The strange (hi)story of particles and waves*

H. Dieter Zeh – www.zeh-hd.de – arxiv:1304.1003v16

Abstract: This is an attempt of a non-technical but conceptually consistent presentation of quantum theory in a historical context. While the first part is written for a general readership, Sect. 5 may appear a bit provocative to some quantum physicists. I argue that the single-particle wave functions of quantum mechanics have to be correctly interpreted as *field modes* that are "occupied once" (that is, first excited states of the corresponding quantum oscillators in the case of boson fields). Multiple excitations lead non-relativistically to apparent many-particle wave functions, while the quantum states proper are defined by wave function(al)s on the "configuration" space of fundamental fields, or on another, as yet elusive, fundamental local basis.

Sects. 1 and 2 are meant as a brief overview of the early history - neglecting all technical details. Sects. 3 and 4 concentrate on some important properties of non-relativistic quantum mechanics that are insufficiently pointed out in most textbooks (including quite recent ones). Sect. 5 describes how this formalism would have to be generalized into its relativistic form (QFT), although this program fails in practice for *interacting* fields because of the complicated entanglement that would arise between too many degrees of freedom. This may explain why QFT is mostly *used* in a semi-phenomenological manner that is often misunderstood as a fundamentally new theory. Sect. 6 describes the application of the Schrödinger picture to quantum gravity and quantum cosmology, while Sect. 7 concludes the paper.

1. Early History

The conceptual distinction between a discrete or a continuous structure of matter (and perhaps other „substances") goes back at least to the pre-Socratic philosophers. However, their concepts and early ideas were qualitative and speculative. They remained restricted to some general properties, such as symmetries, while the quantitative understanding of continuous matter

---

* Free and extended translation of my unpublished German text "Die sonderbare Geschichte von Teilchen und Wellen" – available on my website since October 2011. By the term "(hi)story" I tried to catch the double-meaning of the German word "Geschichte". – V15 has now been published in Z. Naturf. A, **71**, 195 (2016).

and motion had to await the conceptual development of calculus on the one hand, and the availability of appropriate clocks on the other. Quantitative laws of nature and the concept of mass points, for example, were invented as part of classical mechanics.

This theory was first applied to extended "clumps of matter", such as the heavenly bodies or falling rocks and apples. It was in fact a great surprise for Newton and his contemporaries (about 1680) that such very different objects – or, more precisely, their centers of mass – obeyed the same laws of motion.[1] The objects themselves seemed to consist of continuous matter, although the formal concept of mass points was quite early also applied to the *structure* of matter, that is, in the sense of an atomism. Already in 1738, Daniel Bernoulli explained the pressure of a gas by the mean kinetic energy of small objects, but without recognizing its relation to the phenomenon of heat. If one regarded these objects themselves as small elastic spheres, however, the question for their internal structure would in principle arise anew. The concept of elementary particles thus appears problematic from the outset.

At about the same time, Newton's theory was also generalized by means of the concept of infinitesimal massive volume elements that can move and change their size and shape according to their local interaction with their direct neighbors. This route to continuum mechanics formed a mathematical program that did not really require any novel *physical* concepts beyond Newton. The assumption of an unlimited divisibility of matter thus led to a consistent theory. In particular, it allowed for wave-like propagating density oscillations, required to describe the phenomenon of sound. So it seemed that the fundamental question for the conceptual structure of matter had been answered.

As a byproduct of this "substantial" (or "Laplacean") picture of continuum mechanics, based on the assumption of distinguishable and individually moving infinitesimal elements of matter, also the elegant "local" (or "Eulerian") picture could be formulated. In the latter, one neglects any reference to trajectories of individual pieces of matter in order to consider only its spatial density distribution together with a corresponding current density as the kinematical objects of interest. In modern language they would be called a scalar and a vector *field*. In spite of this new form, continuum mechanics thus remains based on the concept of a locally conserved material substance.

The picture of individually moving elements of a substance would prove incomplete, however, if the true elements of matter could move irregularly, as suspected for a gas by Daniel Bernoulli. Since his gas pressure is given by the density of molecular kinetic energy, that is, by the product of the number density of gas particles and their mean kinetic energy, this picture could nonetheless be understood as representing a "chaotic continuum" by means of an appropriately defined simultaneous limit of infinite particle number density and vanishing particle mass. This remained a possibility even when chemists began to successfully apply Dalton's and Avogadro's hypotheses about molecular structures from the beginning of the nineteenth century in order to understand the chemical properties of the various substances. Similar to Auguste Bravais's concept of crystal lattices (about 1849), these structures were often regarded as no more than a heuristic tool to describe the internal structure of a multi-component continuum. This view was upheld by many even after Maxwell's and Boltzmann's explanation of thermodynamic phenomena in terms of molecular kinetics, and in spite of repeated but until then unsuccessful attempts to determine a finite value for Avogadro's or Loschmidt's numbers. The "energeticists", such as Wilhelm Ostwald, Ernst Mach and initially also Max Planck remained convinced until about 1900 that atoms are an illusion, while concepts like internal energy, heat and entropy would describe fundamental continua. Indeed, even after the determination of Loschmidt's number could they have used an argument that formed a severe problem for atomists: Gibbs' paradox of the missing entropy of self-mixing of a gas. Today it is usually countered by referring to the indistinguishability of molecules of the same kind, although the argument requires more, namely the *identity* of states resulting from permutations. Such an identity would be in conflict with the concept of particles with their individual trajectories, while a field with two bumps at points x and y would trivially be the *same* as one with bumps at y and x. Although we are using quite novel theories today, such conceptual subtleties do remain essential (see Sect. 5).

Another object affected by the early dispute about particles and waves is light. According to its potential of being absorbed and emitted, light was traditionally regarded as a "medium" rather than a substance. Nonetheless, and in spite of Huygens' early ideas of light as a wave phenomenon in analogy to sound, Newton tried to explain it by means of "particles of light", which were supposed to move along trajectories according to the local refractive index of matter. This proposal was later refuted by various interference experiments, in particular those of Thomas Young in 1802. It remained open, though, what substance (called the ether) did oscillate in space and time – even after light had been demonstrated by Heinrich Hertz in 1886 to represent an electromagnetic phenomenon in accordance with Maxwell's equations. The possibility of these fields to propagate and carry energy gave them a certain substantial character that seemed to support the world of continua as envisioned by the energeticists. Regarding atoms, Ernst Mach used to ask "Have you seen one?" whenever somebody mentioned them to him. Later in this article I will argue that his doubts may still be justified today – even though we *seem* to observe individual atoms as particles.

At the end of the nineteenth century, the continuum hypothesis suffered a number of decisive blows. In 1897, J. J. Thomson discovered the elementary electric charge; in 1900, Max Planck postulated his radiation quanta with great success for the electromagnetic field; and in 1905, Albert Einstein estimated the value of Loschmidt's number N_L by means of his theory of Brownian motion. Thereafter, even the last energeticists resigned. Einstein even revived the concept of particles of light (later called photons) – although he regarded it merely as a "heuristic point of view" that he was never ready to fully accept himself. For a long time, Plancks radiation quanta were indeed attributed to a discrete emission process rather than to the radiation itself. So in 1913, Niels Bohr replaced the concept of classical motion for atomic electrons by stochastic "jumps" between discrete orbits – in accordance with Planck's and Einstein's ideas about a probabilistic radiation process. These early ideas later led to the insufficient interpretation of quantum mechanics as no more than stochastic dynamics for otherwise classical particles.

However, the development soon began to proceed in the opposite direction again.[2] In 1923, Louis de Broglie inverted Einstein's speculative step from light waves to photons by postulating a wave length $\lambda = c/\nu = h/p$ for the electron, where p is its momentum, in analogy with Planck's relation $E = pc = h\nu$. For him, this could only mean that all microscopic objects must consist of both, a particle *and* a wave, whereby the wave has to serve as a "guiding field" or "pilot wave" for the particle. This field had to be more powerful than a conventional force, since it would determine the velocity rather than merely the acceleration; the initial velocity can according to this proposal not be freely chosen any more. This theory was later brought into a consistent form by David Bohm. Thereby it turned out that the assumed pilot wave cannot be defined in space ("locally"), since it has to be identified with the global entangled wave function to be described in Sect. 4.

2. Wave Mechanics

Inspired by de Broglie's ideas, Schrödinger based his novel wave mechanics of 1926 on the assumption that electrons are *solely* and uniquely described by wave functions (spatial fields, as he first thought). His wave equation allowed him to explain the hydrogen spectrum by replacing Bohr's specific electron orbits in the atom by standing waves (energy eigenstates).

For a special case, the harmonic oscillator, he was furthermore able to construct stable "wave packets" that may imitate particles (see the Figure below for the case of *free* motion, however). Shortly thereafter, interference phenomena in agreement with de Broglie's wave length were observed by Davisson and Germer for electrons scattered from crystal lattices. A wave function can furthermore penetrate a potential barrier and thus explain "quantum tunneling", required for the possibility of α-decay. Does this not very strongly indicate that electrons and other "particles" are in reality just wave packets of some fields that obey Schrödinger's wave equation?

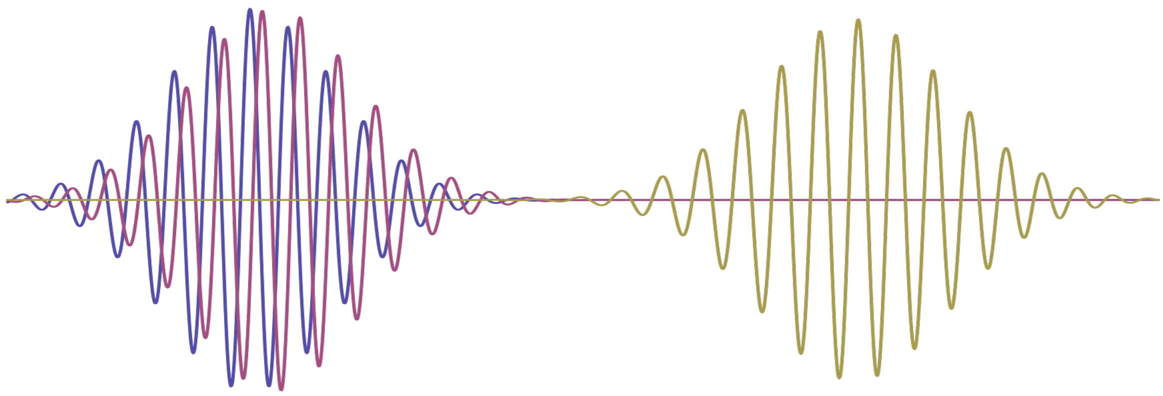

Figure: Real part of a one-dimensional wave packet (the product of a Gaussian with a plane wave $e^{2\pi ix/\lambda}$) moving freely according to the time-dependent Schrödinger equation, depicted at three different times (blue: t=0, red: t=0.04, yellow: t=1 in relative units). If the wave packet describes reality, its size defines a "real uncertainty" for the apparent particle position; it neither represents incomplete information, nor is it related to the measureable particle size (which has to be described by its *internal* degrees of freedom – see Sect. 4). When comparing blue and red, one recognizes that the packet moves faster than its wave crests, while the yellow curve demonstrates a slight dispersion of the packet (in contrast to that of the mentioned harmonic oscillator). The center of the packet moves according to the group velocity $v = p/m := h/m\lambda$, where the mass m is just a parameter of the wave equation. For this reason, momentum is in wave mechanics *defined* by h/λ (not by motion!), although it is mostly *observed* by means of the velocity of such moving "objects". It can be measured even for plane waves (which would not define a group velocity) by means of a conservation law for the sum of wave numbers $k = 2\pi/\lambda$ (giving rise to "momentum transfer") that holds during interactions with other waves that do exist in wave packets, such as Brownian particles. Already for atomic masses and thermal velocities, the de Broglie wave length is clearly smaller than the radius of a hydrogen atom. So one may construct quite narrow wave packets for their center of mass (cms) wave functions. Although the dispersion of the wave packet is reduced with increasing mass m, it becomes always non-negligible after a sufficient time interval. In order to compensate for it, one would need a new dynamical mechanism that permanently reduces the "coherence length" characterizing a packet in order to retain the appearance of a particle (see for "collapse" or "decoherence" in Sect. 4).

A few months before Schrödinger invented his wave mechanics, Heisenberg had already proposed his matrix mechanics. In contrast to Schrödinger, he did not abandon the concept of particles, but in a romantic attempt to revive Platonic idealism and overcome a mechanistic world view, combined with an ingenious guess, he introduced an abstract formalism that was to replace the concept of deterministic trajectories by formal probabilistic rules. Together with Born and Jordan, Heisenberg then constructed an elegant algebraic framework that could be used to "quantize" all mechanical systems. This mathematical abstraction perfectly matched Heisenberg's idealistic philosophy. Later, matrix mechanics was indeed shown to lead to the same observable predictions as wave mechanics when probabilistically applied to closed systems. A year after his first paper, Heisenberg supplemented his formalism by his uncertainty relations between position and momentum of an electron or other "conjugate" pairs of variables. However, such a fundamental uncertainty is clearly in conflict with a consistent concept of particles, while in wave mechanics it would simply be a consequence of the Fourier theorem – without any uncertainty *of the wave function* or an assumption of an unavoidable "distortion" of the electron state during a measurement (as originally suggested by Heisenberg). Another indication of the choice of inappropriate concepts may be the requirement of a "new logic" for them. So it is not surprising that Schrödinger's intuitive wave mechanics was preferred by most atomic physicists – for a short time even by Heisenberg's mentor Max Born. For example, Arnold Sommerfeld wrote only a "Wellenmechanischer Ergänzungsband" to his influential book "Atombau und Spektrallinien".

Some important phenomena, though, remained in conflict with Schrödinger's theory. While his general wave equation $i\hbar\partial\psi/\partial t = H\psi$ would allow various time-dependent solutions, such as the moving wave packets of the figure, bound electrons are usually found in standing waves (energy eigenstates). The latter are solutions of the stationary Schrödinger equation $H\psi = E\psi$ that gives rise to the observed discrete eigenvalues E. Although this equation can be derived from the general one under the assumption of a special time dependence of the form $\psi \propto e^{-iEt/\hbar}$, no general reason for this special form was evident. Instead of obeying the time-dependent wave equation, these eigenstates seemed to be dynamically connected by Bohr's stochastic "quantum jumps", which would thus explain energy quanta of radiation and the hydrogen spectrum by the conservation of energy. Similarly, wave functions seem to "jump" or "collapse" into particle-like narrow wave packets during position measurements. In a Wilson chamber, one could even observe tracks of droplets that can be regarded as successions of such position measurements along particle trajectories.

As a consequence, Schrödinger seemed to resign when Max Born, influenced by Wolfgang Pauli, re-interpreted his new probability postulate, which was originally meant to postulate jumps between different wave functions, in terms of probabilities for the spontaneous *creation of particle properties* (such as positions or momenta). This interpretation turned out to be very successful (and earned Born a Nobel prize) even though it was never quite honest, since the wave function does *not only* describe probabilities. It is also required to define individual observable properties of microscopic objects, such as energy or angular momentum eigenvalues, by means of corresponding "eigenstates" whose spatial structure can often be individually confirmed. Similarly, a spinor (a generalized wave function for the electron spin) describes probabilities for other *individual* spinor states rather than for classical properties.

The impossibility to derive the wave function from any reasonable assumption of "uncertain" *particle* properties (although the opposite is true) was so painful for Heisenberg that he regarded it as "a new form of human knowledge as an intermediary level of reality", while Bohr introduced his, in his own words "irrational", principle of complementarity. It required the application of mutually exclusive ("complementary") classical concepts, such as particles and waves, to the same objects. No doubt – this was an ingenious pragmatic strategy to avoid many problems, but from there on the search for a consistent description of Nature was not allowed any more in microscopic physics. As an answer to the question whether the electron be *really* a wave or a particle (or what else), Bohr insisted that "there is no microscopic reality" – a conclusion that was often regarded as philosophically very deep. Only few dared to object that "this emperor is naked", and the term "complementarity" no more than a name for a conceptual inconsistency. The large number of philosophical or formal "explanations" of this concept in the literature is even the more impressive. In particular, it has always remained open when and where precisely the probability interpretation (or the "Heisenberg cut" between wave functions and classical concepts) has to be applied. Therefore, the Hungarian Eugene Wigner spoke of a "Balkanization of physics" – a traditional (Hapsburgian) expression for the deterioration of law and order.

3. Wave Functions in Configuration Space

So one should take a more complete look at Schrödinger's wave mechanics. When he formulated it, he used Hamilton's partial differential equations as a guiding principle. These equations, the result of a reformulation of classical mechanics, are solved by a function that would describe a whole continuum of independent classical trajectories which differ by their initial conditions – sort of a wave function without interference. Hamilton was mainly interested in the elegant mathematical form of this theory rather than in applications. This turned out to be an advantage for Schrödinger. He assumed that Hamilton's equations were no more than a short wave lengths approximation (corresponding to the limit $h \to 0$) of a *fundamental* wave theory – similar to the approximation of geometric optics that could be applied to Maxwell's theory in order to describe ensembles of trajectories for apparent particles. With respect to Heisenberg's particle concept for the electron, he later remarked ironically that even Newton's particles of light would have been compatible with the observed interference phenomena if one had claimed some "uncertainty relations" for them. However, the short wave length approximation means only that local parts of an extended wave propagate independently of one another roughly along trajectories – not that they represent particles. Similarly, Feynman's path integral represents a propagating wave, while it neither requires nor justifies the existence of individual paths or positions that might then be selected by a mere increase of information.[3] Different partial waves or Feynman paths can interfere with one another (or act coherently) if focused in configuration space. This means that they exist together as *one reality* (one wave function) rather than merely defining a statistical ensemble of *possibilities*.

While light waves propagate in three-dimensional space, Hamilton's waves must according to their construction exist in the configuration space of all possible classical states of the system under consideration. Therefore, Schrödinger, too, obtained wave functions on (what appears to us classically as) configuration spaces of various dimensions. Later, this turned out indeed to be the correct version of his wave mechanics. It can also be understood as a consequence of Dirac's fundamental superposition principle, since the superposition of all classical configurations defines precisely a wave function on configuration space. This concept of a wave function can easily be generalized to include properties that never occur as classical variables (such as spin, neutrino flavor, or even the difference between a *K*-meson and its antiparticle), whose superpositions may again define new *individual* physical states. Dirac himself understood his superpositions in Born's sense as "probability amplitudes" for properties that are formally represented by Heisenberg's "observables", that is, not only for *points* in configuration space (classical states). If these observables are written in terms of dyadic products of their eigenstates (their spectral representation), Born's probabilities can be described as those for jumps of the wave function (stochastic projections in Hilbert space as part of the dynamics). Any proposal for some more fundamental theory would have to *explain* the general applicability of the superposition principle in some sense.

Schrödinger was convinced of a reality in space and time, and so he originally hoped, in spite of the Hamiltonian analogy, to describe the electron as a spatial *field*. Therefore, he first restricted himself with great success to single-particle problems (quantized mass points, whose configuration space is isomorphic to space). Consequently, he spoke of a "ψ-field". Such a spatial wave function can also be used to describe scattering problems – either applied to the center-of-mass wave function of an object scattered from a potential, or to the relative coordinates of a two-body problem. In scattering events, Born's probability interpretation is particularly suggestive because of the usual subsequent position measurement in a detector. This wave function in space is in general meant when one speaks of the *wave-particle dualism*. In spite of its shortcomings, three-dimensional wave mechanics still dominates large parts of most textbooks because of its success in correctly and simply describing many important single-particle aspects, such as atomic energy spectra and scattering probabilities. This limited and hence misleading approach is often supported by presenting the two-slit experiment as *the* key to understanding quantum mechanics (although it is only part of the story).

The generalization (or rather the return) to wave functions in configuration space happened almost unnoticed at those times of great confusion – for some physicists even until today. Although most of them are now well aware of "quantum nonlocality", they remain used to arguing in terms of spatial waves for many purposes. In contrast to fields, however, even single-particle wave functions do not describe additive (extensive) charge or energy distributions, since each piece cut from a plane wave representing a quantum "particle", for example, would describe its full charge and kinetic energy (which is given by the wave number).

Initially, Schrödinger took great pains to disregard or to re-interpret his general wave equation in configuration space, even though it is precisely its application to oscillating field amplitudes rather than mass points that explains Planck's radiation quanta $h\nu$. (Another early example is the rigid rotator, whose wave function depends on the Euler angles.) The spectrum $E = nh\nu$ that one obtains for quantum oscillators q_i (here the amplitudes of fixed field modes that classically oscillate in time with different frequencies ν_i) is known to be proportional to the natural numbers n. Only therefore does it define a concept of *energy quanta* $h\nu_i$ – later identified with photons – regardless of any emission process that had often been made responsible for the quanta. In Schrödinger's wave mechanics, quantum numbers n are defined by the numbers of nodes of the wave functions, which have to obey certain boundary conditions. These nodes have to be distinguished from the *spatial* nodes of the given field mode itself, such as $sin(\boldsymbol{k}_i\boldsymbol{x})$ multiplied with a polarization vector, which may be identified with a "photon wave function" – see Sect. 5.

But where can one find these wave functions if not in space? In contrast to the figure, they are here defined as functions on the abstract configuration space of field amplitudes q_i. Different eigenmodes of a *classical* field $q(\boldsymbol{x},t)$, such as plane waves with their classical frequencies ν_i, can fortunately be quantized separately; their Hamiltonians commute. This means that energy eigenfunctions Ψ for the total quantum field factorize in the form $\Psi=\Pi_i\psi_i(q_i)$, while their eigenvalues simply add, $E = \Sigma_i E_i$. Although the oscillator spectrum $E_i = n_i h\nu_i$ can also be derived formally from Heisenberg's algebra of observables (matrix mechanics) without explicitly using wave functions, the latter's nodes for a fixed field mode q_i have recently been made visible for the first time for various "photon" number eigenstates in an elegant though not quite sufficiently interpreted experiment.[4] The wave functions $\psi_i(q_i)$ on configuration space have thus been confirmed to *exist*, although they cannot be attributed to the traditional "wave-particle" dualism, which would refer to *spatial* waves characterizing "quantum particles". The importance of this fundamental experiment for the wave-particle debate has in my opinion not yet been appropriately appreciated by the physics community or in textbooks (see Sect. 5 for further details).

The difference between Schrödinger's theory and a classical field theory becomes particularly obvious from the fact that the amplitudes of a classical field now appear as *arguments q* in Schrödinger's wave function. Positions occur here only as an "index" that distinguishes field amplitudes at different space points from one another, as they form a spatial continuum of *coupled* oscillators. Since classical fields are usually written as functions on space and time, $q(\boldsymbol{x},t)$, the confusion of their spatial arguments with dynamical quantum variables (particle positions in quantum mechanics) has led to the questionable concept of a "time operator" for reasons of relativistic space-time symmetry (that cannot be manifest in the canonical formalism). However, $\boldsymbol{x}$ and t are here both classical coordinates, while spacetime distances become dynamical variables only as part of the spatial metric of general relativity – see Sect. 6. While a general time-dependent "one-photon wave function" can be understood as a quantum superposition of various spatial field modes (such as different plane waves) that are in their first excited quantum state ("occupied once" – with all others in their ground state), a quasi-classical *field* state has in QFT to be described as a coherent superposition of many *different* excitations $\psi_i^{(n)}(q_i,t)$ (different "photon numbers" n) for each spatial eigenmode i. In contrast to the free wave packet shown in the figure, these "coherent oscillator states" (time-dependent Gaussians, here as functions of the field amplitude) preserve their shape and width *exactly*, while their centers follow classical trajectories $q_i(t)$. For this reason, they imitate oscillating classical fields much better than wave packets in space may imitate particles.

Field functionals Ψ can thus represent classically quite different concepts, such as "particle" numbers and field amplitudes, mutually restricted by a Fourier theorem. For this reason, the *Boltzmann distribution* $e^{-E/kT}$ of their energy eigenstates may describe the Planck spectrum with its particle and wave limits for short and long wavelengths, respectively.

4. Entanglement and Quantum Measurements

Before trying to study *interacting* quantum fields (see Sect. 5), early quantum physicists successfully investigated the quantum mechanics of non-relativistic many-particle systems, such as multi-electron atoms, molecules and solid bodies. These systems could *approximately* be described by means of different (orthogonal) single-particle wave functions for each electron, while the atomic nuclei seemed to possess fixed or slowly moving positions similar to classical objects. For example, this picture explained the periodic system of the chemical elements. On closer inspection it turned out – at first for atoms and small molecules – that *all* particles forming such objects, including the nuclei, have to be described by one common wave function in their *3N*-dimensional configuration space. This cannot generically be a product or determinant of single-particle wave functions – a consequence that must be extended to all composite systems (including the whole quantum universe), and became later known as "entanglement". Similar entanglement must in general exist in QFT between different wave modes q_i when considered as subsystems. David Bohm referred to this property of the wave function as "quantum wholeness", when he began to study its consequences for his theory of 1952, since generically, the state of the whole does not define states of its parts. This is the reason why quantum theory can be consistently understood *only* as quantum cosmology (Sect. 6), while historically, entanglement was mostly misunderstood in a quite insufficient classical picture as a statistical or dynamical correlation between subsystems.

Every physics student is using the entanglement between an electron and a proton in the hydrogen atom when writing the wave function as a product of functions for center-of-mass and relative coordinates. The wave function for the latter defines the size of the hydrogen atom, for example, while the center of mass may be described by a free wave packet as in the Figure. The simplest nontrivial case, the Helium atom, was first successfully studied in great numerical detail by Hylleraas, using variational methods, in a series of papers starting in

1929. Already Arnold Sommerfeld noticed in his *Wellenmechanischer Ergänzungsband* that "Heisenberg's method", which used only the anti-symmetrization of product wave functions by means of "exchange terms", is insufficient. Entanglement is indeed often confused with (anti-) symmetrization, since it is formally equivalent to an entanglement between physical properties and meaningless particle numbers. It merely eliminates the concept of *distinguishability*, and is therefore not required any more in the occupation number representation (see Sect. 5).[†] Genuine entanglement means, for example, that one has to take into account "configuration mixing" as a correction to the independent-particle (Hartree-Fock or mean field) approximation. In the case of long-range interactions, this entanglement may be small in the ground state, since according to the independent-particle picture it describes "virtual excitations" (which are often misinterpreted as "fluctuations" rather than static entanglement).

An important consequence of entanglement is that subsystem Hamiltonians are in general not well defined – thus ruling out local unitarity or an exactly defined Heisenberg or interaction picture for open systems. *Closed* non-relativistic *N*-particle systems, on the other hand, have to be described by *one* entangled wave function in their complete configuration space. Their center-of-mass wave functions may then factorize from the rest, thus leading to free *spatial* wave functions for them (identical to those for mass points or "quantum particles"), while the internal energy quantum numbers are given by the number of nodes (or zeros: now defining hypersurfaces) in the remaining $3(N-1)$-dimensional configuration space. For non-inertial motion, this separability is only approximately valid – depending on the required internal excitation energy.

However, how can the space of all possible classical configurations, which would even possess varying dimensions, replace three-dimensional space as a new fundamental arena for the dynamics of wave functions that may represent physical states? If our Universe consisted of *N* particles (and nothing else), its configuration space would possess $3N$ dimensions – with *N* being at least of the order of 10^{80}. For early quantum physicists – including Schrödinger, of course – such a wave function was inconceivable, although the concept of a space of *possible configurations* fits excellently with Born's probabilities for classical properties. Entanglement can then conveniently be understood as describing statistical correlations between measured variables. But only between *measured* variables! Since macroscopic variables are "perma-

† *Separate* (anti-)symmetrization of spin and orbit parts, however, may lead to real entanglement. The statement "particle at position x_1" (rather than "particle number 1") "has spin-up" – as in a Bell type experiment – *is* physically meaningful.

nently measured" by their environment (see below for decoherence), their entanglement does indeed always appear as a statistical correlation. Only this explains why we are used to interpret the space on which the wave function is defined as a "configuration" space. In the mentioned case of the Helium atom, though, entanglement is responsible for the precise energy spectrum and other *individual* properties – regardless of any statistical interpretation. This conceptual difference is often simply "overlooked" in order to keep up the illusion of a merely epistemic interpretation of the wave function (where probabilities would reflect incomplete information). Even in individual scattering events one often needs entangled scattering amplitudes with well defined phase relations between all fragments. Only after Einstein, Podolsky and Rosen (EPR) had shown in 1935 that the entanglement between two particles at a distance may have non-trivial observable consequences, did Schrödinger regard this property as the greatest challenge to his theory – although he kept calling it a "statistical correlation". EPR had indeed erroneously concluded from their analysis that quantum mechanics cannot represent a complete description of Nature, so that as yet unknown ("hidden") variables should be expected to exist.

Although many physicists speculated that such hypothetical hidden variables might never be observed in an experiment (even though they might exist), it came as a surprise to them when John Bell showed in 1964 that *any* kind of hidden local reality (no matter whether it consists of particles, fields or other local things with local interactions only – observable or not) would be in conflict with certain observable consequences of entangled wave functions. In order to prove this theorem, Bell used arbitrary local variables λ (just a name for something not yet known) for an indirect proof. However, most physicists had by then become so much accustomed to Bohr's denial of a microscopic reality that they immediately accused Bell for having used a "long refuted assumption". The Copenhagen interpretation does indeed clearly go beyond a merely epistemic understanding of the wave function, since, insofar as it refers to ensembles at all, the latter are only meant in a purely formal sense – not in terms of any *elements* (those hidden variables) which would in principle answer the question "Information about what?" In this "operational" approach (supported by Günther Ludwig, for example), the essential problem is therefore "solved" by simply not considering it.

Crucial experiments regarding entanglement had in practice to be restricted to two- or few-particle systems which could be treated as being isolated from the rest until they are measured. They have always confirmed its consequences within a statistical interpretation, but physicists are still debating whether this result excludes locality (in three-dimensional space) or any kind of microscopic reality. For neither those who accept a nonlocal wave function as representing reality nor those who are ready to live without any microscopic reality feel bothered by Bell's theorem. These two camps usually prefer the Schrödinger picture (in terms of wave functions) or the Heisenberg picture (in terms of observables), respectively, and this fact seems to be the origin of many misunderstandings between them. While entanglement, which is generically required by the superposition principle, may also be defined locally, all observable conflicts with the assumption of a local reality are a consequence of *nonlocal entanglement* (a property of the wave function in configuration space).

If one does, therefore, assume the superposition principle to apply universally, one is forced to accept one entangled wave function for the whole universe. Heisenberg and Bohr assumed instead that the wave function is no more than a calculational tool, which "loses its meaning" after the final measurement that concludes an experiment. This "end of the experiment" (the "Heisenberg cut") remains vaguely defined and *ad hoc*, but, if applied too early, it may even exclude the now well established decoherence process (see below). An ontic universal wave function that always evolves according to the Schrödinger equation, however, leads to an entirely novel world view that appears inacceptable to most traditional physicists. For example, if one measures a microscopic object that is initially in a superposition of two or more different values of the measured variable, this gives rise to an entangled state for the microscopic system and the apparatus – the latter including Schrödinger's infamous cat if correspondingly prepared. Since such a state has never been observed, one traditionally assumes according to von Neumann that Schrödinger's dynamics has to be complemented by a stochastic "collapse of the wave function" into a product of narrow wave packets for all macroscopic or mesoscopic variables (such as pointer positions). This hypothetical new dynamics would also permanently re-localize the spreading free wave packet of the Figure. Note that, in the Schrödinger picture, Heisenberg's "observables" are readily defined by the interaction Hamiltonian between system and apparatus rather than forming a fundamental ingredient of the theory; no "eigenfunction-eigenvalue link" is required, since there are no fundamental classical "values" at all. In the Copenhagen interpretation, one would instead pragmatically jump from a description in terms of wave functions to one in classical terms, and back to a new wave function in order to describe a subsequent experiment. Since no collapse dynamics has ever been confirmed in an experiment, though, this unsatisfactory situation is known as the *quantum measurement problem*.

If one is ready, instead, to accept a universally valid Schrödinger equation as describing reality, one must try to understand what an entangled wave function for the microscopic system plus an apparatus might mean. Toward that end one has to include the observer into this description.[5] When he reads off the measurement result, he does himself become part of the entanglement. According to the unitary dynamics, he would thereafter simultaneously exist in different states of awareness – similar to the fate of Schrödinger's cat. Hugh Everett first dared to point out in 1957 that this consequence is not in conflict with our subjective observation of *one* individual outcome, since each "component state" (or "version") of the observer can remember only that outcome which is realized in the corresponding "relative state" of the world (which would also contain consistent versions of all his "friends"). As there must then be many such component states in one *global* superposition, the question which of them contains the "true" successor of the physicist who prepared the experiment has no unique answer: according to the unitary dynamics they all do. But why can these components be regarded as separate "worlds" with separate observers? The answer is that they are dynamically "autonomous" after the measurement in spite of their common origin; each of them describes a quasi-classical world for its macroscopic variables (see the discussion of decoherence below). In contrast to identical twins, however, who also have one common causal root, these different versions of the "same" observer cannot even communicate any more, and thus can conclude each others existence only by means of the dynamical laws they may happen to know. This is certainly an unusual, but at least a consistent picture, and a straightforward consequence of the Schrödinger equation. It only requires a novel but dynamically justified kind of states of individual (subjective) observers that is consistent with a nonlocal wave function under local interactions. Attempts to avoid this conclusion are all based on tradition, and would lead back to the measurement problem.

Until recently one did, therefore, generally believe that some conceptual or dynamical border line between micro- and macrophysics must exist – even though it could never be located in an experiment. Otherwise it should be possible (so it seemed) to observe individual consequences of entanglement between microscopic systems and their macroscopic measurement instruments – similar to the energy or other properties of Hylleraas's entangled Helium atom or of small molecules. However, this bipartite entanglement is not yet realistic. Macroscopic systems must inevitably, very fast, and in practice irreversibly interact with their natural "environment", whereby the entanglement that had resulted from the measurement proper would uncontrollably spread into the "rest of the universe". This happens even before an observer possibly enters the scene. In this way, one may understand how a superposition that extends over different macroscopic pointer positions, for example, would, from the point of view of a potential local observer, inevitably be transformed into an "apparent ensemble" of narrow wave packets which mimic classical states (points in configuration space). Although still forming one superposition, the members of this apparent ensemble of partial waves, which must include any observers, have no chance to meet again in high-dimensional configuration space in order to have coherent consequences for an observer. In this sense they can now be *regarded as forming an ensemble* of different worlds.

This unavoidable entanglement with the environment (that defines the true border line between micro- and macrophysics) is called decoherence,[6] as predominantly phase relations defining certain quantum mechanical superpositions become unavailable – that is, they are irreversibly "dislocalized".[‡] As Erich Joos and I once formulated it, the superposition still exists, but – in contrast to traditional terminology – it "is not there" (somewhere) any more. The time asymmetry of the decoherence process requires a specific cosmic initial condition for the wave function, related to that for the thermodynamic arrow of time.[7] However, without Everett's consequence of splitting local observers ("many minds") as the other non-trivial consequence of universal unitarity, decoherence would not be able to explain the observation of any *individual* measurement outcomes. It has therefore occasionally been claimed to be insufficient to solve the quantum measurement problem, although the subsequent branching of subjective observers amounts for the latter essentially to what Pauli once called the "creation of measurement results outside the laws of Nature"; it is now described as a dynamical consequence of global unitary dynamics *on the observer himself*. Pauli (just as all physicists

---

‡ A mere phase *randomization* ("dephasing") could neither occur under unitary evolution, nor would it solve the issue, as each individual member of an ensemble of local superpositions with different phases would still be a superposition (though possibly with *unknown* phase). A similar argument applies to local phases that are merely assumed to fluctuate rapidly in time because of *stochastic perturbations by* the environment. Nonetheless, phase averaging still forms the most popular misunderstanding of decoherence, which requires entanglement with an uncontrollable environment in the *individual* case. These different concepts are easily confused, in particular, if the initial environment is described as a "thermal bath". However, if such a thermal "mixture" had been caused by earlier *quantum* interactions with a further environment (which is the most plausible assumption), the thus pre-existing entanglement would simply be dynamically extended to the now "dephased" variables, where it would then again lead to their genuine decoherence (a dislocalization of their *individual* relative phases). An application of the reduced density matrix formalism would instead tacitly replace nonlocal entanglement by local ensembles. It is remarkable that many important physicists are still missing the point of decoherence as a "globalization of superpositions", that is, a consequence of the fundamental nonlocality of quantum states. – Historically, the term "decoherence" was invented in the context of "decoherent histories" in about 1985, where it was *assumed* in order to justify "consistent histories" in a probabilistic context, while my approach of 1970 was *derived* from the assumption of a universal validity of unitarity. Ironically, it is precisely this consequence that had led to the prejudice that quantum concepts do *not* apply to the macroscopic world.

at his time) simply did not properly take into account the environment and the role of the observer in a consistent quantum setting.

The experimental confirmation of decoherence as a smooth dynamical process demonstrates that the concept of entanglement does apply beyond microscopic systems. Although this process must remain uncontrollable in order to be irreversible ("real" rather than "virtual"), it has many obvious and important consequences – including apparent quantum jumps and the classical appearance of the world. It also explains why we seem to observe individual atoms as apparent particles in a Paul trap, or tracks in a Wilson chamber as apparent particle trajectories (both are correctly described in terms of narrow wave packets), and why one finds bound microscopic systems preferentially in their energy eigenstates.[6,8] It also allows us to understand the mysterious concept of "complementarity" by the different entanglement of microscopic objects with the environment caused by means of different measurement instruments. Such a choice of "complementary measurement devices" is not available any more for systems that are already strongly entangled with their environment before being measured. The basis "preferred" by such a normal environment defines a quasi-classical *configuration* space for these systems, which may even include major parts of the thus quasi-classical observers (such as human beings). Although *virtual* decoherence had always been known as a consequence of microscopic (reversible and often usable) entanglement, the unavoidable and irreversible effect of the environment on macroscopic systems was overlooked for five decades, mainly since quantum mechanics was assumed *not* to apply beyond microscopic systems. Surprisingly, the apparently reversible dynamics of classical mechanics does in quantum mechanics require the permanent action of irreversible decoherence.

In order to illustrate the enormous number of new "worlds" that are permanently created according to this picture (or must otherwise be permanently annihilated by a collapse mechanism!), let me consider the example of a two-slit experiment. Measuring which slit the "particle" passes would just double the number of worlds, but registration of the particle on the second screen causes a multiplication of worlds by a large factor that depends on the remaining coherence lengths for the positions of the arising spots. (Everett "worlds" are not *exactly* defined, and thus cannot simply be counted; they may even form an overcomplete set.) This understanding of branches means also that quantum computers do *not* simultaneously calculate in parallel worlds (as sometimes claimed) if they are to produce a coherent result that may then be used in "our" world, for example; "real" branches never recohere.

Most “particles” in the two-slit experiment do not even pass the slits, but may be absorbed on the first screen. This absorption corresponds to a position measurement, too – regardless of whether its information is ever extracted. In order to cause decoherence, the “information” may even be thermalized (erased). In contrast, a “quantum eraser” requires a local superposition to be *restored*, that is, re-localized, rather than information to be destroyed, as the name may suggest. Similar considerations apply to all irreversible scattering events between “particles” or between other objects and their environments. For M such measurement-like events in the past history of the universe with, on average, N effective outcomes, one would obtain the huge number of N^M existing branches. Nonetheless, the global configuration space remains almost empty because of its high dimension; the myriads of branching wave packets that have ever been created by *real* decoherence describe separate “worlds” for all reasonable times. Nobody can calculate such a global wave function in detail, but under appropriate (far from equilibrium) initial conditions for the universe, its unitary dynamics can be used consistently to justify (1) quasi-classical properties and behavior for all degrees of freedom that are “robust” under decoherence, (2) statistical methods (retarded probabilistic master equations) for most others,[7] and (3) individual wave functions for appropriately prepared microscopic systems. In the case of controllable non-local entanglement, this kind of preparation can even be applied at a distance – a phenomenon known as “quantum steering”. These three consequences are then also sufficient to construct measurement devices to begin with.

The observation of radioactive decay represents another measurement of a continuous variable (namely, the decay time). Its precision cannot be better than the remaining coherence time (which is usually very much smaller than the half-life, and thus gives rise to apparent quantum jumps). This coherence time depends on the efficiency of the interaction of the decay fragments with their environment, and it would be further reduced by permanent registration of the (non-) decay. If an excited state decays only by emission of weakly interacting photons, decoherence may be slow. One may then even observe interference between different decay times, thus definitely excluding genuine quantum jumps (“events”) in this case.

Many leading physicists who are not happy any more with the Copenhagen interpretation nonetheless prefer to speculate about some novel kind of dynamics (an as yet unknown collapse mechanism) that would avoid the consequence of Many Worlds. This is presently no more than wishful thinking, based on traditionalism, but it could in principle also solve the measurement problem in terms of an ontic (in this case partially localized) universal wave function without requiring Everett’s multiple observers. One should keep in mind, though, that *all* as yet observed apparent deviations from unitarity, such as quantum jumps or measurements, can be well described (and have in several cases been confirmed experimentally) as smooth decoherence processes in accordance with a global Schrödinger equation. Therefore, if a genuine collapse mechanism did exist after all, it would presumably have to be *triggered* by decoherence, but it could then hardly have any observable consequences by its own.

For example, if one of two spatially separated but entangled microscopic systems (such as those forming a "Bell state") was measured, their total state would according to a unitary description become entangled with the apparatus, too, and thus also with the latter's environment. Although autonomous branches have now objectively formed, an observer at the location of the second system, say, becomes part of this entanglement (and therefore "splits") only when he receives a signal about the result. Before this happens, his state factors out, and he may be said still to live identically in all these branches. If he then also measured the second system (at his own location), the state of his memory must depend on the outcomes of both measurements, that is, it must have "split" twice unless there was an exact correlation between the results. Since the order of these two measurements does not matter, in general, this description includes delayed choice experiments. In contrast to the unitary dynamics underlying this description, a genuine collapse caused by the measurement would have to *affect* distant objects instantaneously (whatever that means relativistically) in order to avoid other weird consequences. This would then define the "spooky" part of the story.

In this way, an apparent ensemble of quasi-classical "worlds" is for all practical purposes sufficiently defined by the autonomous branches of the wave function that arise from decoherence: a measurement cannot be undone any more as soon as the global superposition cannot be "re-localized" in practice, while reasonable observer states can exist only separately within different branches. However, neither is this definition of branches exact (as regards their precise separation), nor does it explain Born's rule, since all members of the apparent ensemble are assumed to remain parts of one "real superposition" (the "bird's perspective"). Most of them even describe frequencies of outcomes in series of measurements that are *not* in accordance with Born's rule. What we still need, therefore, is a probabilistic characterization of the permanently branching quasi-classical world in which "we" happen to live.

In all interpretations of quantum mechanics, Born's rule has to be postulated in some form. In principle, this remains true in the Everett interpretation, too, but the situation is now partly solved by decoherence, as the members of an effective ensemble (the branches) have already been sufficiently defined. All we still have to postulate for them on empirical grounds are “subjective” probability *weights*, since, because of their imprecise and time-dependent definition, the branches cannot simply and meaningfully be counted. The only appropriate candidate for consistent probability weights is their squared norm, as it is conserved by the unitary dynamics and, therefore, simply adds in the case of some finer specification of branches. (For example, further branching occurs during subsequent information processing on the retina and in the brain, or by “measurements” somewhere else in the universe.) Only this additivity of the weights gives rise to a concept of “consistent histories” and individual probabilities for apparent “collapse events”. Everett regarded this argument, which is similar to the choice of phase space volume as a probability measure in classical statistical mechanics, as *proof* of Born’s probabilities,[9] since frequencies of measurement outcomes in “most” branches (now defined by means of these weights) are compatible with them. Only *after* postulating these weights does the concept of a density matrix become justified as a tool.

By consistently using this global unitary description, all those much discussed “absurdities” of quantum theory can be explained. It is in fact precisely how they were all predicted – except that the chain of unitary interactions is usually cut off *ad hoc* by a collapse at the last “relevant” measurement in an experiment, where the corresponding decoherence defines a consistent position for the hypothetical Heisenberg cut. Therefore, all “weird” quantum phenomena observed during the last 80 years can only have surprised those who had never accepted the universal validity of the quantum formalism. Absurdities, such as “interaction-free measurements”, arise instead if one assumes the quasi-classical phenomena (such as apparent events) rather than the complete wave function to define “reality”. If the wave function itself represents reality, however, any “post-selected” component cannot describe the previously documented past any more, which would have to be the case if this post-selection was no more than an increase of information about some hidden reality.

So-called quantum teleportation is another example, where one can easily show, using the causal unitary description, that nothing is ever “teleported” that, or whose deterministic predecessor, was not prepared in advance at its intended position in one or more components of an entangled initial wave function.[8] This demonstrates again that nonlocal wave functions cannot describe “just information” – even though the subjective observer *may assume* that an objective global collapse into a non-predictable outcome had already occurred (or that this outcome had come into existence in some other way) as a consequence of the first irreversible decoherence process in a measurement. It is precisely this possibility that justifies the usual pragmatic approach to quantum mechanics (including the Copenhagen interpretation or von

Neumann's collapse during a measurement). However, if one presumes unknown *local* elements of reality (Bell's "beables") to objectively determine measurement outcomes, one has also to believe in teleportation and other kinds of spooky action at a distance. According to the Everett interpretation, the pragmatic restriction of "our quantum world" to one tiny and permanently further branching component of the universal wave function (the apparent collapse) represents no more than a convention rather than a physical process. Such a "collapse by convention" may then even be assumed to apply instantaneously (superluminally), but it should also be obvious that a mere convention cannot be used for sending signals.

If the global wave function does indeed evolve deterministically, the observed quantum indeterminism can evidently *not* represent an objective dynamical law. In Everett's interpretation, it is in principle a "subjective" phenomenon, based on the permanently branching histories of all observers into many different versions ("many minds"). This indeterminism of the observers nonetheless allows them to *prepare* definite pure states of microscopic systems in the laboratory as initial conditions for further studies by selecting the required outcomes in appropriately designed series of measurements. All measurement outcomes are objectivized between those versions of *different* observers (including Wigner's friend or Schrödinger's cat) who live in one and the same Everett branch, and thus can communicate with one another. However, regarding the wave function as representing some fundamental (extraphysical) "quantum information" would be meaningless or inconsistent with empirical facts.

5. Quantum Field Theory

We have seen that quantum mechanics in terms of a universal wave function admits a consistent (even though novel kind of) description of Nature, but this does not yet bring the strange story of particles and waves to an end. Instead of *spatial* waves (fields) we were led to wave functions on a high-dimensional "configuration space" (a name that is justified only because of its appearance as a space of *potential* classical states by means of decoherence). For a universe consisting of N particles, this configuration space would possess $3N$ dimensions, but we may conclude from the arguments in Sect. 3 that for QED (quantum electrodynamics) it must be supplemented by the infinite-dimensional configuration space of the Maxwell fields (or their vector potentials in the canonical formalism). A product of wave functions for the amplitudes of all field modes in a cavity or in free space turned out to be sufficient to explain Planck's quanta by the number of nodes of these wave functions. The spontaneous occurrence of photons as apparent particles (in the form of clicking counters, for example) is then merely a consequence of fast decoherence in the detector.

However, we know from the quantum theory of relativistic electrons that they, too, have to be described by a *quantized field* (that is, by a field functional) – a consequence that must then also apply to the non-relativistic limit. The relativistic generalization of a one-electron wave function is called the *Dirac field*, since it has to be regarded as a function on spacetime. Dirac proposed it at a time when Schrödinger's wave function was still believed to define a spatial field for each electron, but it can *not* be generalized to an N-electron field on a $4N$-dimensional "configuration spacetime", although this has occasionally been proposed; there is only one time parameter describing the dynamics of the whole field or its quantum state. In the Schrödinger picture of QED, the Dirac field is used to define, by its configuration space and that of the Maxwell field, the space on which the corresponding time-dependent wave functionals live. According to the rules of canonical quantization, these wave functionals have to obey a generalized Schrödinger equation again (the Tomonaga equation).[10]

This consequence of QFT avoids a fundamental N-dependence of the relevant configuration spaces for varying numbers N of "particles", as it allows for a concept of "particle creation", such as by raising the number of nodes of the field functional (cf. Sect. 3). Relativistic invariance cannot and need not be manifest in this formalism. For example, the canonical quantization of the Maxwell field leads consistently to a wave functional $\Psi\{\boldsymbol{A}(\boldsymbol{x});t\}$, with a vector field $\boldsymbol{A}$ defined at all space-points $\boldsymbol{x}$ on an arbitrary simultaneity t. Since Schrödinger had originally discovered his one-electron wave function by the same canonical quantization procedure (applied to a single mass point), the quantization of the Dirac field is for this purely historical reason also called a "second quantization". As explained above, though, the particle concept, and with it the first quantization, are no more than historical artifacts.[11]

Freeman Dyson's "equivalence" between relativistic field functionals (Tomonaga) and field operators (Feynman)[12] is essentially based on the (incomplete) equivalence between the Schrödinger and the Heisenberg picture. However, the Heisenberg picture would hardly be able even in principle to describe the hefty, steadily growing entanglement characterizing a time-dependent global wave function. Since relativity is based on the absence of absolute simultaneities, the relativistic generalization of the Schrödinger equation can indeed only be given by the Tomonaga equation with its "many-fingered" concept of time (arbitrary simultaneities). Apparent particle lines in Feynman diagrams, on the other hand, are merely short-hand for certain field modes (such as plane waves, with "particle momenta" representing their wave numbers).[3] These diagrams are only used as intuitive tools to construct terms of a perturbation series by means of integrals over products of such field modes and other factors, mainly for calculating scattering amplitudes. In this picture, closed lines ("virtual particles") describe entanglement between quantum fields. Since high-energy physics is mostly restricted to scattering experiments, unitarity is in many textbooks quite insufficiently explained as describing the "conservation of probability" – thus neglecting its essential consequence for the quantum phases, which may be needed to define superpositions after a scattering process.

The Hamiltonian form of the Dirac equation is unusual as a consequence of its linearization in terms of particle momentum insofar as the classical canonical momenta are not given by time derivatives of the position variables (velocities) any more. Nonetheless, the two occupation numbers 0 and 1 resulting from the *assumption* of anti-commuting field operators[§] are again interpreted as "particle" numbers because of their consequences in the quasi-classical world. Field modes "occupied" once in this sense and their superpositions define again "single-particle wave functions". In contrast to the case of photons, however, one does not observe superpositions (wave functionals) of *different* electron numbers. This has traditionally been regarded as a fundamental restriction of the superposition principle (an axiomatic "superselection rule"), but it may again be understood as a consequence of decoherence: for charged particles, their Coulomb field assumes the role of an unavoidable environment.[13]

---

§ Let me emphasize, though, that the Pauli principle, valid for fermions, does not seem to be entirely understood yet. While the individual components of the Dirac spinor also obey the Klein-Gordon equation, the latter's quantization as a field of coupled oscillators would again require *all* oscillator quantum numbers $n = 0,1,2,\ldots$. Anti-commuting field operators, which lead to anti-symmetric multi-particle wave functions, were postulated quite *ad hoc* by Jordan and Wigner, and initially appeared artificial even to Dirac. Interpreted rigorously, their underlying configuration space (defining a Hilbert space basis again) would consist of a spatial continuum of coupled bits ("empty" or "occupied") rather than a continuum of coupled oscillators. The *n*-th excited state of this bit continuum (that is, *n* occupied positions) would represent *n identical* point-like "objects". Because of the dynamical coupling between bit-neighbors, these objects can move, but only *after* their quantization, which leads to entangled superpositions of different occupied space points, may they give rise to propagating wave functions. The latter must vanish whenever two positions coincide in order to avoid space points to be occupied more than once. This can be achieved by means of antisymmetric wave functions, while the permutation of two occupied positions is now an identity operation ($|x,y\rangle = |y,x\rangle$). No field algebra is explicitly required for this conclusion. In this picture, single-fermion wave functions would represent genuine quantum states (quantum superpositions) rather than wave modes as for bosons. In contrast, coupled oscillators defining a free boson field propagate as spatial waves, and thus obey a *classical* superposition principle (in space rather than in their configuration space) in addition to the quantum superposition principle that is here realized by the field functionals. However, these pre-quantization concepts need not possess any physical meaning by themselves. Moreover, such a fundamental distinction between bosons and fermions may become problematic for *composite* "particles".

In QFT, the formulation that one particle is in a quantum state described by the spatial wave function ψ_1 and a second one in ψ_2 has thus to be replaced by the statement that the two *field modes* ψ_1 and ψ_2 are both in their first excited quantum state ("occupied once"). A permutation of the two modes does not change this statement that is based on a logical "and", so there is only *one* state to be counted statistically. This eliminates Gibbs' paradox in a very natural way. (Schrödinger seems to have used a similar argument in favor of waves instead of particles even before he explicitly formulated his wave equation.[14])

It would similarly be inappropriate to claim that *wave functions* can be directly observed in Bose-Einstein condensates (as is often done). What one does observe in this case are again the (now multiply "occupied") three-dimensional boson *field modes* – even though massive bosons are conventionally regarded as particles because of their appearance under normal conditions. Instead of the *free* field modes used for photons, however, interacting bosons are better described in terms of self-consistent field modes in analogy to the self-consistent Hartree-Fock single-fermion wave functions. Both cases lead to what is regarded as an effective non-linear "single-particle wave equation" – for bosons called the Gross-Pitaevskii equation.[**] In spite of this effective non-linearity, the quantum states proper are, of course, described by the linear Schrödinger equation – relativistically always in the sense of Tomonaga.[10]

As mentioned already in Sect. 3, photon number eigenfunctions $\psi^{(n)}(q)$ in the configuration space of wave amplitudes q – to be distinguished from their three-dimensional field

---

[**] At normal temperatures, "many-particle" systems (that is, multiple quantum field exitations) behave approximately like a gas of classical particles undergoing stochastic collisions because of their permanent mutual decoherence into apparent ensembles of narrow spatial wave packets. This consequence perfectly justifies Boltzmann's *Stosszahlansatz* – but *not* any quasi-deterministic particle trajectories for the wave packets, which would be approximately valid only for macroscopic objects or heavy particles that suffer mainly "pure" decoherence (with negligible recoil). "Open" quantum systems are generally described by similar phenomenological (Lindblad-type) master equations that are usually *postulated* rather than being derived from realistic assumptions for the environment, and often misunderstood as representing fundamental deviations from unitary quantum mechanics. In order to be treated as "macroscopic" in the sense of "always given" or "known" in a context of statistical mechanics, quasi-classical (decohered) variables have furthermore to be redundantly documented in the rest of the universe (see under "fork of causality", "consistency of documents", or "overdetermination of the past" in the first Ref. 7 – for example in its footnote 1 on page 18). In the theory of "quantum Darwinism",[24] these *classical* thermodynamical concepts are intermingled (and perhaps a bit confused) with the quantum concept of decoherence, which represents spreading physical entanglement, but not necessarily any spreading of (usable) information into the environment. Transfer of (necessarily physical) information must always cause decoherence, but the opposite is not true: even an environment in thermal equilibrium does allow entanglement to form.

modes (“single-photon wave functions”, which are fixed modes in a cavity in this case) – have recently been confirmed for various values of n to be part of reality by means of their Wigner functions.[4] For pure states, the Wigner functions are defined as partial Fourier transforms of the dyadic products $\psi^{(n)}(q)\psi^{(n)*}(q')$, and thus equivalent to the wave functions $\psi^{(n)}(q)$ themselves (except for a total phase). The variable q is here the amplitude of the given field mode rather than position as in particle quantum mechanics. The two-dimensional Wigner functions on their apparent phase space q,p were made visible in this experiment, and so allow one to clearly recognize the n nodes of the wave functions $\psi^{(n)}(q)$ (forming circles in phase space). Creation and annihilation operators are defined to change the number of these nodes. Since they occur dynamically only in the Schrödinger equation, they describe *smooth* physical processes (time-dependent wave functionals), while apparent “events” are either meant conceptually, or require a fast decoherence process. The “reality” of field functionals is also confirmed by their ability to form entangled states and, in this way, to contribute to the observable decoherence of their sources without interacting with any absorbing matter.

For relativistic reasons, *all* known elementary physical objects are described as quantum fields (although they are usually called “elementary particles”). The contrast between the first order in time of the Schrödinger equation and the second order of the classical field equations with their negative frequencies then opens the door to the concept of “anti-bosons”. (For fermions this relation assumes a different form – depending on the starting point before quantization, as indicated in the footnote on page 23.) Because of the universality of the concept of quantum fields, one may also expect a “theory of everything” to exist in the form of a unified quantum field theory. At present, though, the assumption that the fundamental arena for the universal wave function be given by the configuration space of some fundamental field(s) is no more than the most plausible attempt. On the other hand, the general framework of Schrödinger’s wave function(al) or Dirac’s superposition principle as a universal concept for quantum states which obey unitary dynamics has always been confirmed, while attempts to derive this framework from some deeper (“hidden”) level have failed and are strongly restricted by various no-go theorems.

Among boson fields, gauge fields play a surprisingly physical role, since gauge transformations appear locally as unphysical redundancies. Their physical role is facilitated by their dynamical entanglement, which thus reveals that the redundancy applies only classically. Gauge variables then appear as purely relational quantities.[15] An important question after quantization is whether gauge symmetries can be broken by a real or apparent collapse.

Unfortunately, *interacting* fields in general require the entanglement of such an enormous number of fundamental degrees of freedom – traditionally interpreted as "quantum fluctuations" even in time-independent states – that they cannot even approximately be treated beyond a questionable (though often successful) perturbation theory in terms of free effective fields. Instead of consistently applying the established concepts of quantum mechanics (namely, entangled superpositions) to the new variables (such as field amplitudes), various semi-phenomenological concepts are therefore used for specific purposes in QFT – mostly for calculating scattering amplitudes between objects that are assumed to be asymptotically free (which never happens for macroscopic objects, though). Stable local entanglement can be regarded as a "dressing" of fields (similar to the entanglement between proton and electron in the bound hydrogen atom – cf. Sect. 4), while chaotic and nonlocal entanglement defines decoherence. Only for individual field modes, as in cavity QED, may one explicitly calculate their entanglement, for example with individual atoms.

Even these semi-phenomenological methods are severely haunted by infinities resulting from local products of field operators that are assumed to appear in the effective Hamiltonians. The construction and interpretation of these methods is mostly based on particle concepts again (such as in Feynman diagrams, or by interpreting clicks and bubbles appearing in detectors as being caused by particles). Therefore, "effective" quantum fields cannot be expected to represent fundamental variables that might be revealed by mere "renormalization" procedures. This opens up quite novel possibilities, perhaps even to understand all fermions as quantum consequences of certain topological defects (such as superpositions of different locations of topological singularities – cf. the footnote on page 23 again).

Similar semi-phenomenological methods as in QFT are also used in condensed matter physics, even when its objects of interest are non-relativistically treated as N-particle systems. They may nontheless give rise to effective phonon fields or various kinds of "quasi-particles". In this description, the wave function for the lattice ions and their electrons, for example, is regarded as fundamental, while the phonon field functional is "emerging" – similar to Goldstone bosons in QFT. Symmetry-breaking effective ground states (such as lattices with fixed position and orientation) and their corresponding "Fock spaces" can be understood as representing Everett branches that have become autonomous by decoherence of their superpositions during the cosmic condensation process (symmetry breaking).[16] Some such "Fock vacua" are characterized by the number of certain particles (such as electrons in a metal) that form a *stable entanglement* in this ground state. Most familiar are pair correlations in the BCS model of superconductivity. A similar model in QFT led to the prediction of the Higgs "particle". However, only in situations described by an effective Hamiltonian that gives rise to an energy gap (defining an effective mass) can the lowest excited states approximately avoid further decoherence within the corresponding Fock space under normal conditions and low temperatures, and thus exhibit the usual phenomena of "single particle" quantum mechanics (cf. Sect. 4).

The BCS (pair correlation) model is appropriate also to understand Hawking and Unruh radiation, which are often interpreted again as describing vacuum fluctuations.[17] The presence of "particles" is here a matter of spacetime perspectives (using non-inertial reference frames for defining "plane" waves representing particles), while the abstract quantum *states*, such as various kinds of "physical vacua", remain *objectively* defined by their physical (such as cosmological) boundary conditions – and thus represent "real" states regardless of their interpretation in terms of particle numbers.

In *microscopic* many-particle systems, for example in small molecules or atomic nuclei, spontaneous *intrinsic* symmetry breaking may even lead to energy eigenstates for collective motions (such as rotations or vibrations). Since electrically neutral microscopic objects may be treated as being isolated from their environments, asymmetric "model ground states" (deformed nuclei or asymmetric, such as chiral, molecular configurations) may then form energy bands or multiplets by means of different superpositions of all their possible orientations.[18] Collective degrees of freedom are often classically visualized as describing slow ("adiabatic") motion, although this would require time-dependent superpositions of different energy eigenstates. Their justification for *macroscopic* objects, where they are indeed found, had to await the invention of the concept of decoherence. Since all particles in one microscopic object are here strongly entangled with one another, their symmetric superpositions are analogous to the bird's perspective of a quantum world, while an external observer of a collective energy eigenstate assumes the role of a "real bird" in this case. In contrast, the quantum world must *contain* its observer, who thus gives rise to "many minds" with their asymmetric frog's perspectives (broken symmetries).[16] In accordance with this analogy, individual particles contributing to collective rotational states are known to feel in first approximation only a *fixed* deformed potential (analogous to a definite measurement outcome), as can be seen from their single-particle spectra (for example Nielson states for deformed nuclei – a variant of the nuclear shell model). In this sense, collective superpositions imitate a "multiverse" of different orientations, but these quantum cosmological implications seem to have delayed the acceptance of the concept of decoherence until its "naïve" interpretation by means of the (insufficient) reduced density matrix formalism became popular and made it acceptable to pragmatists. In the case of a *global* symmetry, collective variables bear some similarity to gauge variables.

On a very elementary level, semi-phenomenological methods were even used already for the hydrogen molecule, namely by separately quantizing its "effective" degrees of freedom (center of mass motion, vibration, rotation and two independent electrons in the Coulomb field of adiabatically moving nuclei) rather than consistently treating it as an entangled four-body problem.

In QFT, the successful phenomenology of apparently fundamental fields ("elementary particles"), such as described by the Standard Model, may be expected to form the major touchstone for any fundamental theory of the future. This may be true even though quantum chromodynamics seems already to be too complex for us to derive nuclear physics phenomena without further assumptions. At present, this Standard Model, which basically consists of linear representations of some abstract symmetry groups whose physical meaning is not yet understood, does not seem to offer any convincing hints for the nature of an elusive fundamental theory. All one may dare to predict is that its Hilbert space must possess a local *basis* (such as the configuration space of spatial fields and/or point-like objects) in order to allow for a definition of dynamical locality or "relativistic causality". This search for a fundamental basis has nothing to do with that for "hidden variables" which were meant to explain quantum indeterminism itself. Novel theories that are solely based on mathematical arguments have to be regarded as speculative unless empirically confirmed – and even incomplete as long as there is no general consensus about the correct interpretation of their quantization. Many quantum field theorists and mathematical physicists seem to regard their semi-phenomenological models, combined with certain methods of calculation, and applied to classical field or particle concepts, as *the* quantum field theory proper. Indeed, why should one expect a consistent theory if there is no microscopic reality to be described – as assumed in the still popular Copenhagen interpretation and its variants? Therefore, most textbooks of QFT do not even *attempt* to present a conceptually consistent and universally valid theory.

Our conclusion that the observed particle aspect is merely the consequence of fast decoherence processes in the detecting media may understandably not be of particular interest for practicing high-energy experimentalists, but it seems to be unknown even to many theoreticians in this field. So they sometimes call the enigmatic objects of their research "wavicles", as they cannot make up their mind between particles and waves. This indifferent language represents another example for Wigner's "Balkanization of physics" (or "many words instead of many worlds" according to Tegmark). The concept of a wave-particle "dualism" is usually understood in the sense of *spatial* waves rather than wave functions in configuration space, although spatial waves should by now be known to be quite insufficient in quantum theory.

6. Quantum Gravity and Quantum Cosmology

I cannot finish this presentation of quantum theory without having mentioned quantum gravity. Although one cannot hope to observe quanta of gravity in the foreseeable future, the formal quantization of gravity can hardly be avoided for consistency in view of the quantization of all other fields. Its dynamical variables must then also appear among the arguments of a universal wave function, and thus be entangled with all other fields – in a very important way, as it turns out.[19]

The Hamiltonian formulation of Einstein's general relativity was brought into a very plausible form by Arnowitt, Deser and Misner in 1962. They demonstrated that the configuration space of gravity can be understood as consisting of the spatial geometries of all possible three-dimensional space-like hypersurfaces in spacetime. These hypersurfaces define arbitrary simultaneities that may form various foliations of spacetime, and which may then be parametrized by a time coordinate t. This Hamiltonian form of the theory is therefore also called "geometrodynamics". Its canonical quantization leads to a (somewhat ambiguously defined) Schrödinger equation in the sense of Tomonaga for the wave functional on all these geometries – known as the *Wheeler-DeWitt equation*. This is another example which demonstrates that the Hamiltonian form of a theory is not in conflict with its relativistic invariance.

In contrast to the normal Schrödinger equation, the WDW equation remarkably assumes the form $H\Psi = 0$. It can also be understood as a constraint, while the Schrödinger equation itself then becomes trivial: $\partial\Psi/\partial t = 0$. The reason is that there is no classical spacetime any more to be foliated. However, the spatial metric that occurs (besides matter variables) as an argument of the wave functional Ψ would determine all proper times ("physical times") along time-like curves which connect it classically (according to the Einstein equations) with any other conceivable spatial geometry, regardless of the choice of a foliation. Therefore, in spite of its formal timelessness, the Wheeler-DeWitt equation *does* define a physical time dependence by means of the entanglement between all its variables – similar to the entanglement $\psi(u,q)$ between a clock variable u and other variables q that defines a physical time dependence in quantum mechanical form. Therefore, the *formal* timelessness of the WDW equation is a genuine quantum property that reflects the absence of trajectories (here classical spacetimes). Classical theories are merely reparametrization invariant. In general, this *physical* time is again many-fingered, that is, it depends on the local progression of the space-like hypersurfaces independently at any space point. In the case of an exactly homogenous and isotropic Friedmann cosmology, it may be represented by just *one* single "finger": the expansion parameter a. If the wave function is regarded as a probability amplitude, it now defines probabilities *for* physical time; it is not a function *of* (some external) time any more.

It is further remarkable in this connection that, for Friedmann type universes, the Hamiltonian constraint $H\Psi = 0$ assumes a hyperbolic form in its infinite-dimensional configuration space – again with a or its logarithm defining a time-like variable. This property is physically very important, since it allows for a global "initial" value problem for the wave functional – for example at $a \to 0$.[20] For increasing a, its solution may then form a superposition of wave packets that "move" through this configuration space as a function of a. Even a drastic asymmetry of Ψ with respect to a reversal of a (an "intrinsic" arrow of time) may be derivable from symmetric boundary conditions (such as the usual integrability condition in a) because of the asymmetry of the Hamiltonian under this reversal.

Claus Kiefer could furthermore derive the time-dependent Schrödinger (Tomonaga) equation with respect to an effective time parameter for the matter wave function under a short wave length approximation for the geometric degrees of freedom. It corresponds to a Born-Oppenheimer approximation with respect to the inverse Planck mass (see Kiefer's Ch. 4 in Joos et al. of Ref. 6 and Sect. 5.4 of Ref. 19). This result emphasizes the fact that the Wheeler-DeWitt equation can only describe a whole Everett multiverse, since each trajectory in the configuration space of spatial geometries would define a classical spacetime. Wave packets for spatial geometry propagating along such trajectories are decohered from one another by the matter variables (which thereby serve as an "environment"). This is analogous to the decoherence of atomic nuclei in large molecules by collisions with external particles – the reason why they appear to move on quasi-classical trajectories according to the frog's perspective of a human observer. In cosmology, decoherence (that is, uncontrollable entanglement rather than the often mentioned "quantum fluctuations") is also important for the origin of "classical" structure in the early universe during the onset of inflation.[21]

If one also allowed for a "landscape" (Tegmark's Level 2 of multiverses[22]), which is suggested to exist in several speculative cosmologies that lead to a drastically inhomogeneous universe on the very large scale, the "selection" (by chance – not by free will) of a subjective observer with his epistemologically important frog's perspective (cf. Sect. 4) may be roughly characterized by a hierarchy of five not necessarily independent steps: (1) the selection (in the sense of Level 3, that is, Everett) of an individual landscape from their superposition that must be part of a global quantum state, (2) the selection of a particular region in this three or higher dimensional landscape (a causally separate "world" that may be characterized by specific values of certain "constants of nature" – Level 2), (3) the selection of a quasi-classical spacetime from the Wheeler-DeWitt wave function as indicated above (Level 3 again), (4) the selection of one individual complex organism from all those that may exist in this world, including some "moment of awareness" for it (giving rise to an approximate localization of this observer in space and time: a subjective "here-and-now" – thus including Level 1), and (5) the selection of one of his/her/its "versions" that must have been created by further Everett branching based on the decoherence of matter variables according to Sect. 4 (Level 3).

Each step may create its own unpredictable initial conditions characterizing the further evolution of the resulting individual "worlds". Most properties characterizing our observed world can thus not be *explained* by the objective theory; they have to be empirically determined as part of an answer to the question: *Where* do *we* happen to live in the real and objective "configuration space"? The unpredictability of certain "constants of nature", complained about by some mathematical physicists and cosmologists, is by no means specific for a multiverse. It would similarly apply to any kind of stochastic dynamics (such as in collapse theories), or whenever statistical fluctuations are relevant during the early cosmic evolution. Only step 4 can *not* be objectivized in the usual sense, namely with respect to different observers in the same quasi-classical world. Some of these steps may require an application of the weak anthropic principle in some sense (although I would not recommend to *rely* on it for the future by playing "Russian quantum roulette"!), while none of their individual outcomes experienced by us should have *exceptionally* small probability weight – still a strong condition. Entropy may *decrease* during most of these steps (depending on its precise definition).[5,7,23]

Let me add for completeness that Tegmark's Level 1 and 2 multiverses are classical concepts, and thus unrelated to Everett's branches, as they merely refer to separate regions in conventional space rather than in "configuration" space. It appears somewhat pretentious to speak of "parallel worlds" or a "multiverse" in this case; these names were originally invented for Everett branches (separate wave packets in configuration space), and are here simply misused. The reason may be that many cosmologists are not particularly familiar with the role of entangled superpositions on the macroscopic scale, and therefore prefer to tacitly replace them by statistical correlations that would characterize a collapse mechanism. In this case, different outcomes can be simultaneously realized only at different places in a sufficiently large universe, while different Everett "worlds" would exist even for a closed universe of finite size. However, while such landscapes are conceivable in most stochastic theories without making use of Everett at all (similar to locally varying order parameters resulting from symmetry breaking phase transitions[16]), almost *identical* local situations occurring by chance in an infinite quasi-homogeneous world (Level 1) may be regarded as something between trivial (entirely irrelevant for us) and ill-defined. Although the required double exponentials, needed to describe the required distances, can easily be formulated, an extrapolation of properties (such as approximate flatness) from the observed universe with its size of 10^{10} *ly* to something like $10^{10^{100}}$ *ly* appears at least questionable. Statistical estimates of probabilities for such kinds of Doppelgänger would in any case apply only to chance fluctuations (such as "Boltzmann brains"), but not to situations resulting from evolution. Moreover, probabilities calculated by using some *physical* (that is, additive) entropy would not explain the existence of "consistent documents" in the form of consistent correlations (also known as an "overdetermination of the past" – see footnote on page 24 and Sect. 3.5 of the first Ref. 7), while *unstructured* initial conditions (such as the initial homogeneity of a gravitating universe) would represent even lower entropy values – in spite of their "plausibility".

The role of Tegmark's (as yet unmentioned) Level 4 universes is even entirely questionable, since mathematics, although known to provide extremely useful conceptual tools for theoretical physics because of its analytical (tautological) nature and, therefore, the undeniable *formal truth* of its theorems, cannot by itself warrant the *applicability* of specific concepts to the empirical world. Only if, and insofar as, such kinematical concepts have been empirically verified to be universally and consistently applicable, can we consider them as candidates for a description of "reality". (This seems to be a point that many mathematicians working in theoretical physics and cosmology have problems to understand, since they are used to define their concepts freely and for convenience.) Different mathematical frameworks can therefore not be regarded as indicating the existence of corresponding physical "worlds" – or different parts of one world. While Everett's "many worlds" (just as all scientific cosmology) result from hypothetical extrapolation of the observed world by means of empirical laws, there are no arguments supporting the physical existence of Level 4 worlds. The *mathematical* concept of "existence", for example, means no more than the absence of logical inconsistencies, that is, a necessary but not a sufficient condition for being "realized" in Nature.

7. Conclusions

These remarks about quantum gravity and quantum cosmology may bring the strange story of particles and waves in principle to a (preliminary) end. While the particle concept has been recognized as a mere illusion, the observed wave aspects of microscopic objects can be consistently understood only as part of a universal wave function in a very high-dimensional (if not infinite-dimensional) space. Matrix mechanics with its formal concept of "observables" thus turns out to be essentially no more than an effective probabilistic description in terms of not consistently applicable (hence "complementary") particle or other classical concepts. Many physicists are busy constructing absurdities, paradoxes, or no-go theorems in terms of such traditional concepts in order to demonstrate the "weirdness" of quantum theory. Even Alice and Bob are classical concepts, quantum mechanically to be based on decoherence! "Quantum Bayesianism", presently much *en vogue*, does not even do that; it replaces the whole physical world by a black box, representing an abstract concept of "information" that is assumed to be available to some vaguely defined "agents" rather than to observers who may be consistent parts of the physical world to be described. Obviously, such a "non-theory" can never be falsified (it is "not even wrong").

Although concepts like particles and spatial fields remain important for our every-day life, including that in physics laboratories, their limited validity must deeply affect a consistent world model (cosmology, in particular). It is always amazing to observe how the love affair of mathematical physicists and general relativists with their various classical fields often prevents them from consistently accepting, or even from understanding, elementary quantum mechanics with its conceptual "inconsistencies". If quantum unitarity applies universally, *our observed* quantum world, that is, the "relative state" of the world with respect to the quantum states representing our subjective states as observers, can be no more than a very small (but dynamically autonomous in its future) *partial* wave of the global wave function: one of its "branches". In contrast, the Wheeler-DeWitt wave function seems essentially to be meaningful only from a bird's perspective. We have to accept, however, that the precise structure of a fundamental Hilbert space basis, which is often assumed to be given by the configuration space of some fundamental fields, remains elusive. Because of the unavoidable entanglement of all variables, one cannot expect the *effective* quantum fields, which describe apparent "elementary particles", to be related to these elusive fundamental variables in a simple way. This conclusion puts in doubt much of the traditional approach to QFT, which is based on concepts of renormalization and "dressing" that would suffice to explain the effective fields from the elusive fundamental point of view.

There are indeed excellent arguments why even emergent ("effective") or quasi-classical fields may be mathematically elegant – thus giving rise to the impression of their fundamental nature. Novel mathematical concepts might nonetheless be required for finding the elusive ultimate theory, but their applicability to physics has to be demonstrated empirically, and can thus never be proven to be *exactly* valid. This may severely limit the physical value of many "abstract" (non-intuitive) mathematical theorems. Fundamental physical laws and concepts have so far mostly turned out to be mathematically relatively simple, while their applications are highly complex. This may explain why mathematicians have dominated theoretical physics preferentially *after* completion of a new fundamental theory (such as Newton's), or at times of stagnation, when mere reformulations or unconfirmed formal speculations (such as strings at the time of this writing) are often celebrated as new physics.

References:

[1] E. Segrè, *From Falling Bodies to Radio Waves – Classical Physicists and their Discoveries* (Freeman, NY 1984)

[2] M. Jammer, *The Conceptual Development of Quantum Mechanics* (McGraw-Hill 1966)

[3] H. D. Zeh, Feynman's interpretation of quantum theory, Eur. Phys. J. **H36**, 147 (2011) – arxiv:0804.3348

[4] S. Deléglise et al., Reconstruction of non-classical cavity field states with snapshots of their decoherence, Nature **455**, 510 (2008) – cf. also Fig. 3.14 in E. Joos et al. from Ref. 6

[5] H. D. Zeh, The role of the observer in the Everett interpretation, NeuroQuantology **11**, 97 (2013) – arxiv:1211.0196

[6] H. D. Zeh, On the interpretation of measurement in quantum theory, Found. Phys. **1**, 69 (1970); E. Joos and H. D. Zeh, The emergence of classical properties through interaction with the environment, Z. Phys. **B59**, 223 (1985); W. H. Zurek, Decoherence and the transition from quantum to classical, Physics Today **44** (Oct), 36 (1991); W. H. Zurek, Preferred states, predictability, classicality, and the environment-induced decoherence, Progr. Theor. Phys. **89**, 281 (1993); H. D. Zeh, What is achieved by decoherence? arxiv:quant-ph/9610014; L. Diósi and C. Kiefer, Robustness and Diffusion of Pointer States, Phys. Rev. Lett. **85**, 3552 (2000); E. Joos et al., *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer 2003), Chs. 1-4; M. Schlosshauer, *Decoherence and the quantum-to-classical transition* (Springer 2007)

[7] H. D. Zeh, *The Physical Basis of the Direction of Time,* 5th edn. (Springer, 2007); H. D. Zeh, Open questions regarding the arrow of time, arxiv:0908.3780

[8] H. D. Zeh, Quantum discreteness is an illusion, Found. Phys. **40**, 1476 (2010); H. D. Zeh, There are no quantum jumps, nor are there particles, Phys. Lett. **A172**, 189 (1993)

[9] P. Byrne, *The Many Worlds of Hugh Everett III* (Oxford UP, 2010), Ch. 27

[10] S. Tomonaga, On a Relativistically Invariant Formulation of the Quantum Theory of Wave Fields, Progr. Theor. Phys. **1**, 27 (1946); E. C. G. Stückelberg, Die Wechselwirkungskräfte in der Elektrodynamik und in der Feldtheorie der Kernkräfte, Helv. Phys. Acta **11**, 225 (1938)

[11] H. D. Zeh, There is no 'first' quantization, Phys. Lett. **A309**, 329 (2003)

[12] F. J. Dyson, The Radiation Theories of Tomonaga, Schwinger and Feynman, Phys. Rev. **75**, 486 (1949)

[13] D. Giulini, C. Kiefer, and H. D. Zeh, Symmetries, superselection rules, and decoherence, Phys. Lett. **A199**, 291 (1995)

[14] C. Joas and C. Lehner, The classical roots of wave mechanics: Schrödinger's transformations of the optical-mechanical analogy, Stud. Hist. Phil. Mod. Phys. **40**, 338 (2009)

[15] C. Rovelli, Why gauge? Found. Phys. **44**, 91 (2014)

[16] H. D. Zeh, Symmetry-breaking vacuum and state vector reduction, Found. Phys. **5**, 371 (1975)

[17] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space,* (Cambridge UP, 1983)

[18] R. E. Peierls and J. Joccoz, The Collective Model of Nuclear Motion, Proc. Phys. Soc. London **A70**, 381 (1957); H. D. Zeh, Symmetrieverletzende Modellzustände und kollektive Bewegungen, Nucl. Phys. **202**, 38 (1967)

[19] C. Kiefer, *Quantum Gravity*, 3rd edn. (Oxford UP, 2012)

[20] H. D. Zeh, Emergence of classical time from a universal wave function, Phys. Lett. **A116**, 9 (1986); H. D. Zeh, The nature and origin of time-asymmetric spacetime structures, In: *Springer Handbook of Spacetime Physics*, A. Ashtekar and V. Petkov, edts. (Springer, 2013)

[21] C. Kiefer and D. Polarski, Why cosmological fluctuations look classical to us, Adv.Sci.Lett. **2**, 164 (2009) – arxiv:0810.0078

[22] M. Tegmark, The Multiverse Hierarchy, arxiv:0905.1283

[23] M. Tegmark, How unitary cosmology generalizes thermodynamics and solves the inflationary entropy problem, Phys. Rev. **D 85**, 123517 (2012)

[24] W. H. Zurek, Quantum Darwinism, Nature Physics, **5**, 181 (2009)