

String theory: a perspective over the last 25 years

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ABSTRACT: This article provides some historical background and then reviews developments in string theory over the last twenty-five years or so. Both perturbative and non-perturbative approaches to string theory are surveyed and their impact on how we view quantum gravity is analysed.*

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1. Origins

String theory was conceived partly by accident and partly by design. It arose in the study of scattering amplitudes for hadronic bound states. Lacking a fundamental formulation (that today is believed to be Quantum Chromodynamics), hadronic amplitudes were studied in the 1960's using symmetry, consistency and some observed properties such as the asymptotic rate of growth of cross-sections with energy. A beautiful formula proposed in 1968[1, 2] embodied the expected properties of 4-point amplitudes. The Veneziano formula eventually turned out to describe the scattering of open relativistic strings. These were later generalised[3] in a way that is now understood to describe closed-string scattering¹.

The proposed hadronic amplitudes depended on the familiar Mandelstam variables

$$s \equiv (k_1 + k_2)^2, \quad t \equiv (k_1 - k_3)^2, \quad u \equiv (k_1 - k_4)^2 \quad (1.1)$$

where k_i are the momenta of the two incoming and two outgoing particles, and also on a parameter α' with dimensions of (length)². α' is phenomenologically defined as the slope of “Regge trajectories”, approximately linear plots of the spin of different hadronic resonances versus their (mass)². A basic reason why Regge trajectories should be linear was not known, nor was it clear what α' means in fundamental terms.

Over the next couple of years, Susskind[5] and independently Nambu[6] proposed a way to understand the Veneziano formula, and thereby the strong interactions, from a novel starting point. They postulated the existence of a “fundamental” relativistic string to represent the confining flux between quarks. Quantising this string would lead to physical excitations that could be identified with the baryonic and mesonic states formed by bound quarks.

The world-sheet action² describing a free string propagating in flat spacetime can be written:

$$S = \int d\sigma dt \sqrt{-\det(\partial_a X^\mu \partial_b X^\mu)} \quad (1.2)$$

¹Details on the early history of string theory, along with references, can be found in Ref.[4].

²For derivations and explanations of the results in this section, with references, see for example Ref.[7].

where (σ, t) are the parameters labelling points on the string world-sheet, X^μ are the space-time coordinates of the string and $\partial_a = (\partial_\sigma, \partial_t)$. The integral is the invariant area of the world-sheet in the pull-back of the flat Minkowski metric of space-time. The action possesses reparametrisation invariance on the world-sheet and by a suitable gauge-fixing of this invariance it can be brought to the simpler form:

$$S = \frac{1}{2} \int d\sigma dt \partial_a X^\mu \partial^a X_\mu \quad (1.3)$$

subject to constraints. This action is conformally invariant at the classical level.

The first success of string theory was in explaining Regge behaviour. This follows from the fact that the oscillator modes $a_n^\dagger{}^\mu$ that create excitations of the string, being Fourier modes of the string coordinate $X^\mu(\sigma, t)$, carry a spacetime index μ . Applying a particular oscillator n times to the ground state $|0\rangle$ creates a state:

$$|\mu_1 \mu_2 \cdots \mu_p\rangle = a_n^{\dagger \mu_1} a_n^{\dagger \mu_2} \cdots a_n^{\dagger \mu_p} |0\rangle \quad (1.4)$$

with n symmetrised spacetime indices. This means that the spin of the state is n . At the same time the $(\text{mass})^2$ operator for a string is the number operator for these oscillators in units of the string tension. Hence the above state has $(\text{mass})^2 \sim nT$ with T being the string tension. Regge behaviour then follows immediately and we see that the Regge slope is $\alpha' = 1/T$. Note that in units where $\hbar = c = 1$, tension indeed has dimensions of $(\text{length})^{-2}$ and hence α' has dimensions of $(\text{length})^2$. It is a fundamental constant of string theory.

Quantising the fundamental string carefully using different methods confirmed the expectations: strings display Regge behaviour and the scattering amplitudes of low-lying strings states are, at tree level, those postulated by Veneziano and Virasoro along with their generalisations.

Some surprises also emerged. At the quantum level, conformal invariance of the string world-sheet action turns out to have a quantum anomaly that vanishes only in a spacetime of 26 dimensions. In the presence of the anomaly, standard quantisation of the string is inconsistent (a mode describing scale fluctuations of the world-sheet fails to decouple). Therefore string theory is, in particular, inconsistent in four space-time dimensions.

There is also another problem. Even in 26 dimensions, the particle spectrum of the string starts with a state of tachyonic (imaginary) mass. This arises from the combined effect of the zero-point energies of infinitely many oscillators describing the independent modes of the string. Tachyons are present in the spectrum of both open strings, which are interpreted as carrying quarks at their ends, and closed strings, which describe the quark-less sector of the theory of strong interactions – what we would today call the glueball spectrum.

A tachyon in the particle spectrum of a theory does not necessarily mean the theory is inconsistent. This is familiar in ordinary quantum field theory, where a mass

term of the “wrong” sign (equivalent to making the mass imaginary) is interpreted as an instability of the theory *when expanded above a vanishing expectation value of the field*. Such theories are generally consistent when we expand the field about its true (non-zero) vacuum expectation value. This phenomenon is well-understood in quantum field theory, and is the basis of the Higgs mechanism crucial to the Standard Model of particle physics. Unfortunately it is much harder to understand explicitly in the context of string theory. Shifting a field by its own expectation value in string theory requires knowledge of the “off-shell” dynamics of strings, which was certainly not known at the time string theory was first studied. In the last decade or so, however, more powerful techniques[8, 9] have taught us that the tachyonic string theory described above (now known as the “bosonic string”) is indeed inconsistent, because the analogue of the Higgs potential is unbounded below and the expectation value of the field therefore “runs away” to infinity.

Analysis of the spectrum of open and closed strings reveals that while both contain tachyons, both also contain conventional massless and massive particles. In particular open strings produce a spin-1 massless particle while closed strings produce a massless particle of spin 2 (spin is of course measured in units of $\hbar$). Such particles do not have any obvious counterpart in the hadron spectrum. As we will see below, this fact led to a radical new role for string theory: as a theory of fundamental processes.

The consistency requirements of 26 dimensions as well as the presence of a tachyon and a massless spin-2 particle proved rather discouraging for the string approach to hadronic physics. The focus of strong interactions therefore soon shifted, with the quark model slowly becoming a reality and being described successfully by a gauge field theory, quantum chromodynamics. In this period the string idea of Susskind and Nambu remained a useful qualitative picture to model the force between quarks. The fact that under normal conditions quarks are permanently confined fits nicely with the notion that they are connected by strings of fixed tension, because then increasing the inter-quark separation gives rise to a linearly growing potential energy which is naturally expected to confine.

2. Superstrings

2.1 The fermionic string

A small group of physicists continued to think about strings in the 1970’s[4] although by then the focus of particle physics had shifted to gauge theory. They investigated the formalism of string theory with a view to addressing its negative features, primarily the tachyon and the requirement of 26 dimensions. Also the original string theory did not contain fermions in its spectrum, so the inclusion of fermions became one of the goals.

It was a natural step to introduce fermionic degrees of freedom on the string world-sheet. These were to be thought of as “fermionic coordinates” $\psi(\sigma, t)$ partnering the usual bosonic string coordinates $X^\mu(\sigma, t)$. A proper treatment of the fermionic string took some time to evolve but by the end it was clear that some of the excitations of this string were indeed fermions in space-time. Many remarkable features emerged from this somewhat formal exercise. The world-sheet theory of the fermionic string exhibited an entirely novel symmetry (at the time) called “supersymmetry”, that related bosons to fermions. Moreover the particles that arose by quantising this string were also related to each other by supersymmetry, but now in space-time. The fermionic degrees of freedom modified the condition that had required 26 space-time dimensions in the original string theory. The required number of dimensions was now 10 – still far from the real-world value of 4, but considerably closer.

Space-time supersymmetry has an immediate and striking consequence. In supersymmetric systems the contributions of bosonic and fermionic degrees of freedom to the zero-point energy cancel each other out. This result immediately implies the absence of tachyons in the new “superstring” theory. Thus three distinct problems: a very high critical dimension of 26, the presence of tachyons and the absence of fermions, were all simultaneously solved by the advent of superstrings.

2.2 Gravity and gauge symmetry from superstrings

Given that the original motivation for string theory had been to explain the forces between quarks and the formation of hadrons, the advent of superstring theory seemed to take the subject in an entirely new direction. It could not be applied to the strong interactions for several “obvious” reasons: it was still not consistent in 4 spacetime dimensions, and it had spacetime supersymmetry which is not a property of the strong interactions at least at low energies. But the most undesirable property was that, upon quantisation, the free closed superstring had a massless spin-2 excitation, while the free open superstring had a massless spin-1 excitation. These were features of the original bosonic string theory that, unlike the tachyon, did not go away by introducing supersymmetry, and they seemed to have little to do with hadrons.

In quantum field theory, fields of integer spin very generally have a problem with negative-norm states. The reason is that the norm of a state carrying vector indices can only be Lorentz invariant if it is expressed in terms of the Minkowski metric. For example a vector state $|\mu\rangle$ or a tensor state $|\mu\nu\rangle$ will have norms proportional to:

$$\begin{aligned}\langle\mu|\nu\rangle &\sim \eta_{\mu\nu} \\ \langle\mu\nu|\rho\sigma\rangle &\sim \eta_{\mu\rho}\eta_{\nu\sigma} + \dots\end{aligned}\tag{2.1}$$

where in the second line there can be extra terms to reflect the symmetry/antisymmetry of the state if any. Because $\eta_{\mu\nu}$ has a negative component, the above formula always

leads to a negative-norm state. As a consequence, unitarity of the theory becomes problematic.

The solution to this problem is that fields of integer spin must have a local invariance. This can project out the negative-norm states and restore unitarity. In the case of spin-2, the local invariance is that of general coordinate transformations and the resulting theory is general relativity. With this symmetry, unphysical modes of a spin-2 particle that would have had null or negative norm decouple and the particle is interpreted as a graviton. This line of thought motivated the suggestion that a similar mechanism may hold for superstrings. If so, one could interpret the spin-2 particle in the closed string spectrum as a graviton. Closed string theories would then be theories of gravity. By the same token, since open strings have massless spin-1 particles in their spectrum and these are consistent only in the presence of a local gauge symmetry, one would expect that open strings describe gauge particles.

These rather audacious proposals could be subjected to very stringent tests. If closed strings described gravity and open strings described gauge interactions, it must be that the string-string interactions contain a tightly correlated set of terms dictated by the symmetries of these interactions. For example, general coordinate invariance predicts a unique interaction vertex among four gravitons at two-derivative level (i.e. having two factors of momentum in the interaction). This is unique both in its coefficient (once the propagator and three-point function are normalised) and in its tensorial structure.

Now, unlike in quantum field theory where one is free to add any interaction one likes (at least at tree level), in string theory there is a unique prescription[7, 10] to compute scattering amplitudes for any fixed background. For four-point amplitudes at tree level, this amounts to expanding the superstring analogue of the Virasoro amplitude and keeping the leading term in the parameter α' . From this one reads off the interaction term in an effective Lagrangian for the corresponding massless particle. Remarkably this turns out to have exactly the structure predicted by general relativity.

The expansion of the Einstein-Hilbert Lagrangian

$$\sqrt{|g|}R \tag{2.2}$$

in powers of the fluctuation field $h_{\mu\nu}$ defined by:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{2.3}$$

is well-known to contain infinitely many powers $(h_{\mu\nu})^n$, with definite index contractions, corresponding to contact interactions of n gravitons for every n . Therefore to establish rigorously that general relativity is reproduced by the tree-level scattering of closed strings, one would need to calculate n -point closed-string amplitudes for

all n , a rather difficult task. Nevertheless every term that has been computed leads, after performing allowed field redefinitions, to the correct expression in the expanded Einstein-Hilbert action. So there can no longer be any doubt that closed superstrings describe gravity³.

It is not just Einstein-Hilbert gravity that is described by strings, however. The scattering amplitudes for closed strings have an expansion in powers of α' . Because α' has dimensions of $(\text{length})^2$, these terms must have additional powers of momenta, or in position space, *derivatives*. These higher-derivative terms too are found to be general-coordinate-invariant. Since they can be neglected for sufficiently slowly varying fields, the correct statement is that closed string amplitudes reproduce Einstein-Hilbert gravity for slowly varying fields, with calculable corrections expressed in terms higher-order variations of the fields.

A very similar calculation for open strings reveals that to lowest order in α' one finds a non-Abelian gauge theory of Yang-Mills type, with its characteristic cubic and quartic self-interactions. Again there are higher derivative corrections that are suppressed for slowly varying fields. In the simplest (Abelian) case, an infinite subset of these turns out to be in correspondence with the old non-linear electrodynamics theory of Dirac and of Born and Infeld[10].

The more general non-Abelian calculation was initially performed using a somewhat ad hoc prescription, introducing matrices called “Chan-Paton factors” at the ends of the open string. In the presence of such matrices, one automatically obtained a $U(N)$ gauge group for $N \times N$ matrices. The introduction of unoriented open strings enabled one to realise orthogonal and symplectic groups as well. Much later it became clear that these Chan-Paton matrices were labelling dynamical objects on which open strings end, or “D-branes”, about which we will say more below.

Thus the proposal that closed and open strings describe gravity and gauge theory was convincingly supported by amplitude calculations. Because all known fundamental interactions fall into these two types, it is no surprise that string theory emerged as a natural framework to describe the world.

2.3 Dilaton and the genus expansion

So far we have spoken only about tree amplitudes. In field theory, “tree level” refers to the lowest order in an expansion in a coupling constant. In string theory, by contrast, there is no parameter in the theory other than the dimensional constant α' . Therefore “tree level” as discussed above refers simply to a computation where the world-sheet has a tree-like configuration without any holes or handles. For a

³For bosonic strings, on ignoring the tachyon one finds that they also describe gravity in more or less the same way. Now the tachyon can be meaningfully ignored at tree level, but once quantum (loop) corrections are included then the tachyon renders the theory inconsistent. Hence we confine our main remarks to the tachyon-free superstring.

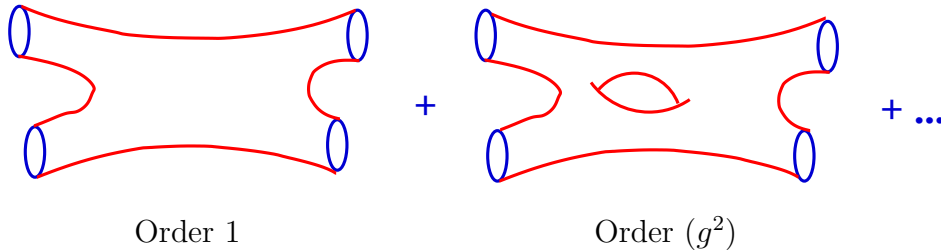
beautiful reason, this in fact turns out to be the lowest order in a “hidden” string coupling.

This arises as follows. Along with a graviton, quantisation of the closed string gives rise to a massless scalar particle called the dilaton[7, 10] denoted Φ . In perturbation theory it turns out that this scalar has no potential, and is therefore the first of many scalars in string theory (called “moduli”) that can take arbitrary vacuum expectation values.

From the unique structure of string interactions one can infer that in tree diagrams the dilaton couples to the low-energy Lagrangian by an overall multiplicative factor of $e^{-2\Phi}$. The vacuum expectation value $\langle\Phi\rangle = \Phi_0$ can be taken outside action and gets identified with $\frac{1}{g^2}$ where g is a coupling constant, since that factor (in some frame) is what multiplies the whole action of a field theory. Thus by virtue of the dilaton VEV, string theory acquires a coupling constant:

$$g = e^{\Phi_0} \tag{2.4}$$

Now if we compute scattering amplitudes on world-sheets of some higher genus $h > 0$, it can be shown that the power of the dilaton VEV multiplying the action is $e^{(2h-2)\Phi_0}$. It follows that world-sheets of genus h are weighted by g^{2h-2} and therefore the perturbation expansion of string theory is an expansion in world-sheets of increasing genus:



In each order one has to sum over all surfaces of the given topology. This sum is subtle for various mathematical and physical reasons, but has by now been thoroughly investigated.

2.4 Quantum corrections

Once it was clear that superstring theory is consistent at tree level, it became extremely urgent to check for its consistency after introducing perturbative quantum corrections, known in field theory as “loop” corrections[11, 12]. The rules for tree level superstring amplitudes are particularly easy to extend to the next order in perturbation theory, namely one-loop level corresponding to genus-one Riemann surfaces. Here the proposal that closed strings describe gravity can receive its first test at the quantum level.

Einstein-Hilbert gravity suffers from ultraviolet divergences in every order of perturbation theory. From this it follows that at least in a perturbation expansion the theory is non-predictive. String theory could well turn out to have the same problem. However, there are physical grounds for optimism. One may hope that the spatial extent of the string (parametrised by α') could provide an ultraviolet cutoff. Then graviton scattering in superstring theory would have finite loop corrections. This hope was dramatically realised with the computation of one-loop graviton amplitudes in superstring theory, which indeed turned out to be ultraviolet finite.

In field theory, it is known that combining supersymmetry with gravity leads to the theory of “supergravity” in which gauged or local supersymmetry is present. Therefore just on symmetry grounds, one can show that the low-energy effective action of superstring theories must correspond to a field theory of supergravity, possibly coupled to supersymmetric matter (and augmented by higher-derivative corrections). Now, important phenomenological work using supergravity theories was done in the late 1970’s and early 1980’s (see for example Ref.[13]), ignoring the fact that – as field theories – they were probably not ultraviolet complete. In a remarkable convergence of streams of research, superstrings provided a sound justification for this work: the interesting physical applications of supergravity at low energy could co-exist with an ultraviolet-finite high-energy completion coming from string theory.

Thus superstring theory was understood to be, in 10 dimensions, a quantum theory of gravity incorporating non-Abelian interactions and supersymmetry in a unified framework. With gravity and non-Abelian gauge symmetry being undisputed properties of nature, and supersymmetry also a likely property, superstring theory exhibited the potential to answer all the important questions about fundamental interactions that had remained unsolved for over half a century. Moreover it was a theoretical formulation of unmatched beauty and power. However it was very far away from being able to reproduce even gross properties of the real world such as the observed gauge groups and parity-violating particle spectrum.

There were few people working on superstrings at the time this state of affairs was reached. Particle theorists were more excited, and with good reason, over the recent successes of the electro-weak theory as well as quantum chromodynamics. Moreover, the calculational techniques of string theory as understood at the time were unfamiliar and difficult to grasp. And of course gravity was totally irrelevant to the particle-physics experiments which these theories could explain so convincingly.

2.5 Conformal invariance and the sigma-model approach

Stepping briefly out of historical sequence, we mention here a slightly different approach to studying strings that provides more direct evidence of the presence of gravity and general coordinate invariance. In this approach, instead of studying strings propagating in flat spacetime one writes down the world-sheet action for

strings propagating in an arbitrary curved space-time, and more generally in arbitrary background fields. In the specific case of a nontrivial space-time with metric $g_{\mu\nu}(X)$ (in some coordinate system), the world-sheet theory reduces to a non-linear sigma-model in two dimensions, with Lagrangian:

$$\mathcal{L} = \frac{1}{2} g_{\mu\nu}(X) \partial_a X^\mu \partial^a X^\nu + \dots \quad (2.5)$$

where as before $X^\mu(\sigma, t)$ is the position coordinate of the string and the world-sheet derivative is $\partial_a = (\partial_t, \partial_\sigma)$. The terms represented by $+\dots$ are those which depend on the fermionic coordinates of the superstring.

When $g_{\mu\nu}(X)$ is a non-trivial function of X , the above action describes an interacting two-dimensional field theory in which the coordinates X^μ play the role of scalar fields. Because it has no dimensional coupling⁴, this theory is classically conformal invariant just like the one for strings propagating in flat space-time that we discussed earlier. We already saw that conformal invariance is generically violated and renders the theory inconsistent unless the dimension is critical. In the present case there are additional sources of anomalies, so not only does anomaly-freedom fix the critical dimension to a critical value (10 for superstrings) but it also imposes *conditions on the allowed metric* $g_{\mu\nu}(X)$. Indeed, to first order in α' , a short calculation[14, 15] reveals that the theory remains conformally invariant if:

$$R_{\mu\nu} = 0 \quad (2.6)$$

Notice that this is Einstein's equation in a vacuum! This resemblance is no coincidence. It has been convincingly argued[16, 17] that the condition for conformal invariance imposed on the string sigma-model is equivalent to the equations of motion of the effective low-energy field theory arising from the string theory. In particular for backgrounds with non-trivial stress-energy, the conformal invariance condition becomes the standard Einstein equation with the stress-energy on the right hand side:

$$R_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + \frac{1}{2-D} g_{\mu\nu} T \right) \quad (2.7)$$

Thus, conformal invariance of the string world-sheet provides a new principle, without an analogue in particle mechanics, to derive space-time actions. In particular it shows that the low-energy effective action of closed string theory, to lowest order in α' , is given by the Einstein-Hilbert action Eq. (2.2). This term is universal for all superstring (and even bosonic string) theories. But from the next order in α' there are

⁴This is potentially confusing because the coupling is really $\sqrt{\alpha'}$ which multiplies each occurrence of X^μ . The explanation is that from the space-time point of view both α' and X^μ are dimensional, with $\sqrt{\alpha'} X^\mu$ being dimensionless. But from the world-sheet point of view both α' and X are classically dimensionless, this being true for X^μ by virtue of its identification with a scalar field in 2 dimensions.

differences depending on whether one does or does not have supersymmetry. Some corrections that arise in the bosonic string do not arise for the superstring[15, 18]. This has crucial consequences for the physics of the theory: in the superstring, space-times that solve the equations of motion to lowest order in α' can continue to solve them when higher-order corrections are included. This is not so in the bosonic string, where higher-order corrections rule out a large class of potentially interesting solutions.

2.6 New insights about gravity: I

The developments reviewed above gave rise to several important insights into the nature of quantum gravity. Many were properties of gravity that had already been suspected or conjectured but for which string theory provided a concrete realisation and/or tangible new evidence.

(i) It is possible to have a well-defined ultraviolet finite theory of gravity, with the characteristic minimum size providing a UV cutoff. Even if the idea is not totally new, string theory provides a precise mechanism and allows one to dissect precisely how the cutoff works. The world-sheet of the string is a Riemann surface on which certain diffeomorphisms called “modular transformations” act. While summing over all surfaces, the dangerous ones for UV divergences are those that become very “thin”, corresponding to high momentum flowing through. But a modular transformation relates such “thin” surfaces to others that are not degenerating, so we never actually reach infinitely thin surfaces, or equivalently infinitely high momenta.

(ii) Higher derivative terms are generic. Any effective action will have this property, but in string theory one sees for the first time a concrete computation of these terms given a particular space-time background.

(iii) Space-time is an option. Classical solutions are configurations that give a conformal-invariant world-sheet, and these need not resemble space-time (for example, a tensor product of “minimal” conformal field theories with total central charge equal to the critical value but each component having central charge less than 1 would be a valid solution).

(iv) Gravity and gauge symmetry have a common origin. With closed and open strings, the identical quantisation procedure yields respectively gravitons and gauge bosons. Moreover gravitons arise by combining a “vector” state from the left- and right- movers of the closed string, making them in some sense the “square” of gauge fields. Present-day research on gluon and graviton amplitudes (see for example Refs.[19, 20] has revealed more and more structures suggesting this “square” relationship between gravity and gauge theory. A deep understanding of this could revolutionise our understanding of the gravitational force.

3. Structure and varieties of superstring theories

3.1 Five types of superstrings

String theories, like field theories, have supersymmetry generators that are organised as spinors of the relevant local Lorentz group. Thus in 10 dimensions, a supersymmetry generator (or “supercharge”) Q_α is a real 16-component Majorana-Weyl spinor of the local Lorentz group $SO(9,1)$. Now it was found (for details, see for example Refs.[7, 12]) that string theory could accommodate either one or two such spinor generators as a symmetry⁵, leading to type I and type II superstring theories respectively.

It was also shown that type II theories can only have closed strings, because open-string boundary conditions break supersymmetry⁶. Moreover, the two Majorana-Weyl supersymmetries can correspond to a pair of spinors of opposite chirality or the same chirality, leading to two physically inequivalent theories called type IIA and type IIB. The latter is parity-violating, since both supercharges have the same chirality.

In type I theories one has open strings which, via the Chan-Paton mechanism discussed above, are associated to a gauge group⁷. The Chan-Paton mechanism is able to accommodate only gauge groups of the unitary, orthogonal or symplectic types. Thus, at least classically, one can have infinitely many type I superstring theories in 10 dimensions, one for each choice of gauge group. Having one chiral supercharge, they are all parity-violating in 10 dimensions.

An exotic way to achieve a “type I-like” theory is to match the left-moving sector of the type II superstring to the right-moving sector of the bosonic string. This leads to the so-called “heterotic” superstring theory[21]. Because it uses only half the type II superstring, in a precise technical sense, it has half the supersymmetries and therefore has $\mathcal{N} = 1$ supersymmetry in 10 dimensions. For historical reasons heterotic strings are not called “type I” because the latter terminology is reserved for string theories having both open and closed string excitations about the vacuum.

As one would expect, the low-energy limits of type I/II string theories are type I/II supergravity field theories in 10 dimensions. The field theories contain the massless fields associated to the string. Because supersymmetry is so tightly constraining, one can actually write out the entire low-energy action for type I/II strings at the classical level and to lowest order in α' purely on grounds of supersymmetry.

⁵The same is true in 10d field theory.

⁶Much later it was understood that there are actually *sectors* in type II theories that admit open strings. The physical interpretation of these sectors is in terms of dynamical objects called D-branes that we will discuss below.

⁷In a modern interpretation the presence of open strings is interpreted in terms of “condensed D-branes in the vacuum”.

Quantum mechanically, it is essential to check that the parity-violating string theories are free of anomalies. Because anomalies do not depend on the ultraviolet behaviour of a theory, the anomaly structure is the same for superstring theories and for their low-energy supergravity field theories. A search was therefore launched in the early days for possible anomaly-free supergravity theories (possibly coupled to supersymmetric Yang-Mills theories) in 10 dimensions. Type IIA is trivially anomaly-free, while type IIB supergravity theory was found[22] to be anomaly-free due to non-trivial cancellations among a number of anomalous contributions.

Later Green and Schwarz[23] studied the type I case. They made the remarkable discovery that there were precisely two anomaly-free field theories in 10 dimensions, one with gauge group $SO(32)$ and the other with the exceptional gauge group $E_8 \times E_8$. On trying to match these with string theories they found that an open-and-closed type I theory could be constructed to have gauge group $SO(32)$. However, no such theory with Chan-Paton factors can have the gauge group $E_8 \times E_8$. Heterotic strings, however, can incorporate either of the two gauge groups. Thus at the end we have five superstring theories in 10 dimensions: type IIA, IIB, type I ($SO(32)$), heterotic ($SO(32)$), heterotic ($E_8 \times E_8$).

The “almost uniqueness” of superstring theory, including a tight restriction on gauge groups in the type I case, was very exciting. A selection principle governing the gauge groups that can occur in nature would be quite unprecedented. Such a principle has never existed in four dimensional field theory, where the constraints of anomaly freedom are still present but they restrict the representations that can appear rather than the groups themselves.

The elegant classification into five distinct superstring theories was somewhat tempered by the later discovery of non-supersymmetric strings that, like superstrings, are tachyon-free and whose critical dimension is still 10. These theories were found by extracting different space-time dynamics from a common world-sheet theory by imposing different projections on it. These discoveries presaged the advent of duality, which made it very clear that essentially all string theories are different vacua of a common theory.

3.2 Tensors and Ramond-Ramond fields

In addition to the graviton, the ground state of the closed string contains a whole supermultiplet of massless fields in 10 dimensions. With maximal or $\mathcal{N} = 2$ supersymmetry, this supermultiplet includes the graviton, the dilaton and several tensor fields of the type $C_{\mu_1\mu_2\cdots\mu_p}$ that are totally antisymmetric in their indices (see for example Ref.[12]). These are referred to as “ p -forms”. Some of these p -forms are special in the way that they arise from string quantisation and the nature of their associated gauge invariance (a point that is too technical to discuss here). They carry the name “Ramond-Ramond (RR) fields”. Type IIA strings have RR 1-form

and 3-form fields while type IIB strings have RR 0-form (scalar), 2-form and 4-form fields.

To illustrate the structure of the low-energy theory, we quote here the action of type IIA supergravity keeping only the RR 1-form and ignoring the other tensor fields and fermions:

$$S_{type\ IIA} = \frac{1}{(2\pi)^7 \ell_s^8} \int d^{10}x \sqrt{-\|G\|} \left[e^{-2\Phi} (R + |d\Phi|^2) - \frac{2}{8!} |dA|^2 \right] \quad (3.1)$$

where we have used the notation $\ell_s = \sqrt{\alpha'}$. Here $G_{\mu\nu}$, Φ are the graviton and dilaton while A_μ is the RR 1-form. We will return to this action in a subsequent section.

3.3 Perturbative dualities

String amplitudes possess the attribute of “channel duality” which can be summarised as follows[4, 7]. In particle physics, two-particle to two-particle scattering takes place in one of three ways, corresponding to the s , t and u channels (associated to the three possible ways of connecting up four participating particles in a Feynman diagram). However, in string theory the corresponding scattering process has just one contribution, which embodies within it all three channels. This is a manifestation of the fact that string scattering is described via a world-sheet, which can be smoothly deformed at will so that the process resembles any of the s , t or u -channel processes for particle scattering.

Another duality in string theory arises naturally when the spacetime contains non-contractible circles (a detailed review can be found in Ref.[24]). The string, being an extended object, can wrap around a circle of radius R giving rise to a “winding state” with energy:

$$E \sim n \frac{R}{\alpha'} \quad (3.2)$$

where n is the winding number. But the spectrum of the theory also contains familiar states of quantised momentum when the string propagates as a whole along the compact direction. Such states have a typical energy:

$$E \sim \frac{m}{R} \quad (3.3)$$

where m is the quantised mode number.

We see that for a large radius R of the circular direction, the winding states are heavy due to their spatial extent, while the momentum states are light. But if R is sufficiently small then the situation is reversed: winding states are light while momentum states become heavy due to the uncertainty principle. In fact under the replacement $R \rightarrow \alpha'/R$, the spectrum of winding and momentum states gets

interchanged. Moreover it is known that all interactions of the string are preserved, with winding and momentum states simply getting interchanged. Therefore

$$R \rightarrow \frac{\alpha'}{R} \tag{3.4}$$

is an exact symmetry of string theory. This symmetry is called “target-space duality” or “T-duality” for short.

Both channel duality and T-duality are intrinsic and remarkable properties unique to string theory.

3.4 Superstrings and 4d physics

As a starting point to describe the fundamentals of the real world in 4 space-time dimensions, string theory had all the desirable properties one might want – partial uniqueness, gauge and gravitational interactions, fermions and parity violation. But one had to derive 4-dimensional physics, specifically the Standard Model at low energies, from this structure.

It has long been known in field theory that gauge fields in four dimensions can be obtained by compactifying a theory of pure gravity in higher dimensions. This was the Kaluza-Klein mechanism, in which gauge fields arise as isometries of the compactification manifold. However, it was shown by Witten[25] that a parity-violating spectrum in four dimensions could not be obtained in this way by starting with a purely gravitational theory in higher dimensions. Applied to string theory, this ruled out type IIA and IIB string theories and focused attention on the type I and heterotic versions which had non-trivial gauge groups in 10 dimensions. The Type I open-closed string theory admitted only the $SO(32)$ gauge group and it gradually became evident that an anomaly-free compactification starting from this gauge group, under quite general conditions, also failed to give parity violation in 4d.

That left only the $E_8 \times E_8$ gauge group, which was realised exclusively in heterotic string theory. The low energy field theory of such a string, compactified to 4 dimensions on a suitable class of 6-dimensional manifolds, was shown in a classic paper of Candelas, Horowitz, Strominger and Witten[26] to yield qualitatively correct phenomenological properties, including parity violation. The 6-manifolds with favourable four-dimensional phenomenology were known as “Calabi-Yau” spaces. This development sparked off an explosion of interest in string theory worldwide.

Subsequently a class of simplified compactification models were discovered[27] that make use of “orbifolds” rather than smooth manifolds. Typically orbifolds are singular limits of smooth manifolds. In appropriate orbifold limits the entire curvature of a manifold can be concentrated at a discrete set of points while the rest of the space is flat. This considerably simplifies the analysis of the spectrum and

interactions. Of course such an approach will not work if the singular curvatures give rise to singular answers for amplitudes. However it, emerged that under fairly general conditions string propagation at a singularity is smooth, in part due to the extended nature of strings.

In hindsight, despite the defining historical role played by the Candelas et al paper, its central premise – that a realistic string theory incorporating and generalising the Standard Model, plus gravity, was round the corner – has not proved to be correct. One of the fondest hopes implicit in this work, that of starting with an “almost unique” theory and obtaining 4d physics via an “almost unique” compactification dictated by elegant mathematical considerations, is now believed to be far from the truth. One of the key issues unresolved issues of the time was how to deal with the plethora of massless scalars, or “moduli” inherent in almost any compactification. By the time this problem was effectively addressed, the scenario had changed considerably, as we will see, with the discovery of many non-perturbative features of string theory.

The study of string compactifications led to the discovery of a new and unexpected mathematical property[28] called “mirror symmetry”. This was the occurrence of pairs of 6-manifolds of Calabi-Yau type with a remarkable property: while the members of the pair have little geometric resemblance to each other, strings propagate in precisely the same way on both of them. The interchange of the two “mirror” manifolds is an exact symmetry of string theory. Thus in the domain of stringy quantum geometry (a concept yet to be completely unravelled), these manifolds would be completely indistinguishable from each other. Besides the input to mathematics, mirror symmetry provides a useful physical result. Upon varying moduli, examples have been found where a particular manifold undergoes topology change while its mirror remains smooth. This makes it clear that topology change can be a perfectly smooth phenomenon in string theory. It is now understood that mirror symmetry is a sophisticated version of T-duality, which has made it slightly less mysterious.

3.5 New insights about gravity II

Despite the absence of compactification schemes that could reproduce the real world, some remarkable new properties of space-time emerged from these investigations. Most of the illuminations stemmed from T-duality, described above. By relating short (sub-string-scale) distances to long distances, this challenged the very nature of geometry. It gradually emerged that a new “stringy geometry” is a more appropriate concept for the way space-time behaves in string theory.

An immediate consequence of T-duality was the proposal[29] that when the universe contracts to a big bang in the far past, one should simply perform a T-duality when stringy sizes are reached and thereafter the universe will expand in

the new coordinates. More generally, the ability of the string to effectively resolve singularities and the possibility of topology change potentially provide important inputs into cosmology.

4. Non-perturbative string theory

The latter half of the 1980's saw efforts to systematise (even classify) two-dimensional conformal field theories, as these were now understood to correspond to possible string vacua. In particular, it was hoped that the right compactification 6-manifold, of “Calabi-Yau” type, would reproduce the Standard Model in a unified framework that contained gravity and was UV finite. Apart from many other complexities, one of the most frustrating problems encountered in this process was that of massless scalars or moduli fields. To avoid seriously conflicting with experiment they needed to be “stabilised” or given a mass by some mechanism. There were early hints[30] that the inclusion of fluxes over the compactification space would help, and these turned out correct. However it proved impossible to stabilise all moduli using known *perturbative* mechanisms.

Therefore it was hoped that non-perturbative effects in string theory would lift these moduli. Perhaps non-perturbative phenomena could also rule out some or most of the possible compactifications, whose ever-growing variety was proving to be an embarrassment. However, such effects could not even be studied within the existing formalism of string theory, essentially a set of rules to produce a perturbative S-matrix. At this stage string theorists began to seek non-perturbative information about the theory. A number of routes were explored and all led to valuable insights.

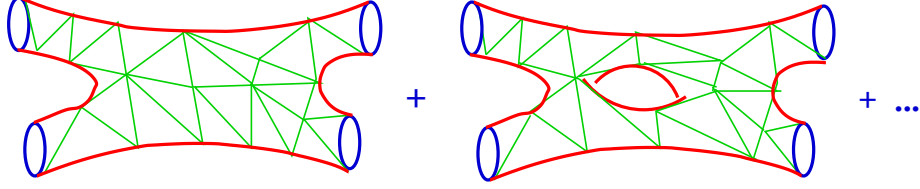
4.1 Random matrices and noncritical strings

The first successful approach to non-perturbative string theory was to use random matrices to model the string worldsheet as a sort of random lattice (for a review, see Ref.[31]). In a suitable continuum limit, the Feynman diagram expansion for the matrix theory would include all topologies of string worldsheets and one might hope that results could be extracted beyond perturbation theory. This approach worked only for vacua of string theory in very low spacetime dimensions (in particular, one space and one time) and moreover with a coupling constant that varies along the space dimension⁸. The continuum limit could also be analysed using standard worldsheet techniques (for a review of the continuum approach, see Ref.[32]).

We saw that the perturbation series for strings is described by a set of surfaces. For each topology, we have to sum over all conformally inequivalent metrics on the

⁸The varying coupling successfully evaded the D=26 consistency condition, but unfortunately replaced it with the new condition $D \leq 2$.

surface. This description is intrinsically perturbative in the string coupling. The matrix approach works instead by discretizing the worldsheet.



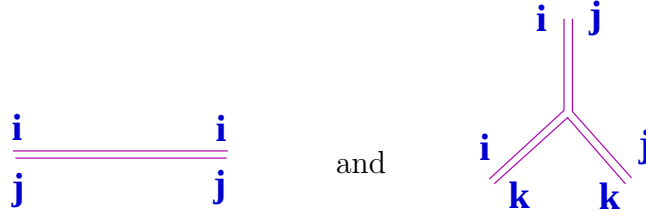
Thus the surface is divided into triangles and summing over all ways to do this implements the sum over all surfaces. This in turn is done by writing a random matrix integral, a simple example being:

$$\int dM e^{-N \text{tr} \left(\frac{1}{2} M^2 + g M^3 \right)} \quad (4.1)$$

where M_{ij} is an $N \times N$ Hermitian matrix. This can be expanded in a Feynman diagram expansion:

$$\int dM e^{-\frac{N}{2} \text{tr} M^2} \sum_{n=0}^{\infty} \frac{1}{n!} (-gN \text{tr} M^3)^n \quad (4.2)$$

which, as usual, is drawn in terms of propagators and vertices. The propagator for a matrix, $\langle M_{ij} M_{ji} \rangle$, and the vertex $\text{tr} M^3 = M_{ij} M_{jk} M_{ki}$ are represented pictorially by:



respectively. Connecting propagators and vertices, one constructs graphs that have cubic intersections, and their dual graphs are then triangulated random surfaces. In the process one finds that the string coupling constant is effectively:

$$g \sim \frac{1}{N} \quad (4.3)$$

where N is the rank of the matrix.

In these “non-critical” strings it proved possible to go beyond just the finite-order diagrammatic expansion. Here one could compute scattering amplitudes to all orders in perturbation theory, an unprecedented achievement, and also identify non-perturbative effects⁹. The key consequence of these studies was the unexpected

⁹The original non-critical string theories were later found to be non-perturbatively inconsistent, but this problem was overcome by introducing world-sheet supersymmetry[33]. Therefore in the end this development did lead to non-perturbatively consistent and well-understood string backgrounds.

discovery that non-perturbative effects in string theory scale differently from those in field theory[34]. While the latter are typically of order:

$$\sim \exp\left(-\frac{1}{g^2}\right) \quad (4.4)$$

where g is the coupling constant, stringy non-perturbative effects were found to scale as:

$$\sim \exp\left(-\frac{1}{g}\right) \quad (4.5)$$

which is a considerably larger effect at weak coupling.

This observation was at the root of the discovery, a few years later, of D-branes. These objects, viewed as classical solutions of type II string theory, give rise to effects that scale as in Eq. (4.5) because their masses (or, in the case of instantons, their actions) scale as:

$$M \sim \frac{1}{g} \quad (4.6)$$

This makes them lighter, and their effects correspondingly more important, compared to the traditional heavy solitons/instantons in field theory whose masses/actions scale like:

$$M \sim \frac{1}{g^2} \quad (4.7)$$

and which are therefore responsible for conventional non-perturbative effects as in Eq. (4.4).

4.2 Non-perturbative dualities

Studies of magnetic monopoles and dyons in quantum field theory reveal a great deal of similarity between these magnetic objects and the usual “fundamental” excitations of the quantum field which carry electric charge. Indeed, the only significant physical difference between the two classes of objects is that in a weakly coupled field theory, fundamental electric states are relatively light (with masses independent of the coupling constant) while solitonic magnetic states are heavy, their masses scaling as in Eq. (4.7).

Similarities in the dynamics of these objects motivated the proposal of “electric-magnetic duality” – that there exists a transformation on a field theory which interchanges the roles of electricity and magnetism (for a review of this concept and much of the material in this section, see Ref.[35]). In order to exchange fundamental and solitonic objects of the type described above, this duality must act as:

$$\begin{aligned} g &\rightarrow \frac{1}{g} \\ M(g) &\rightarrow \frac{1}{g^2} M\left(\frac{1}{g}\right) \end{aligned} \quad (4.8)$$

Clearly this takes a weakly coupled to a strongly coupled quantum field theory. So initially it was difficult to imagine how it could ever be tested.

The resolution came from the “quantum BPS bound” or Witten-Olive[36] bound. In the original formulation due to Bogomolny[37] and Prasad and Sommerfeld[38] (BPS) this was a classical lower bound for the mass of solitons and magnetic monopoles in terms of their charges and parameters of the theory. When the bound was saturated, it became easier to obtain classical solutions of the field theory. But it was realised that generically quantum corrections would affect the mass, and the bound would therefore no longer be saturated in the quantum theory. Witten and Olive showed that the situation is very different in field theories with extended supersymmetry.

In such theories they noted that the “central charge” in the supersymmetry algebra, evaluated on charged states, evaluates their electric/magnetic charges. Moreover when the BPS bound between mass and charge is saturated, the representations of supersymmetry become shorter – the multiplets have a reduced dimensionality. As a typical example, in a theory with 16 supersymmetry generators a generic state has a multiplicity of $2^8 = 256$ but a BPS state has a multiplicity of just $2^4 = 16$. As long as supersymmetry is not violated by quantum corrections, this ensures – as an exact operator statement – that states saturating the bound classically continue to saturate it at the quantum level, since quantum corrections cannot continuously deform a 16-dimensional vector into a 256-dimensional one.

This discovery provided unprecedented control over quantum corrections in theories with extended supersymmetry. In such cases (the most famous being $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in 3+1 dimensions) one could now make some definite statements about the strongly coupled theory. For example the spectrum of BPS states had to be isomorphic to that at weak coupling, something that is not true generically without the BPS condition since field theory states can decay as the coupling changes.

Such considerations led Ashoke Sen in a classic 1994 paper[39], to propose a test for strong-weak coupling or electric-magnetic duality in a compactification of superstring theory to four dimensions which at low energies (and ignoring α' corrections) reduced to $\mathcal{N} = 4$ super-Yang-Mills theory. He showed explicitly that the spectrum of this theory fulfils the requisite conditions for strong-weak duality. This result sparked off a wave of interest in non-perturbative duality. Following it, a variety of dualities were discovered in different field theories, notably Seiberg-Witten theory[40], and in compactifications of string theory[41, 42]. These dualities were non-perturbative in nature and could only be argued for using properties of BPS states as explained above.

The role of duality in string theory was a unifying one. Apparently different compactifications of different 10-dimensional string theories were related by con-

tured duality transformations for which stringent tests were proposed. String theory passed all the tests, and in each case did so by strikingly different dynamical mechanisms. During this period, it became more clear than ever that the underlying structure of string theory was very rigid and constrained, and that dualities were an intrinsic and deep property built into the theory.

Let us briefly describe how duality operates in uncompactified type II string theories. In type IIB, the strongly coupled theory is dual to the weakly coupled one but with an important change: the fundamental string gets interchanged with a D1-brane (alternatively called “D-string”). Something quite different happens in the type IIA superstring – it gets mapped not onto a weakly coupled string theory, but into a theory with an extra dimension[42]. The large coupling is exchanged for the size of the extra (circularly compactified) dimension. In the limit of infinite coupling we find an 11-dimensional theory that has 11-dimensional supergravity as its low-energy limit.

Under this duality the string of type IIA string theory reveals itself to be a membrane of the 11-dimensional theory, which has been dubbed “M-theory”[43]. This brings 11d supergravity, the most beautiful and unique of all supergravity theories, into the framework of strings¹⁰. It also enlarges the scope of string theory by relating it to a theory where the important excitations are not strings at all.

Before duality, it was thought that there were five different superstring theories in 10 dimensions and from them a large variety of lower-dimensional theories could be obtained via compactification. However these compactifications now turn out to be duality transforms of each other, and a path exists (via compactification, duality and de-compactification) between all the 10-dimensional string theories. Moreover they can all be linked to M-theory in 11 dimensions.

4.3 D-branes

Besides membranes, other extended “brane”-like solitons are known to exist in various supergravity theories. These were extensively explored in the late 1980’s and early 90’s. The typical strategy for finding such branes is to postulate that they exist and saturate the quantum BPS bound. This leads to simpler equations of motion whose solutions, at least when there is a high degree of supersymmetry, are guaranteed to solve the full supergravity equations of motion. In this way, several explicit solutions have been found. Among such “brane” solitons, some of them in type II string theory carry charges under the Ramond-Ramond gauge fields, a striking fact given that no states previously known in the theory carried such charges. Moreover these have tensions that scale according to Eq.(4.6) rather than Eq.(4.7), which makes them lighter than conventional solitons at weak coupling. Finally, they exist

¹⁰11 is the highest space-time dimension in which supergravity exists.

in pairs whose tensions are related by electric-magnetic duality according to:

$$T(g) = \frac{1}{g^2} T\left(\frac{1}{g}\right) \quad (4.9)$$

Let us examine some of these solitonic brane solutions in a little detail (see for example Refs.[12, 44]). For this purpose we will need the supergravity action including the Ramond-Ramond sector, which was written down in a previous section for type IIA string theory, retaining only the graviton, dilaton and 1-form Ramond-Ramond gauge field A_μ :

$$S_{type\ IIA} = \frac{1}{(2\pi)^7 \ell_s^8} \int d^{10}x \sqrt{-\|G\|} \left[e^{-2\Phi} (R + |d\Phi|^2) - \frac{2}{8!} |dA|^2 \right] \quad (4.10)$$

Now we start by looking for a classical solution corresponding to a point particle that is charged under A_μ . This will be given by a spherically symmetric gravitational field along with an electric flux of A_μ . Coulomb's law for field strengths in 10 dimensions has a $1/r^8$ fall-off, so the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ takes the form:

$$F_{0r} \sim \frac{N}{r^8}, \quad r \rightarrow \infty \quad (4.11)$$

where we anticipate that there will be N quantised units of this flux.

The gravitational field is specified by writing the metric:

$$ds^2 = - \left(1 + \frac{r_0^7}{r^7}\right)^{-\frac{1}{2}} dt^2 + \left(1 + \frac{r_0^7}{r^7}\right)^{\frac{1}{2}} \sum_{a=1}^9 dx^a dx^a \quad (4.12)$$

where $r = \sqrt{x^a x^a}$. This is like an extremal Reissner-Nordstrom black hole in 10 dimensions (in these coordinates the horizon is at $r = 0$). To complete the solution we have to specify the dilaton and gauge potential:

$$\begin{aligned} e^{-2\Phi} &= e^{-2\Phi_0} \left(1 + \frac{r_0^7}{r^7}\right)^{-\frac{3}{2}} \\ A_0 &= -\frac{1}{2} \left[\left(1 + \frac{r_0^7}{r^7}\right)^{-1} - 1 \right] \end{aligned} \quad (4.13)$$

where we recall that $g_s = e^{\Phi_0}$.

We can compute the mass of this object from the classical solution, and the result comes out to be:

$$M = \frac{1}{d g_s^2 \ell_s^8} (r_0)^7, \quad (4.14)$$

where d is a constant. Using the Dirac quantisation condition one can argue that this object can only occur in integer multiples of a minimally charged object, where the integer is:

$$N = \frac{1}{d g_s \ell_s^7} (r_0)^7 \quad (4.15)$$

This can be thought of as the charge of the object in units where the minimum charge is unity. The supergravity solution is valid only when N is large, i.e. $r_0 \gg \ell_s$, otherwise the curvatures will be large and we are not entitled to use the lowest-order action in α' .

From the above formulae we see that:

$$M = \frac{1}{g_s \ell_s} N \quad (4.16)$$

Now, just using supersymmetry one can prove that states charged under A_μ in this theory obey a mass bound:

$$M \geq \frac{1}{g_s \ell_s} N \quad (4.17)$$

Since the above soliton saturates this bound, it must correspond to a stable particle state in the theory. Also we see clearly that although there are no particles in the perturbative string spectrum carrying RR charges, the soliton exhibited above precisely carries such charges.

So far the physical interpretation has been quite conventional. This changed when Polchinski, in a landmark paper[45] in 1995, observed that such branes admit an alternate description as the end points of open strings. The idea is that within a closed string theory, there are dynamical objects with the property that open strings can end on them. In fact the open strings ending on this object provide an alternative description of the object itself.

To end on a fixed object, open strings must have Dirichlet boundary conditions in all directions transverse to the object. These partially break the Lorentz invariance and supersymmetry of the underlying closed string background. This is understandable because the brane in question is an excited state of the closed string theory and therefore (like any excited state) should spontaneously break some of the original symmetries.

As a special example corresponding to a point-like particle, consider an open superstring with Dirichlet boundary conditions on all 9 space directions on each of its two endpoints. To be specific, we restrict both ends to lie at the origin in 9-dimensional space. Being thus nailed down, such a configuration clearly has no centre-of-mass degree of freedom. Therefore the excitations of this open string are all bound to the location of the string end-points. In this situation the effective field theory for the open string is not a 10d field theory at all, but just quantum mechanics on a “world-line” fixed at the origin of space. The claim is that this quantum mechanics provides an alternate description of the pointlike soliton described above in Eqs.(4.12),(4.13).

Let us see why 9 Dirichlet boundary conditions describe a particle. These boundary conditions clearly break Lorentz as well as translation invariance in 10d. However,

$SO(9)$ rotational invariance around the origin is preserved:

$$SO(9, 1) \rightarrow SO(9) \quad (4.18)$$

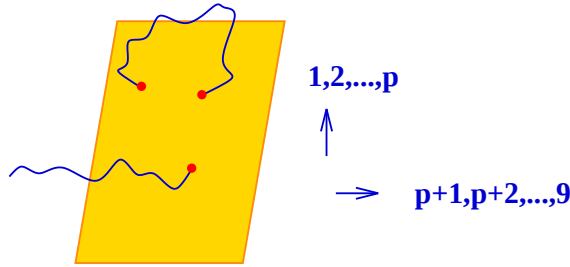
Moreover, in the world-line theory the would-be gauge field A_μ (which would have been present had the ends of the string been free to move) is re-interpreted as a (non-dynamical) gauge field A_0 along with nine scalar fields ϕ^a . Now in field theory, a particle state breaks translational invariance, since translations move the particle to another point. But it preserves rotational invariance around the location of the particle. We see that the endpoint of the Dirichlet open string described above has the right properties to be a dynamical particle. This interpretation provides a nice interpretation for the 9 scalar fields on its worldline: they would be the 9 spatial coordinates of this particle!

With this interpretation, the mass and charge of the string endpoint (which we now refer to as a “D-particle”) can be computed within string theory[45]. The result is that the D-particle carries precisely one unit of charge under the Ramond-Ramond gauge field A_μ . Moreover its mass is:

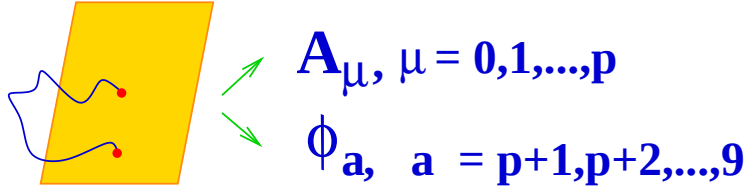
$$M = \frac{1}{g_s \ell_s} \quad (4.19)$$

These properties are consistent with, and support, the notion that the open string endpoint describes a unit-charged version of the RR soliton exhibited as a classical solution in Eqs.(4.12),(4.13).

The above discussion can be generalised to the case where the string has Neumann boundary conditions at each end in some of the 9 space directions, say $1, 2, \dots, p$, and Dirichlet conditions in the remaining $9 - p$ directions.



This defines a p -dimensional hypersurface in spacetime. Generalising the “D-particle”, such a wall is called a “ Dp -brane”. We find that the massless states are now a photon A_μ in $p + 1$ dimensions, as well as $9 - p$ scalar fields ϕ_a (plus, of course, fermions). Clearly the low-energy effective field theory on a Dp -brane is a $(p + 1)$ -dimensional field theory.

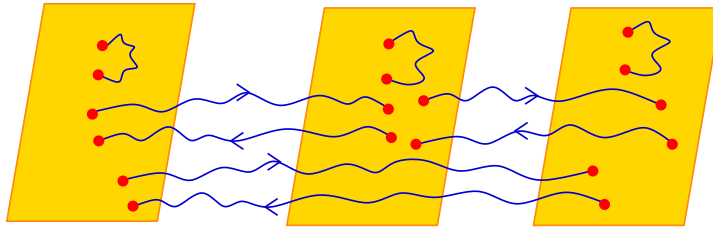


In particular, there is one scalar field for each direction transverse to the brane. As before, the vacuum expectation value of these scalars are naturally interpreted as the transverse locations of the branes.

It was perhaps misleading to refer to a D-brane as a “wall” as we did above. Since it has a fixed tension (which matches that of its dual description as a soliton) it can be deformed in all possible ways simply by providing appropriate amounts of energy. So it is simply an extended dynamical object. A planar D-brane is of course the most symmetric allowed configuration and therefore also the simplest to find as a classical solution.

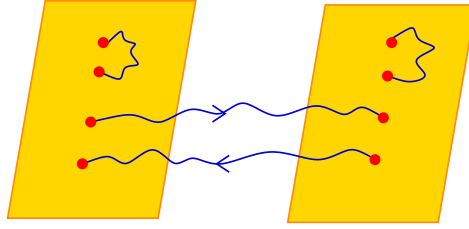
There are stable D-branes charged under each of the RR fields of type IIA/B supergravity. Over the years, each type of brane has provided rich new insights into quantum field theory in the corresponding space-time dimension. By far the most profound insight comes from the AdS/CFT correspondence which we discuss in the following section. Here we briefly describe a simpler physical insight that is a precursor to the correspondence: the origin of non-Abelian gauge symmetry.

For this, let us assemble a collection of N parallel planar D-branes. As gravitating objects one might expect them to attract each other, but due to supersymmetry there are extra exchange forces between them besides the gravitational one, and these neatly cancel so this configuration is stable.



In this array, an open string can start on any one of the branes and end on any other. Thus there are N^2 species of open strings. That means their lowest excitations, each one of them a massless gauge field, can be collected into an $N \times N$ matrix $A_\mu^{\alpha\beta}$.

Let's consider the simplest example, a pair of D3-branes of type IIB string theory:



We see that there are four species of strings. Of these, two are localised on individual branes, so they clearly represent the Abelian gauge field for that brane. Together, these two strings provide $U(1) \times U(1)$ gauge fields. However the two strings stretching across the branes are electrically charged under $U(1) \times U(1)$. Thereby they provide the extra gauge fields to enhance:

$$U(1) \times U(1) \rightarrow U(2) \quad (4.20)$$

If the two D-branes are precisely coincident, then the strings stretching from one to the other can shrink to zero length under their own tension. At this point, all the four gauge fields are massless. If we now separate the branes, two of the four strings acquire a minimum length and therefore a classical energy. So the corresponding gauge fields must be massive. Since transverse motion of the branes is represented by giving a VEV to the transverse scalar fields, this is string theory's geometric realisation of the Higgs mechanism!

Quantising the N^2 strings on a stack of N D3-branes and performing amplitude calculations, one finds that the low-energy effective theory is the Yang-Mills theory of a $U(N)$ gauge field $A_\mu^{\alpha\beta}$, $\alpha, \beta = 1, 2, \dots, N$, coupled to scalars ϕ^a and fermions ψ^A in the adjoint representation of $U(N)$, with the action:

$$\begin{aligned} \mathcal{L} = \text{tr} \left\{ -\frac{1}{4g_{YM}^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} D_\mu \phi^a D^\mu \phi^a - \frac{g_{YM}^2}{4} [\phi^a, \phi^b]^2 \right. \\ \left. + \frac{i}{2} \bar{\psi}^A \gamma^\mu D_\mu \psi^A - g_{YM} \bar{\psi}^A \Gamma_{AB}^a [\phi^a, \psi^B] \right\} \end{aligned} \quad (4.21)$$

where $a, b = 1, 2, \dots, 6$; $A, B = 1, 2, \dots, 4$ and $g_{YM} = \sqrt{g_s}$. All fields in the action are matrices. This action has the maximal supersymmetry allowed for a gauge theory in 4 dimensions, namely $\mathcal{N} = 4$ supersymmetry, and is completely dictated by supersymmetry.

A special property of this theory, arising from the constancy of the dilaton in the classical solution, is that it is conformally invariant. Its β -function in fact vanishes to all orders in g_{YM} , which renders it scale invariant. As often happens in field theory, scale invariance automatically gets promoted to conformal invariance. One consequence is that supersymmetry is enhanced: by commuting special conformal transformations with the usual 16 supersymmetries, one generates 16 new supersymmetries. Among Dp -brane theories for $p = 1, 2, \dots, 9$, this is the only conformally invariant theory.

Amplitude calculations with multiple D-branes explicitly reveal the famous “three-gluon” and “four-gluon” interactions, the signature of Yang-Mills theory:

$$\text{tr } \partial_\mu A_\nu [A^\mu, A^\nu], \quad \text{tr } [A_\mu, A_\nu] [A^\mu, A^\nu] \quad (4.22)$$

In addition, they correct the Yang-Mills action written above with α' corrections involving higher derivatives of the fields.

Besides introducing non-Abelian gauge symmetries into string theory, D-branes also help reformulate familiar notions from field theory and mathematics in a new way. This leads to several novel insights about gauge theory and gravity. Indeed the very conception of string theory and its role as a theory of quantum gravity has undergone a fairly radical change after Polchinski’s discovery. Being intrinsically solitonic, D-branes are non-perturbative objects like magnetic monopoles in field theory (and heavy at weak coupling like these monopoles). Nevertheless, perturbative techniques in open string theory can be brought to bear on their dynamics. In this way one acquires the power to reliably analyse non-perturbative excitations. String theory is no longer just a theory of strings, but of extended branes of all kinds, and their gravitational dynamics is, via open strings, inextricably linked to gauge dynamics.

4.4 M-theory

As already mentioned above, dualities led to the discovery of a new theory called M-theory. It has similar properties to string theory in that it possesses extended objects. However it has no stable strings. It is well-defined only in 11 dimensions, though like string theory it can be reduced to any dimension $d < 11$ by compactification. One reason to believe M-theory is fundamental is that, as noted above, 11 is the highest allowed number of dimensions for a consistent supersymmetric theory.

A significant difference from string theory is that M-theory has no dilaton and consequently no perturbative expansion. However, on compactifying the low-energy action of M-theory from 11d to 10d, we recover the low-energy action of type IIA string theory. Indeed, the dilaton of string theory emerges as a scalar mode of the 11-dimensional metric. This is one more reason why M-theory seems more basic than string theory.

M-theory is not very well-understood, but even at the level at which we understand it, it “explains” many interesting features of string theory. In string theory there are several p -form fields and the objects charged under them are strings and branes. Just considering stable objects, type IIA string theory for example has three different p -form fields and as many as seven different types of strings and branes. However M-theory has a single 3-form field and a basic brane, a membrane (extended in two space dimensions) electrically charged under it. There is also its dual,

a 5-brane that is magnetically charged under the 3-form field (for details, see for example Ref.[44]).

On compactifying M-theory on a circle, these two branes can wrap – or not wrap – the circle and as a result, they give rise to all the branes of string theory[46, 47], including the fundamental string.

Since it is the membrane of M-theory that gives rise to the fundamental string upon compactification, we may guess that this is the closest to a fundamental object in M-theory¹¹. Given that there are no strings in the theory, a fascinating question is what generates interactions between parallel membranes. The answer is that when one has parallel membranes then they can be smoothly be connected by other membranes into a single larger object. After compactifying the theory on a suitable circle, this object reduces to a configuration of type IIA D2-branes and strings running between them. These considerations have led to a new understanding, though still incomplete, of membrane interactions in M-theory[48, 49, 50].

4.5 Black Holes

Being a consistent theory of quantum gravity, string theory can be used as a testing ground to delve into the nature of black holes and the various potential paradoxes surrounding these objects. The nature of black hole evaporation, the entropy of black holes and the possibility of information being lost in black holes were all issues that had been widely discussed for many years. With the discovery of string dualities and D-branes, reliable and precise tests became possible.

The basic idea is to consider string states carrying a fixed set of charges at weak coupling, where they can be thought of as string/D-brane excitations. The number Ω of microscopic states of these excitations can be counted and their logarithm is the “statistical entropy” of the system. By contrast, at strong coupling these states fall inside their own Schwarzschild radius and turn into a black hole[51, 52]. The arguments of Beckenstein and Hawking can then be used to compute the entropy of these black holes in terms of their famous area formula. Thus one had two independently computed quantities:

$$S_{\text{statistical}} = \ln \Omega, \quad S_{\text{black hole}} = \frac{A}{4} \quad (4.23)$$

(both evaluated in suitable units where all dimensional constants are set equal to 1). In principle there might have been little connection between these quantities, arising respectively from the spectrum of states at extremely weak and strong coupling. However in supersymmetric theories it is possible to relate the two extremes under some circumstances due to the remarkable quantum BPS property alluded to earlier.

¹¹In fact, M stands for Membrane among other things.

In a major breakthrough, Strominger and Vafa[53] in 1996 computed both quantities for dyonic extremal charged black holes in 5 dimensions¹². The black-hole calculation is standard, requiring knowledge of the black-hole metric which can then be integrated over the horizon to obtain the area. However the microscopic calculation, as well as the reason why it should be relevant, is special to superstring theory. This computation is based on a configuration of intersecting D-branes of different dimensionalities wrapped around different cycles of the space-time manifold. The system carries fixed electric and magnetic black hole charges Q_e, Q_m , and is described by a field theory on the uncompactified part of the D-brane world-volume, which is two dimensional and also turns out to be conformally invariant. Standard techniques based on the Virasoro algebra then allow one to estimate the degeneracy of states of this theory, at least in the limit that some of the charges are taken large (recall that the black hole picture also makes sense for large charges).

If Q_e, Q_m are the (integer) electric and magnetic charges of the system, the result of Ref.[53] is:

$$S_{\text{statistical}} = 2\pi \sqrt{Q_m \left(\frac{1}{2} Q_e^2 + 1 \right)}, \quad Q_e \text{ fixed, } Q_m \text{ large} \quad (4.24)$$

$$S_{\text{black hole}} = 2\pi \sqrt{\frac{1}{2} Q_m Q_e^2} \quad Q_e, Q_m \text{ both large}$$

Note that the second formula is valid in a more limited range of parameters than the first, so agreement is expected only upon taking Q_e large in the first line. In this limit the two expressions agree perfectly, including the numerical coefficients. This calculation strongly supports the notion that a black hole is fundamentally a statistical ensemble and its entropy is due to the microstates that make it up.

It also suggests a class of generalisations: if one could compute both quantities in the above equation for finite charges, or at least up to some finite order in an expansion in inverse charges, one should find agreement in each order. On the statistical side this requires greater control over the conformal field theory, allowing the computation of the degeneracy of states to higher accuracy as an expansion in large charges. On the black hole side the relevant quantity is the Wald entropy, defined via a formula that generalises the Beckenstein-Hawking formula to theories with higher-derivative terms in the action. It is the Wald entropy rather than the Beckenstein-Hawking entropy that satisfies the second law of thermodynamics in general theories. In string theory we have seen that the higher-derivative terms are uniquely fixed by the background and therefore the Wald entropy sensitively probes the stringy origin of the gravity action. Phenomenal agreement has been found between the corrections to $S_{\text{statistical}}$ and to $S_{\text{black hole}}$ and the field of “precision

¹²The dimensionality was merely a convenience, and analogous results were soon obtained for extremal black holes in four dimensions.

counting” of black hole states has now come into being (for a review see Ref.[54], while more recent results can be found for example in Ref.[55]).

4.6 New insights into gravity III

The triangulation of Riemann surfaces by random matrices correctly reproduces, in the continuum limit, what we expect from summing over continuous surfaces, the latter being a calculation in two-dimensional gravity. It is found explicitly that in the domain of overlap, the continuum approach agrees with the results from matrices. However the matrix approach is actually much more powerful, allowing the computation of amplitudes to all orders in the string coupling. These insights about two-dimensional gravity arose as a kind of by-product of string theory.

But it is D-branes that have brought about the most major revolution in our understanding of quantum gravity through string theory. They are massive solitonic objects, excitations of the closed-string theory, so they of course gravitate and the metric of space-time around them is precisely known via supersymmetry and BPS equations. On the other hand they are described by open strings on their world-volume and the low-energy limit of this theory is a gauge theory of Yang-Mills type. These two descriptions of D-branes amount to a duality, crucial to the black hole entropy calculations sketched above: the gravitating description of D-branes provides the black-hole entropy while the open-string description of the same objects counts the microscopic degrees of freedom. This is an example of what one could call “open-closed duality”. The most profound example of such a duality is the AdS/CFT correspondence, to be discussed in the following section.

5. AdS/CFT correspondence

5.1 Precise statement via heuristic derivation

The discovery of D-branes made it natural to consider a system of open string states inside a closed-string theory. A particular example of such a system is a stack of N D3-branes, on whose volume, as we have seen, the open strings generate an $\mathcal{N} = 4$ supersymmetric gauge theory in four space-time dimensions.

In 1997, Maldacena[56] considered this system in the limit of large N . He observed that on the one hand, at low energies this is a conventional gauge theory (even though supersymmetric and with a gauge group of large rank). On the other hand a brane, viewed as a solitonic object, deforms the spacetime around it so that the geometry near the brane is very different from the flat spacetime far away. For N D3-branes, this local geometry has the form of five-dimensional Anti-deSitter space-time $\text{AdS}_5 \times$ a 5-dimensional sphere. In a path-breaking work using symmetries and dynamical arguments, Maldacena proposed that the two descriptions are exactly

equivalent, so that a four-dimensional gauge theory is the same as a closed string theory (including gravitation!) in a particular 10-dimensional spacetime. This, the latest and perhaps most dazzling of string dualities, has revolutionised our notions about gravity and gauge theory: previously thought of as two totally distinct types of theories, we must now accept that under certain circumstances they can be one and the same theory. Moreover the correspondence is “holographic” in the sense that gravity degrees of freedom are encoded in a gauge theory that lives in a lower number of dimensions.

In concrete terms, consider the Lagrangian of type IIB supergravity (that arises as the low-energy limit of the type IIB string). This time we include the self-dual 4-form Ramond-Ramond gauge field that is present in this theory:

$$S_{type\ IIA} = \frac{1}{(2\pi)^7 \ell_s^8} \int d^{10}x \sqrt{-\|G\|} \left[e^{-2\Phi} (R + |d\Phi|^2) - \frac{2}{5!} |dD^+|^2 \right] \quad (5.1)$$

(technically the self-duality condition makes $|dD^+|^2$ vanish, so we impose that condition after computing the equations of motion). Now we can write the metric for a classical solution corresponding to a “black 3-brane”:

$$ds^2 = \left(1 + \frac{R^4}{r^4}\right)^{-\frac{1}{2}} \left(-dt^2 + \sum_{i=1}^3 dx^i dx^i\right) + \left(1 + \frac{R^4}{r^4}\right)^{\frac{1}{2}} \sum_{a=1}^6 dx^a dx^a \quad (5.2)$$

where $r = \sqrt{x^a x^a}$. Here R is a constant parameter of the solution (not to be confused with the scalar curvature!).

The dilaton is constant in this solution:

$$e^{-2\Phi} = e^{-2\Phi_0} \quad (5.3)$$

while the 4-form potential is:

$$D_{0123}^+ = -\frac{1}{2} \left[\left(1 + \frac{R^4}{r^4}\right)^{-1} - 1 \right] \quad (5.4)$$

The quantised charge N of this solution is easily found to be:

$$N = \frac{R^4}{4\pi g_s \ell_s^4} \quad (5.5)$$

Notice that in 10 dimensions, a 3-brane is enclosed by a 5-sphere and the integral of the field strength dD^+ over this 5-sphere measures the total charge N .

We now examine physics of a test particle in this field. The coefficient of $-dt^2$ tells us there is a redshift between the energy measured at some radial distance r and at ∞ :

$$E_\infty = \left(1 + \frac{R^4}{r^4}\right)^{-\frac{1}{4}} E_r \quad (5.6)$$

This means that a given object near $r \rightarrow 0$ has a very small energy when measured from infinity. Define:

$$U \equiv \frac{r}{\ell_s^2} \quad (5.7)$$

which is a spatial coordinate with dimensions of energy. Then, multiplying through by ℓ_s , we find:

$$E_\infty \ell_s = \left(1 + \frac{4\pi g_s N}{(U \ell_s)^4}\right)^{-\frac{1}{4}} E_r \ell_s \quad (5.8)$$

This shows that from the point of view of an observer at infinity, low energy $E_\infty \ell_s \ll 1$ means:

$$U \ell_s \ll 1 \quad \text{or} \quad E_r \ell_s \ll 1 \quad (5.9)$$

Indeed this low energy limit can be thought of as $\ell_s \rightarrow 0$ with energies held fixed.

In the first regime, the metric of the D3-brane becomes:

$$ds^2 = \sqrt{4\pi g_s N} \ell_s^2 \left[\frac{U^2}{4\pi g_s N} (-dt^2 + dx^i dx^i) + \frac{dU^2}{U^2} + d\omega_5^2 \right] \quad (5.10)$$

This is the metric of the spacetime $\text{AdS}_5 \times S^5$. There is also an RR field strength in the classical solution. The second regime instead describes states of small proper energy in units of ℓ_s^{-1} . Such states correspond to the $\ell_s \rightarrow 0$ limit of supergravity, which is free in this limit.

Next one uses the dual description of D3-branes as open-string endpoints. In this description, the system is described by an effective action for open strings plus an action for closed strings plus an action describing open-closed couplings:

$$S = S_{\text{open}} + S_{\text{closed}} + S_{\text{open-closed}} \quad (5.11)$$

Taking $\ell_s \rightarrow 0$ keeping energies fixed, the closed-string part (supergravity) becomes free and the open-closed couplings also vanish. Finally, in the open-string part, the higher-derivative terms disappear since they are proportional to powers of ℓ_s . The surviving action is the $\mathcal{N} = 4$ supersymmetric Yang-Mills field theory written in Eq. (4.21), with gauge group $U(N)$ and coupling constant $g_{YM} = \sqrt{g_s}$.

Thus comparing the two sides we see that each one has a free supergravity action, which can be equated. The remaining part, which can also be equated, is (i) string theory in the curved background $\text{AdS}_5 \times S^5$, and (ii) $\mathcal{N} = 4$ supersymmetric Yang-Mills field theory. The AdS/CFT correspondence is the conjecture that these two theories are the same.

Unlikely as it may seem, this conjecture says that string theory in a particular bulk spacetime is equal to a conformal-invariant field theory in a (conformally) flat space-time that corresponds to the boundary of the original bulk space-time. Moreover the dimensions of the two theories are 10 and 4 respectively.

The above was a heuristic derivation following Ref.[56], but the AdS/CFT correspondence has not yet been rigorously proven. It has been tested in many different ways though, and some of these tests are briefly described below.

5.2 Matching symmetries: isometries

To test the AdS/CFT correspondence, one can first check that the symmetries match on both sides. The isometries of $\text{AdS}_5 \times S^5$ are:

$$\begin{aligned} \text{AdS}_5 : & \quad SO(4, 2) \\ S^5 : & \quad SO(6) \end{aligned} \tag{5.12}$$

On the other side of the correspondence we have a conformally invariant field theory, $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. It clearly possesses $SO(3, 1)$ symmetry, namely Lorentz invariance. Another symmetry we see right away is global $SO(6)$ invariance which rotates the six scalar fields ϕ^a that describe transverse motions of the D3-brane. The remaining desired symmetries arise from the fact that whenever a field theory has conformal invariance, this symmetry combined with Lorentz invariance gives rise to an enhanced symmetry group:

$$SO(d, 1) \rightarrow SO(d + 1, 2) \tag{5.13}$$

Thus indeed, $\mathcal{N} = 4$ supersymmetric Yang-Mills theory has $SO(4, 2) \times SO(6)$ symmetry, just like the isometries of $\text{AdS}_5 \times S^5$. This is a successful test of the AdS/CFT correspondence.

Another test follows from matching supersymmetry. Closed superstrings propagating in flat spacetime have $\mathcal{N} = 2$ supersymmetry in 10 dimensions. The supercharges have 16 components each, making a total of 32 components. The only other 10 dimensional space-time with the same number of supersymmetry charges is $\text{AdS}_5 \times S^5$. $\mathcal{N} = 4$ SYM theory has 4 spinor supercharges, each with 4 components. Therefore there are apparently just 16 supersymmetries. However, as we mentioned earlier, taking the commutator of special conformal transformations with supersymmetries gives rise to a new set of supersymmetries, also 16 in number.

Thus at the end, both sides have 32 supersymmetries. In fact one can show that:

$$SO(4, 2) \times SO(6) \times \text{susy} \subset SU(2, 2|4) \tag{5.14}$$

where the RHS is a particular super-algebra, which is a symmetry of both sides of the AdS/CFT correspondence.

5.3 Parameters and gravity limit

The proposed duality is so nontrivial that, beyond symmetries, it is not immediately obvious how to test it or use it. One major obstacle is that string theory on $\text{AdS}_5 \times S^5$

has Ramond-Ramond flux. We do not know how to study strings propagating in the presence of such backgrounds. Thus we are forced to restrict ourselves to the low-energy effective action of string theory, namely supergravity. This is valid in the weakly curved case:

$$R \gg \ell_s \quad (5.15)$$

which amounts to:

$$\lambda \equiv g_{YM}^2 N \gg 1 \quad (5.16)$$

At the same time we must restrict to tree-level, since supergravity is a non-renormalisable theory so loop diagrams will not make sense. Therefore we must have:

$$g_s \ll 1 \implies g_{YM} \ll 1 \quad (5.17)$$

It follows that the gauge theory must have $N \gg 1$. Indeed, the behaviour of gauge theories at large N (and the simple example of random matrices discussed above) were among the earliest indications that field theory is related to string theory!

5.4 Gravity-CFT dictionary

There is a precise dictionary between gravity variables and gauge theory variables, that is known explicitly in many cases. The general proposal[57, 58] is that to each gauge-invariant operator $\mathcal{O}(x^\mu)$ in the SYM theory, there corresponds a field $\phi(x^\mu, U)$ in supergravity such that:

$$\left\langle \exp \left(\int d^4x J(x^\mu) \mathcal{O}(x^\mu) \right) \right\rangle_{\text{gauge theory}} = \mathcal{Z}_{\text{supergravity}} \left(\phi(x^\mu, U) \Big|_{\phi(x^\mu, U \rightarrow \infty) = J(x^\mu)} \right) \quad (5.18)$$

Here the LHS is a gauge theory correlation function in 4 dimensions. The RHS is the gravity partition function evaluated on 5-dimensional fields $\phi(x^\mu, U)$ in AdS_5 , but with their values constrained to be equal to the source $J(x)$ on the boundary of AdS_5 . We can generalise this to supergravity fields that depend on the S^5 coordinates, by Fourier decomposing them on S^5 and treating each Fourier mode as an independent field on AdS_5 .

As a relatively simple example, consider the marginal operator which changes the gauge theory coupling constant. This is just the entire Lagrangian of the gauge theory! In the gravity dual the corresponding field in 5 dimensions is the dilaton operator $\Phi(x^\mu, U)$. Its value on the boundary of AdS_5 determines the coupling of the gauge theory. Thus in this case the correspondence is:

$$\begin{array}{ll} \text{Operator in gauge theory} & \iff \text{Field in supergravity} \\ -\frac{1}{4} \text{tr} F_{\mu\nu} F^{\mu\nu}(x^\mu) + \dots & \iff \Phi(x^\mu, U) \end{array} \quad (5.19)$$

We see that the extra holographic dimension on the gravity side is the radial direction U . This can be shown to correspond to an energy scale in the field theory. Conformal invariance of the field theory is natural in this interpretation. The dilaton background is constant in the AdS classical solution, therefore in particular it is independent of U . Therefore the dual field theory is independent of energy scale, which in turn implies conformal invariance.

If we want to generalise AdS/CFT to have a scale-dependent theory like QCD on the gauge theory side, then the dual spacetime must be different from AdS in the interior, and the dilaton must be a non-trivial function of U .

5.5 An application of the correspondence

Since one side of the correspondence is classical gravity, which is relatively easy to study, we can use it to deduce properties of quantum gauge theories at large N [59]. In nature we do not want to know about the gauge theory called $\mathcal{N} = 4$ supersymmetric Yang-Mills, but about Quantum Chromodynamics, however at finite temperature the $\mathcal{N} = 4$ SYM can be shown to resemble Quantum Chromodynamics in some ways.

We start by placing the gauge theory not on $R^{3,1}$ but on $S^3 \times S^1$. This in particular requires us to make the theory Euclidean, corresponding to finite temperature. If β is the radius of S^1 , then the temperature is:

$$T = \frac{1}{\beta} \tag{5.20}$$

We also define the radius of S^3 to be β' .

Conformal invariance then tells us the theory depends only on the dimensionless ratio β/β' . It has been shown[59] that there are two candidate gravity duals to this theory. One is a spacetime called thermal AdS (like AdS₅ but at finite temperature). The other is a Schwarzschild black hole which asymptotically becomes AdS. Which of these two is the correct gravity dual depends on the temperature, more precisely on β'/β . At small values of this parameter (low temperature) the thermal AdS dominates the path integral. At high temperatures instead it is the AdS black hole.

Now the gravity description can be used to compute the entropy in each case. At low temperatures it is found that:

$$S \sim 1 \tag{5.21}$$

while at high temperatures, the Bekenstein-Hawking formula for black holes gives us:

$$S \sim R^3 \times R^5 \sim R^8 \sim N^2 \tag{5.22}$$

The jump from one to another AdS dual of the field theory as we vary temperature is a phase transition, and is interpreted as the deconfinement phase transition of the

gauge theory! The numbers fit beautifully with the fact that at low temperature the gauge theory has only singlet states but at high temperature the gluon degrees of freedom, which are N^2 in number, are liberated¹³.

We see the power of the AdS/CFT correspondence in extracting analytic information about confinement. Though the gauge theory in this discussion is not a realistic one, there have been many attempts to generalise the above discussion to more realistic confining examples and this remains an active direction of research.

5.6 Gravity and fluid dynamics

Gravitational theories which allow asymptotically AdS space-times can be truncated to pure Einstein gravity with a (negative) cosmological constant. This implies a certain universality property for the dual gauge theory on the boundary, which in turn can be encoded in the dynamics of the gauge-invariant fields, of which the most important class are the single-trace operators of the type:

$$\mathcal{O}_n = \text{tr } f_n(F_{\mu\nu}, \phi^a, \dots) \quad (5.23)$$

It has been shown[60] that one can use the AdS/CFT correspondence to obtain universal predictions for the transport coefficients of a gauge-theory plasma, qualitatively (in some ways) similar to the quark-gluon plasma produced in relativistic heavy-ion collisions.

If we consider boosted extended black holes (really branes) in AdS space-time having a definite temperature and velocity. Via holography these get related to the temperature and velocity of asymptotically AdS solutions to the Einstein equations. As a result one finds a beautiful relation between the Einstein equations and Navier-Stokes equations, the equations of fluid dynamics (see for example Refs.[61]).

This approach has practical utility in understanding strongly coupled plasmas that are hard to study using other more conventional formalisms. But its most striking feature is that it correlates fundamental properties of hydrodynamics and gravitation – a remarkable synthesis of disparate ideas with a long history.

5.7 New insights about gravity IV

The AdS/CFT correspondence tells us that gravity and gauge theory are not two different types of forces but instead are equivalent to each other. This may seem very bizarre at first sight. However the different dimensionalities of the gravity and gauge theory suggest that the transcription between them is not going to be straightforward, and locality of the bulk theory is a particularly striking mystery¹⁴. Despite

¹³This discussion may seem somewhat confusing given that $\mathcal{N} = 4$ supersymmetric Yang-Mills theory does not exhibit confinement. This and other subtleties of the present discussion are explained in Ref.[59].

¹⁴On which steady progress is being made, see Ref.[62].

useful examples, the full dictionary between gravity and gauge theory concepts is complicated and not fully understood. However one key illumination is that the radial direction in AdS gravity is related to the energy scale of the gauge theory. Therefore from the gauge theory point of view, motion along this direction is seen as a renormalisation group flow! (see Ref.[63] and for very recent work in this direction, Ref.[64]).

The correspondence described here is a realisation of an older conjecture known as the “holographic principle”, which says that the degrees of freedom of quantum gravity propagating in a bulk region can be encoded on the boundary of that region. Just as the comparison of microscopic and macroscopic entropy of black holes in string theory provided a concrete realisation of something that had previously been conjectured, here too we have for the first time a concrete realisation of a principle that had previously been articulated on physical grounds by ’t Hooft[65], and Susskind[66].

To date, the AdS/CFT correspondence has primarily been used to gain information about strongly coupled gauge theories using the dual classical, weakly-coupled supergravity. This is of physical interest because of QCD and speculative theories beyond the Standard Model, and also because of strongly coupled systems in condensed matter theory, which for reasons of space we have not been able to discuss here¹⁵. Information in the reverse direction would be highly desirable in order to grapple with conceptual problems of quantum gravity. This has been slightly less forthcoming because the domain in which gauge theory is relatively easy to study is that of weak coupling, in which region the dual AdS superstring propagates in a highly curved space-time of sub-stringy size. This is not directly relevant to the world today.

However as we briefly discuss below, a sub-stringy holographic world would be very relevant for studies of the early universe. And of course the work on black hole entropy described earlier makes use of information from the gauge-theory side in important ways. Though some of the initial ideas for deriving microscopic black hole entropy in string theory were developed before the AdS/CFT correspondence had been precisely articulated, today these ideas have found a natural place in the context of holography which has also provided many generalisations. It is likely that the next decade will see increasing feedback from the AdS/CFT correspondence and its generalisations to address conceptual issues in quantum gravity.

6. Flux compactifications

The old problem of string theory compactifications, namely the large number of undetermined moduli or flat directions, also benefitted from the AdS/CFT correspon-

¹⁵But see Ref.[67] for a review.

dence which led to a new line of attack. It had been known that nontrivial fluxes in the internal manifold could fix some of the moduli by providing a potential for them. Using inspiration from AdS spacetime, it was then realised that such fluxes typically lead to “warped” backgrounds of string theory: backgrounds in which the metric at a given point of the noncompact spacetime depends on the point in the internal space. Such warped backgrounds can have an AdS-like “throat” region in the internal space and one can thereby obtain a desirable separation of scales. Flux compactifications use combinations of D-branes, anti-branes and orientifolds¹⁶ to generate, among other things, a positive cosmological constant and broken supersymmetry[68] (see also earlier work on flux stabilisation in Refs.[30, 69, 70]).

Flux compactifications are the closest we have come to realising the Standard Model of particle physics, as well as a realistic gravitational sector including a cosmological constant, in the context of string theory. Their success has paradoxically raised a problem: it seems likely that string theory admits an immense number of vacua, known as the “landscape”, and many of them lie arbitrarily close to the real world. The problem of finding “the right vacuum” in this situation looks very daunting. We have ended up very far from the original hope that a simple, almost unique compactification of string theory would lead to a viable description of the real world.

7. String Cosmology

The initial analyses of D-branes focused on charged, stable branes, and for a while it was not noticed that string theory admits other D-branes that are uncharged and unstable. One way to think of them is to consider a pair of a D-brane and an anti-D-brane, the net charge of this system being zero. Such a system would generically be unstable and decay.

It has been shown (a review can be found in Ref.[71]) that the decay of these branes can be understood in terms of a tachyonic particle that arises from quantising the open string ending on the brane (this differs from the closed-string tachyon that we encountered in bosonic string theory). The rolling of the tachyon down to the bottom of its potential well is the world-volume description of the process in space-time wherein the brane decays. This “rolling tachyon” paradigm has proved very influential in string-inspired cosmology.

Cosmology involves the study of regions of space-time (particularly relevant in the early universe) where conventional physics breaks down because of the possible occurrence of singularities. Holography can become a conceptual necessity in situations where the geometric description of space-time itself is in question – one may then have no choice but to shift from a macroscopic picture (gravity) to a dual microscopic picture. Finally, an ultraviolet completion of general relativity is required

¹⁶Singularities similar to orbifolds but which carry RR charge.

when energies reach the Planck scale. Because of the many different properties of string theory described in this article, it is a prime candidate to address and potentially solve all these problems (these observations and the ones that follow are elaborated in the review article Ref.[72]).

Discussions about cosmology in string theory are typically made in the framework of non-supersymmetric configurations of multiple D-branes, referred to above, with the tachyon playing an important role in the dynamics. Of course such discussions require a sensible background space-time, for which one needs at least a semi-realistic model. One of the most important constraints on such backgrounds is that the moduli associated to the string compactification be completely stabilised.

The primary goals of string cosmology would be to provide a precise mechanism for inflation and an explanation of dark energy – the latter being arguably the single biggest puzzle in our microscopic picture of the world. Importantly, there can be conflicts between such mechanisms and the mechanism for moduli stabilisation. Also, inflation can potentially screen from observation the kind of microscopic information about the early universe that a particular string model might provide.

In addition to addressing basic open questions about the origin of the universe, string cosmology potentially provides an experimental testing ground for string theory itself. This would require identifying observational features of string theory that are hard, or impossible, to reproduce in field theory. The nature of corrections to the cosmic microwave background provides a possible class of examples. Despite (or perhaps because of) these difficulties, string cosmology is currently among the most fertile fields of research within string theory.

8. Conclusions

It may not be wrong to say that the relevance of string theory to the real world it still to be completely determined. However the question of “relevance” in this context has evolved far beyond what anyone could have imagined in 1985.

As a candidate for a unified theory of all fundamental interactions, string theory is extremely compelling but a precise theory with a set of unambiguous predictions is indeed not yet available. One key obstacle is the the landscape problem. Far from there being no accurate string description of the real world, it looks likely that there could be so many of them as to make the discovery of the “right” one virtually impossible. This is an issue on which new ideas are awaited.

However, string theory is also a potent and precise formalism that generalises and probably supplants quantum field theory. It provides an ultraviolet consistent theory of quantum gravity wherein singularities are typically smoothed out. At low energies it reduces to ordinary Einstein gravity and its perturbation expansion can

be used to compute quantum corrections to gravitational processes. Moreover it has a non-perturbative structure of which many key aspects are well-understood. This allows it to be used as an unparalleled and accurate testing ground for concepts in quantum gravity.

String theory has provided the provided the most convincing evidence that black holes have a large number of microscopic states that account for their entropy. This has essentially settled in the affirmative the fundamental question of whether the thermodynamic nature of gravity arises from microscopic states, just like the relation between usual thermodynamics and statistical mechanics. It is reasonable to expect that the information-loss paradox will similarly be addressed, and perhaps resolved, using string theory in the near future.

The usefulness of strings in providing a holographic description of the strong force and associated strongly coupled fluids is demonstrated and steadily growing. It is widely expected that the actual QCD string, the very basis for the invention of string theory, will be found in the near future.

The application of string models to study the early universe promises to uncover the secrets of the big bang. Given the increasing body of observations pertinent to the early universe, it is just conceivable that this direction could also yield the long sought-for experimental tests of string theory.

Perhaps the single most important point is that all the above applications of string theory do not exist independently of each other. Gravity, gauge symmetry, compactification, unification, holography, hydrodynamics, strongly correlated systems and cosmology, and probably many more aspects of physics, are inextricably linked to each other in this remarkable and profound edifice.

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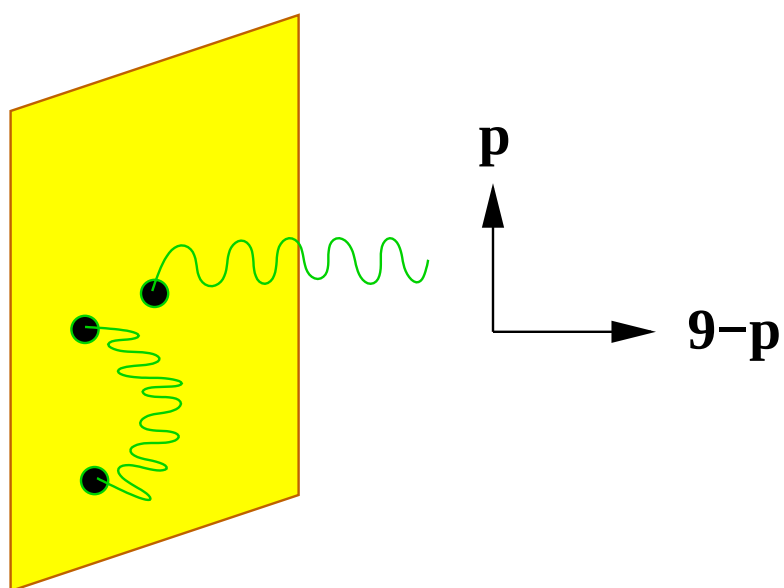
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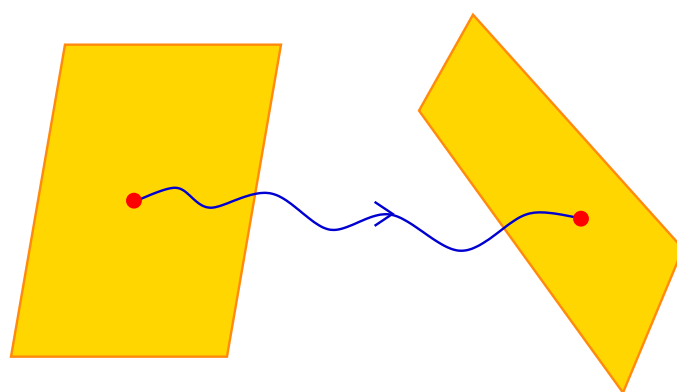
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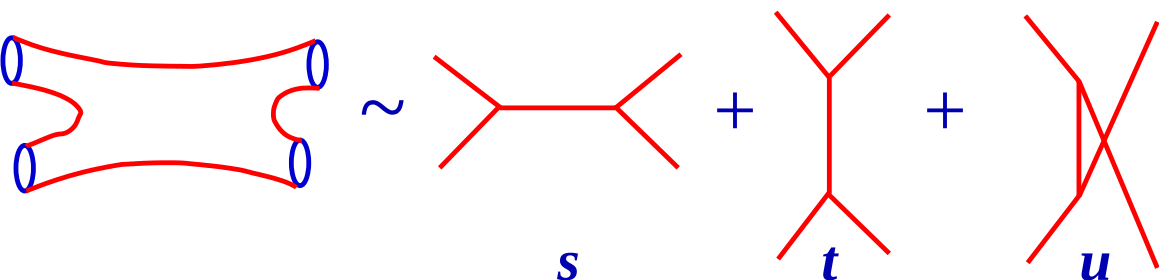
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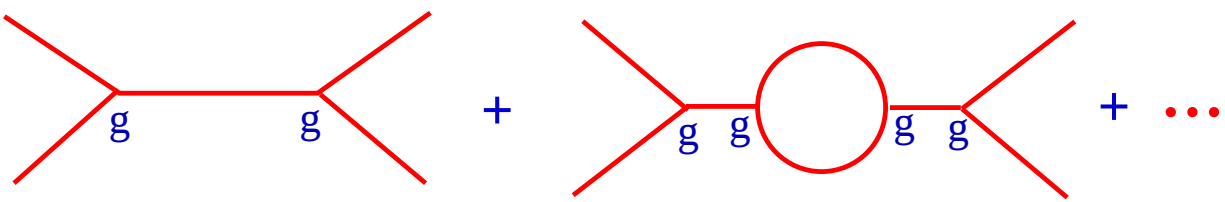


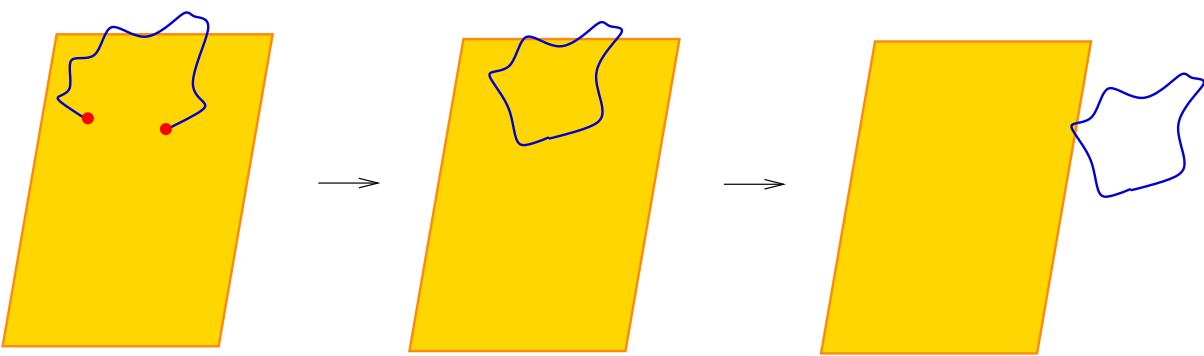
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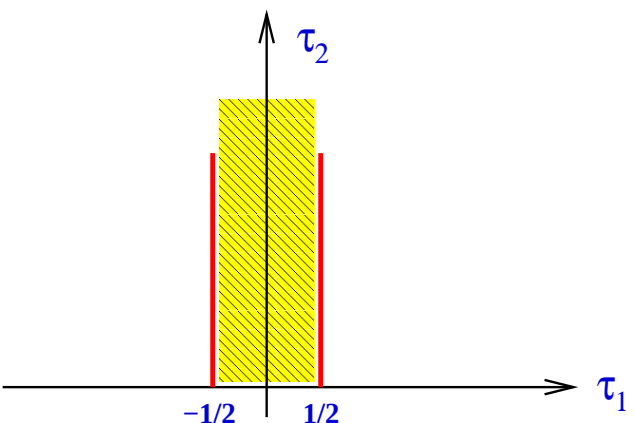


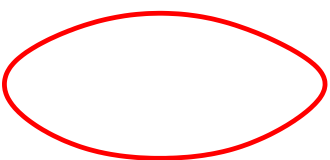




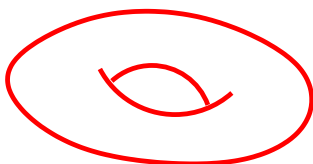




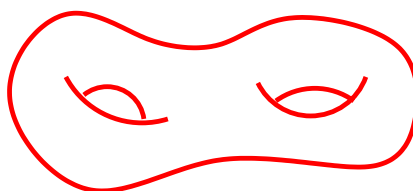




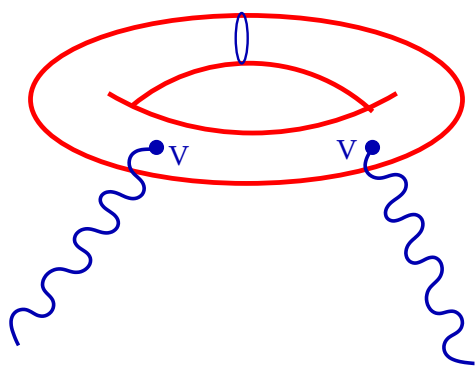
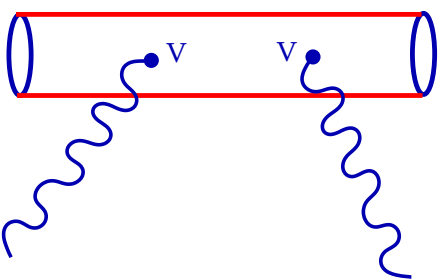
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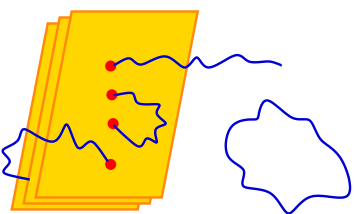


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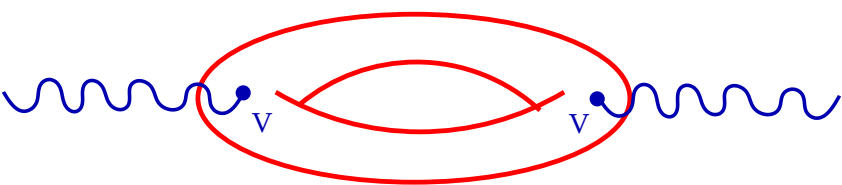


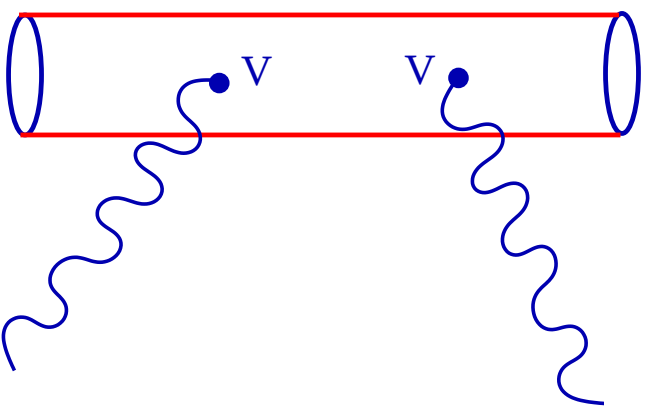
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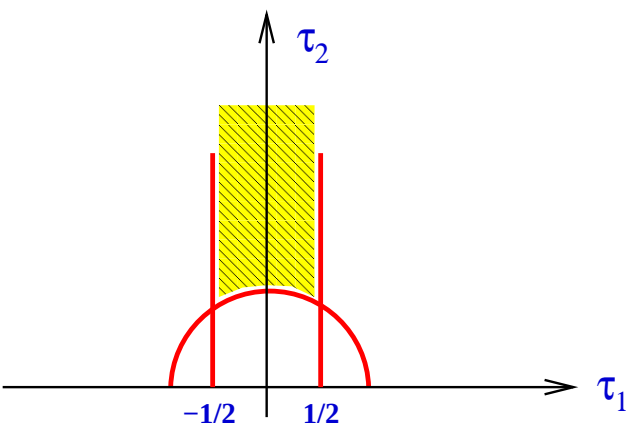


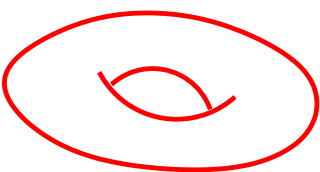


N D-branes

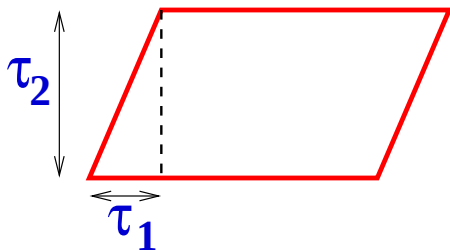


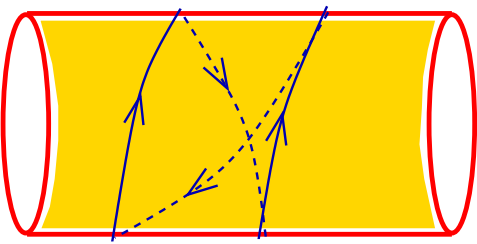






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