, and Hamiltonian constraint respectively. In Ashtekar's formalism, the Einstein-Hilbert-Ashtekar Hamiltonian of GR is written as [1 -5, 23].

$$\mathcal{H}_{EHA} = N^c\mathcal{C}_c + N\mathcal{H} + T^j\mathcal{G}_j = 0 \tag{5}$$

Where, $\mathcal{C}_c, \mathcal{H}, \mathcal{G}_j, N^c$ and N are vector constraint, the scalar constraint, and Gauss constraints, shift and lapse, respectively. The T^j is a Lie algebra valued function over spatial surface [1 -5, 23].

The unit imaginary i.e. $i = \sqrt{-1}$ appeared in the equation (1) makes the formalism complex valued. Therefore, some restrictions in terms of reality condition on the possible solutions of the theory must be applied to achieve tangible physical results relevant to a metric valued in $\mathbb{R}$ instead of in $\mathbb{C}$ [1 -5, 23].

For example, If $\dot{Z}$ is used to represent the time derivative of Z; then, the reality condition and constraints i.e., $\mathcal{G}_j = D_c\tilde{E}_j^c$ must be satisfied by solutions. In this case, there are two reality condition and the second condition is the time derivative of first condition. thus [1 -5, 23],

$$\tilde{E}_j^c\tilde{E}_k^d\delta^{jk} \in \mathbb{R} \tag{6}$$

$$\{\tilde{E}_j^c\tilde{E}_k^d\delta^{jk}\}^\bullet \in \mathbb{R} \tag{7}$$

In standard form, the Ashtekar variables is given as [1 -5, 23],

$$\tilde{E}_j^c \to \frac{1}{\gamma}\tilde{E}_j^c, K_c^j \to A_c^j = \Gamma_c^j - \gamma K_c^j \tag{8}$$

Where, the γ is the Barbero-Immirzi parameter. If, $\gamma = i$; then, the equation takes the original form. The Poisson brackets is written as [1 -5, 23],

$$\{K_c^j(x), \tilde{E}_k^d(y)\} = \{A_c^j(x), \tilde{E}_k^d(y)\} = k\delta_k^j\delta_c^d\delta(x,y) \tag{9}$$

In standard form [1 -5, 23],

$$\{A_c^j(x), \tilde{E}_k^d(y)\} = 8\pi G\gamma\delta_k^j\delta_c^d\delta^3(x,y) \tag{10}$$

Where, $k = 8\pi G\gamma$.

The reality condition is not necessary for real value and as a result new variable and constraints are also real [1 -5, 23].

The form of Hamiltonian constraint becomes complicated with the real value of the γ i.e.,

$$\mathcal{H} = \varepsilon_l^{jk}\tilde{E}_j^c\tilde{E}_k^d F_{cd}^l - 2(1+\gamma^2)\tilde{E}_j^{[c}\tilde{E}_k^{d]}K_c^j K_d^k \approx 0 \tag{11}$$

If, $\gamma = i$; then the form of Hamitonian constraints becomes simple [1 -5, 23].

1.3 The area operator and the γ

The spectrum of area operator and its eigen value can only be understood with the real valued γ. As mentioned, complex valued γ makes area operator complex valued and the significance of complex valued area operator is ambiguous. The expression of area operator is [1 - 5, 23]

$$\hat{A}_S \Psi_\Theta = l_P^2 \Sigma_k \sqrt{j_k(j_k+1)} \Psi_\Theta \tag{12}$$

In standard form, the area operator with the γ can also be written as [1 - 5, 23]

$$\hat{A}_S \Psi_\Theta = 8\pi\gamma l_P^2 \Sigma_k \sqrt{j_k(j_k+1)} \Psi_\Theta \tag{13}$$

$$\therefore \hat{A}_S = 8\pi\gamma l_P^2 \Sigma_k \sqrt{j_k(j_k+1)} \tag{14}$$

1.4 Black hole entropy calculation in LQG and the γ

The expression of entropy of black hole in Planck units calculated semiclassically by Hawking is written as [1 - 5, 23],

$$S = \frac{A}{4} \tag{15}$$

In 1997, Ashtekar et al [33] showed that spin networks explains spacetime geometry outside a black hole. Some edges of this spin network puncture the event horizon, provides the value of area through this contribution. The $U(1)$ ChernSimons theory explains the quantum geometry of the horizon. In this formalism, the rotation of $SO(2)$ describes $2D$ geometry, that is isomorphic to $U(1)$. The entropy of black hole is calculated through counting of spin network states relevant to an event horizon. Thus, the expression of black hole entropy in LQG is [1 - 5, 23],

$$S = \frac{\gamma_0 A}{4\gamma} \tag{16}$$

Where, the value of γ_0 is either $\gamma_0 = \frac{ln(2)}{\sqrt{3}\pi}$ or is $\gamma_0 = \frac{ln(3)}{\sqrt{8}\pi}$ that relies on the choice of the gauge group. By taking $\gamma_0 = \gamma$, one gets actual black hole entropy formula calculated by Hawking [1 - 5, 23] i.e.

$$S = \frac{\gamma_0 A}{4\gamma_0} = \frac{A}{4} \quad (17)$$

This calculation is true for each sort of black hole [1 - 5, 23]. Since, the minimal values of the spin is essential; in other proposals, some corrections are introduced that will be introduced in the next section .

2 Various proposals on the physical significance of the γ

In this section, various proposals on the physical significance of the γ are briefly given [24 - 65].

2.1 Why the γ was introduced in LQG?

As mentioned, complex valued Ashtakar's variables simplified constraints of quantum gravity based on canonical quantization i.e., LQG. Thereafter, Barbero came up with new stretegy to tackle Ashtekar's variable with real value for Lorentzian signature space-times. In his paper, Barbero wrote down Ashtekar's variable with a free parameter (eqn (8)); namely, γ (in his paper he denoted it as β). Ashtekar used $SU(2)$ and $SL(2,\mathbb{C})$ groups of Yang - Mill theory to deliver complex valued constraints i.e., Ashtekar's variables. While, Barbero showed that one can use $SO(3)$ Yang - Mill phase space to write that modified Hamiltonian constraint with Lorentz signatures without complex variable to elaborate space-times without lossing the features of Ashtekar's variables [1-5, 25].

Barbero also explained that for simple form of Hamitonian constraint, complex variable is required; while, for complicated form, this constraint could be written with real variables. For instance, loop variable of LQG. Barbero derived Hamiltonian constraint with $\gamma^2 = 1$ (real valued and Euclidean signature). While, with Lorentzian singature yields again complex valued form of equation. Barbero also derived hamiltonian constraint with $\gamma = -1$. Hamiltonian constraint could also be written with real Ashtekar variables for Lorentzian general relativity with $SO(3)$ ADM formalism [25].

Thereafter, Immirzi [28 -29] explained the importance of the γ in his paper. In these papers, Immirzi explained canonical quantization of gravity i.e., LQG with Regge calculus briefly. In his paper, Immirzi elaborated basics of LQG briefly with the discussion on the γ. Immirzi discussed various possibilities of the value of the γ and named this arbitrariness of the γ as the γ crisis [28 - 29].

Since, Barbero introduced this free parameter and Immirzi used it to explain canonical quantuzation method along with Regge calculus; the γ is known as the Barbero - Immirzi parameter. In short, Barbero used one-parameter scale transformation to generalize the Ashtekar canonical transformation to a $U(\gamma)$; While, Immirzi observed that such a transformation modifies the spectra of geometrical quantities of LQG [1 -5, 25, 28-29].

2.2 Proposals on the γ as Immirzi ambiguity

In LQG, geometrical obeservable such as area and volume are quantized and they exhibits discrete spectrum. In 1996, Immirzi noticed that LQG does not determine the complete scale of such spectra. Immrizi also observed that one can have different spectra for the same geometrical quantities if he begins with scaled elementary variables. The algebra of holonomy relies on a free parameter that gives family of one parameter of quantum theories with inequivalance. The γ is this family of one parameter [31].

There is a certain symmetry under study according to which classical theory is identified as a canonical transformation; but, one can can not identify it as a unitary transformation of quantum theory. Since, holonomy is operator of LQG and because of weird sort of representation of LQG one has to consider the γ as ambiguity. the γ as ambiguity is also due to background independence and diffeomorphism symmetry of LQG [31].

In LQG, there are two connection i.e., A and Γ. Therefore, one has to create the γ scaled connection namely, A_γ by interpolation between different connections. thus the elementary excitation of LQG namely Wilson loop of A_γ results into different for various value of the γ. Therefore, some physical spectrum of quantity of LQG relies on the γ [31].

Another thing is metric information resides in the E(conjugate variable). Since, E is conjugate to connection; in quantum formalism, it is written as derivative operators that acts on functions over the group. Over the group manifold, any geometrical quantity that is function of E behaves as elliptic operator that results into discrete spectrum. Such elliptic operators possess non-vanishing scalar dimension relative to the affine scaling of the connection. Hence, in the elliptic geometric operators spectrum, ambiguity is introduced i.e., the Immirzi ambiguity. This ambiguity influences discreteness of space in LQG [31].

In year 2001, Samual [35] commented that Interpretations as the Immirzi ambiguity is not clear and does not give any agreement on its origin and significance.

In 2017, Veraguth and Wang [57] published a paper in which they explained LQG wihtout Immirzi ambuguity using conformal LQG.

2.3 Proposals on the γ and black hole entropy calculation in LQG

The black hole entropy calculation in LQG is quite enriched research area. The value of the γ and black hole entropy formula in LQG is a topic whose implications are far - reaching [1-5, 23].

In 1996, Rovelli [26] calculated black hole entropy within LQG using the statistical framework. As mentioned in section (1.4), Ashtekar et al [33] also calculated for $\gamma_0 = \frac{ln2}{\pi\sqrt{3}}$ black hole entropy within LQG in 1998. In 2002, Dreyer [36] Fixed the value of γ i.e., $\gamma = \frac{ln3}{2\pi\sqrt{2}}$ using classical quasinormal mode spectrum of a black hole and gave black hole entropy formula in LQG with $SO(3)$ group instead of $SU(2)$. Thereafter, Meissner [37] fixed the value of Area in LQG by fixing γ_M (either $\gamma_M = 0.23753295796592....$ or $\gamma_M = 0.273985635....$) and comparing black hole entropy formula with Bekenstein Hawking formula [1 - 5].

In 2004, Domagala and Lewandowski [38] gave microscopic degrees of freedom to count the black-hole entropy. They predicted different value of the γ i.e., $\frac{ln2}{\pi} \leq \gamma \leq \frac{ln3}{\pi}$, since, the spin greater then $\frac{1}{2}$ to the entropy also contributes and this contribution can not be neglected. In 2007, Jacobson [43] studied renormalization and black hole entropy in LQG $\left(S_{LQG} = \frac{b}{\gamma}\frac{A}{\hbar G}, \therefore S_{LQG} \propto A, S_{LQG} \propto \frac{1}{\gamma G}\right)$. He found that one should compare this formula with actual Bekenstein - Hawking entropy formula after accounting the scale dependence of Newton's constant and area. For any value of the γ, if some property of renormalization is followed than one can compare both entropy formulas [1 - 5].

In 2013, Frodden et al [52] found that by taking complex valued Ashtekar variables, black hole entropy formula in certain condition is achieved. This paper shows that the number of microstates $N_\Gamma\left(A, \gamma \to \pm i\right)$ acts as $exp\left(\frac{A}{(4l_P^2)}\right)$ for certain case i.e. for large area A in the large spin semiclassical limit.

There are many other papers available on black hole entropy in LQG [30, 43, 53, 55, 58, 62 -65].

2.4 On the constant that fixes the area spectrum in canonical gravity

As mentioned in the section (1.3), the proportionality coefficient in the formula of area operator in LQG includes the γ. In 1998, Krasnov [32] found that the multiplicative factor of the area operator is $8\pi\gamma$. Hence, the equation is

$$A = 8\pi\gamma l_P^2 \sqrt{j(j+1)} \tag{18}$$

2.5 Origin of the γ

In 2005, Chou et al [39] found a technique through which a ratio is obtained which equals the γ. They used quadratic spinor techniques and found that the γ can be written as ratio between scalar and pseudoscalar contributions in the theory i.e., $\gamma = \frac{\langle\bar{\psi}\psi\rangle}{\langle\bar{\psi}\gamma_5\psi\rangle}$. This ratio can be seen as a measure of how gravity differs from covariant gravity. Such a techniques permits the renormalization scale regarding the γ via spinor's expectation value in quantization process.

2.6 The γ in covariant formulation

In 2007, Fatibene et al [42] gave a proposal on covariant formulation of the Barbero - Immirzi connection in which they defined a global covariant $SU(2)$ - connection over whole spacetime that limits to general the Barbero - Immirzi connection on a given slice of space.

2.7 The γ as a scalar field

In 2008, Taveras and Yunes [45] gave a proposal on the γ as a scalar field. they studied LQG based generalization of general relativity in which they modified the Holst action. In this formalism, the γ as a dynamical scalar field.

2.8 Topological interpretation of the γ

In 2008, Date et al[46] gave a proposal on Topological interpretation of the γ. In this proposal , in the time gauge, Nieh-Yan topological density of a theory of gravity permits to explain gravity as a real $SU(2)$ connection. In the Hamiltonian formalism, for $\eta = 1$ the set of constraints is equal to the Barbero formulation. The Immirzi formulation with $\gamma = \eta^{-1}$ is for rest real value of this parameter. Similar to the topologically non-trivial vacuum structure of Quantum chromodynamics, that gives in the form of the θ -parameter; in the gravity theory for η parameter suggests enriched gravity vacuum structure.

2.9 The Peccei-Quinn Mechanism in Gravity and the Nature of the γ

In 2009, Mercuri [47] gave a proposal on nature of the γ using Peccei-Quinn mechanism in gravity in which the γ is taken as a field. In this proposal, the particular form effective action of geometry corresponds the value of the γ with the other quantum ambiguities through the Peccei-Quinn mechanism.

2.10 Proposals on The Kodama state and the γ

In year 2006, Randono [59 -60] wrote two paper on generalization of Kodama states in which he used real valued γ to generalize these states and derived physical interprtation of these states.

In year 2011, Wieland [49] gave a proposal namely, "Complex Ashtekar variables, the Kodama state and spinfoam gravity"; in which he used complex valued Ashtekar variable and real valued γ. He used $SL(2,\mathbb{C})$ Kodama state and proposed a spinfoam vertex amplitude.

2.11 The Quantum Gravity γ A General Physical and Topological Interpretation

In year 2013, El Naschie [50] gave a proposal on general physical and topological interpretation of the γ. In this paper, he explained the γ of LQG as a definite quantum entanglement correction.

2.12 A Correction to the γ of SU(2) Spin Networks

In year 2014, Sadiq [54] gave a correction to the γ of $SU(2)$ spin networks. In this paper, by taking $j = 1$ and to preserve SU(2) symmetry of theory; twice value of the γ is proposed. Previously in LQG, the γ is fixed by $j = 1$ transitions of spin networks as the dominant processes instead of $j = 1/2$ transitions. It means $SO(3)$ should be a gauge group instead of $SU(2)$.

2.13 The holographic principle and the γ of loop quantum gravity

In year 2015, Sadiq [56] gave a proposal that correlates the γ in LQG and the hologrphic principle. In this proposal the γ is fixed using the equipartition theorem based on LQG at holographic boundary in such a way that Unruh - Hawking law of temperature holds and follows. such derived value of the γ exhibits validity universely. in that way this approach correlates the value of the γ in LQG and the holographic principle.

3 Concluding remarks

- In this paper, initially, a short introduction of the Barbero - Immirzi parameter the γ along with the historical timeline of research in the physical significance of the γ in LQG are given.
- Thereafter, Ashtekar's formalism on which LQG is based on is explained.
- Since, the value of the γ and its implication is very important in the area spectrum and black hole entropy calculation in LQG; afterwards, these are elaborated.
- At last, various proposal on the physical significance of the γ in LQG are very briefly given.
- Hence, the γ, whether complex-valued or real-valued is a crucial free parameter of LQG.

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