Unconventional Supersymmetry via the Dressing Field Method

J. François*
University of Graz (Uni Graz), Heinrichstrase 26/5, 8010 Graz, Austria, and
Masaryk University (MUNI), Kotlářská 267/2, Veveří, Brno, Czech Republic, and
Mons University (UMONS), 20 Place du Parc, 7000 Mons, Belgium.

L. Ravera†
Politecnico di Torino (PoliTo), C.so Duca degli Abruzzi 24, 10129 Torino, Italy, and
Istituto Nazionale di Fisica Nucleare (INFN), Section of Torino, Via P. Giuria 1, 10125 Torino, Italy, and
Grupo de Investigación en Física Teórica (GIFT),
Universidad Católica De La Santísima Concepción, Concepción, Chile.

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In this letter, we show that the so-called *matter ansatz* at the basis of the three-dimensional model of unconventional supersymmetry by Alvarez-Valenzuela-Zanelli (AVZ) is to be understood as an instance of the *Dressing Field Method*. The latter is a systematic way to exhibit the gauge-invariant, *relational* content of general-relativistic gauge field theories. It is shown here to be foundational to (unconventional) supersymmetry: The spin-1/2 Dirac field of the AVZ model being obtained as a supersymmetry invariant variable, encoding physical relational degrees of freedom.

Keywords: Unconventional supersymmetry, Dressing Field Method, Gauge invariance, Relationality.

I. INTRODUCTION

Supersymmetry (susy) is commonly conceived as a candidate unifying fundamental symmetry in particle physics, relating bosons and fermions and packing them in supermultiplets. This view implies the existence of superpartners, which can be accommodated only through the notion of Spontaneous Supersymmetry Breaking. Yet, until now they fail to show up in colliders and probability is low for susy to be confirmed in the foreseeable future.

Nonetheless, the framework of supersymmetric field theory – which does not necessarily require the matching of bosonic and fermionic degrees of freedom (d.o.f.) [1] – can be applied to established physics. One may recall that Berezin pioneered the introduction of supergeometry in physics to describe particles with half-integer spin [2]. Supersymmetric field theory has also been applied to QCD [3] and Condensed Matter Physics (CMP) [4–6]. Another, more recent, prominent application in this direction is given by the so-called *unconventional supersymmetry* (ususy) [7–12], originally called "supersymmetry of a different kind" – see also [13] (and [14]) for its relation with supergravity. In ususy both the bosonic and the fermionic fields are components of a (super)connection – which is often phrased as "the fermions transforming in the adjoint representation under susy".

The first ususy model, presented in [7] by Alvarez, Valenzuela and Zanelli (AVZ model hereafter), is a Chern-Simons (CS) theory in $2+1$ dimensions for the supergroup OSp(2|2). The only propagating field is a spin-1/2 Dirac spinor χ with a mass term given by the three-dimensional negative cosmological constant. The Dirac field is minimally coupled to the background gravitational field and a U(1) gauge field. The model was shown to have important applications in CMP. In particular, it was shown to describe graphene-like systems near the Dirac points in a generic spatial lattice with curvature and torsion [15–25].

In the AVZ model, the fermionic 1-forms $\psi^\alpha{}_A = \psi^\alpha{}_{A|\mu} dx^\mu$, $\alpha = 1,2$, $A = 1,2$, $\mu = 0,1,2$, in the bi-fundamental of the $\mathrm{Sp}(2) \times \mathrm{SO}(2)$ group, are written in terms of spin-1/2 fields $\chi^\alpha{}_A$ (0-forms) through of the so-called *matter ansatz* [11] – here and in the following, we frequently omit spinor and R-symmetry indices, to lighten the notation:

$$\psi = i\gamma_a e^a \chi, \tag{1}$$

where e^a, $a = 0,1,2$, are the soldering forms, whose components are the dreinbein of the three-dimensional spacetime, and γ^a are the corresponding gamma matrices.

In [26, 27], the matter ansatz (1) was said to correspond to a "projection" of the vector-spinor ψ, in which its gamma-traceless component vanishes, while later, in [28], it was claimed to be the result of a gauge fixing – more precisely, a BRST-covariant gauge fixing of the odd symmetries.

In this work, we show that (1) is actually a case of application of the so-called *Dressing Field Method* (DFM) of "symmetry reduction" [29] to construct gauge-invariant variables.

The DFM is best understood within the formalism of the bundle differential geometry of field space [30–32], but it also has a simple field-theoretic framing [33]. For the conceptual and mathematical difference between gauge fixing and dressing, we refer the reader to [34, 35]. The DFM has a natural *relational* interpretation, as gauge-invariance is achieved by extracting the *physical* d.o.f. representing *relations* among field variables

* jordan.francois@uni-graz.at
† lucrezia.ravera@polito.it

[31, 36, 37]. The pivotal role of relationality in supersymmetric theories has been surprisingly overlooked in the literature. In [35, 37] it was shown for the first time that the DFM, together with its relational interpretation (i.e., in terms of *relational observables*), is foundational to general-relativistic Gauge Field Theory, including supersymmetric field theories. Here we show that it is foundational also to ususy.

The goal of the present investigation is threefold:

1. To prove that the matter ansatz is a case of dressing.
2. To elucidate the origin and meaning of the spin-1/2 field χ, as a *relational variable*.
3. To clarify the (residual) symmetries and the physical d.o.f. of the AVZ model.

The first objective will be achieved via DFM. This result can be extended to any number of spacetime dimensions, in principle, beyond the AVZ model. Through the second and third points it will be evident that in ususy models supersymmetry is actually "reduced", via dressing, by creating relational variables. So, we give for the first time a *relational interpretation of ususy*. This provides a new first principle guide to properly study (u)susy theories.

The remains of this letter is organized as follows: In Section II we provide a dense streamlined presentation of the AVZ model of ususy, stressing key conceptual points regarding its geometric underpinning. In Section III we explain the DFM and show that the AVZ ansatz is a case of dressing, highlighting its relational character. In Section IV we discuss the residual gauge symmetries of the AVZ model, also clarifying the status of the Nieh-Yan-Weyl (NYW) symmetry [38–42]. In Section V we conclude and discuss future developments.

II. THE AVZ MODEL OF USUSY

In the following, we shall first detail the kinematics and then the dynamics of the AVZ model of unconventional supersymmetry, highlighting the core points of its geometric underpinning.

A. Kinematics of the model

The field-theoretic presentation of the AVZ model [7] is based on a supergeometric framework in three spacetime dimensions, with rigid supergroup $H = \mathrm{OSp}(2|2)$, whose associated Lie superalgebra is $\mathfrak{h} := \mathfrak{osp}(2|2)$. We shall not exploit much of this supergeometric framework, save to remind that the vertical automorphism group of the superbundle induces, at the field-theoretic level – i.e. on the base (bosonic) spacetime manifold M – the gauge group $\mathcal{H} = \mathcal{OS}p(2|2) := \{g, g' : U \subset M \to H \,|\, g'^g = \mathrm{Conj}(g^{-1})\, g' := g^{-1}g'g\}$, i.e. the gauge group acts on its own elements via the conjugation action. Correspondingly, the gauge algebra is Lie$\mathcal{H}$ =Lie$\mathcal{OS}p(2|2) := \{\lambda, \lambda' : U \to \mathfrak{h} \,|\, \delta_\lambda \lambda' = \mathrm{ad}(-\lambda)\,\lambda' = [\lambda', \lambda]\}$, i.e. the gauge algebra elements are ad-tensorial 0-forms. We have $g(x) = e^{\lambda(x)}$. We shall use the compact matrix notation

$$\lambda = \begin{pmatrix} \beta & \frac{1}{\sqrt{\ell}}\varepsilon \\ -\frac{1}{\sqrt{\ell}}\bar{\varepsilon} & \alpha \otimes J \end{pmatrix}, \tag{2}$$

where β takes values in $\mathfrak{sp}(2, \mathbb{R}) \simeq \mathfrak{sl}(2, \mathbb{R}) \simeq \mathfrak{spin}(1, 2)$, $\varepsilon \in \mathbb{C}^2$ is a spinor field with $\bar{\varepsilon} := \varepsilon^\dagger \gamma_0 = -\varepsilon^\dagger i J$, α is $\mathfrak{u}(1) = i\mathbb{R}$-valued and $\alpha \otimes J$ is to be treated as a scalar in the matrix λ even though J is the symplectic matrix (in component $J \sim \epsilon_{AB}$) acting on ε as $J\varepsilon$ and on $\bar{\varepsilon}$ as $\bar{\varepsilon}J$. The defining transformation property of the infinitesimal gauge parameters is then

$$\delta_\lambda \lambda' = \begin{pmatrix} \delta_\lambda \beta' & \frac{1}{\sqrt{\ell}}\delta_\lambda \varepsilon' \\ -\frac{1}{\sqrt{\ell}}\delta_\lambda \bar{\varepsilon}' & \delta_\lambda \alpha' \otimes J \end{pmatrix} \tag{3}$$
$$:= \begin{pmatrix} [\beta', \beta] - \frac{1}{\ell}\left(\varepsilon' \bar{\varepsilon} - \varepsilon \bar{\varepsilon}'\right) & \frac{1}{\sqrt{\ell}}\left[(\beta' - \alpha' \otimes J)\varepsilon - (\beta - \alpha \otimes J)\varepsilon'\right] \\ * & \frac{1}{2\ell}\mathrm{Tr}(\varepsilon' \bar{\varepsilon} - \varepsilon \bar{\varepsilon}') \otimes J \end{pmatrix},$$

the factor $\frac{1}{2}$ in the (2,2) entry being given by prescription. This then encodes the commutators between generators of $\mathfrak{h} = \mathfrak{osp}(2|2)$, in terms of which a Lie$\mathcal{H}$ element reads $\lambda(x) = \frac{1}{2}\beta^{ab}(x)\mathbb{J}_{ab} + \bar{\mathbb{Q}}\varepsilon(x) - \bar{\varepsilon}(x)\mathbb{Q} + \alpha(x)\mathbb{T}$, with $\mathbb{J}$, $\mathbb{Q}$, $\bar{\mathbb{Q}}$, and $\mathbb{T}$ the generators of Lorentz, susy and U(1) transformations, respectively.

The superconnection adopted in the AVZ model is the 1-form $\mathbb{A} = \mathbb{A}_\mu dx^\mu = \frac{1}{2}\omega^{ab}\mathbb{J}_{ab} + i\,\bar{\mathbb{Q}}\,\gamma\chi + i\,\bar{\chi}\gamma\,\mathbb{Q} + A\,\mathbb{T}$. In the above compact matrix form,

$$\mathbb{A} = \begin{pmatrix} \omega & \frac{i}{\sqrt{\ell}}\,\gamma\chi \\ \frac{i}{\sqrt{\ell}}\,\bar{\chi}\gamma & A \otimes J \end{pmatrix}, \tag{4}$$

where χ is a spin-1/2 0-form field and γ denotes the gamma matrix 1-form $\gamma = \gamma_\mu dx^\mu = \gamma_a e^a{}_\mu dx^\mu = \gamma_a e^a$, with $e^a = e^a{}_\mu dx^\mu$ the soldering form on M inducing a metric with signature $(+, -, -)$. Our convention for the gamma matrices in $2+1$ dimensions is $\gamma_0 = \sigma_2 = -iJ$, $\gamma_1 = -i\sigma_1$, $\gamma_2 = -i\sigma_3$. The parameter $1/\ell$ is related to the negative cosmological constant Λ by $\Lambda = -1/\ell^2$. The associated curvature 2-form is, by Cartan structure equation, $\mathbb{F} = d\mathbb{A} + \frac{1}{2}[\mathbb{A}, \mathbb{A}] = d\mathbb{A} + \mathbb{A}^2$. In matrix form,

$$\mathbb{F} = \begin{pmatrix} \Omega & \frac{i}{\sqrt{\ell}}\varrho \\ * & \tilde{F} \otimes J \end{pmatrix}$$
$$:= \begin{pmatrix} R - \frac{1}{\ell}\gamma\chi\,\bar{\chi}\gamma & \frac{i}{\sqrt{\ell}}\left(\gamma_a T^a \chi - \gamma \nabla \chi\right) \\ * & \left(dA - \frac{1}{2\ell}\bar{\chi}\gamma\,\gamma\chi\right) \otimes J \end{pmatrix}, \tag{5}$$

where $R := d\omega + \omega^2$ and $T^a := de^a + \omega^a{}_b e^b = T^a{}_{bc} e^b \wedge e^c$ are the spacetime curvature and torsion 2-forms, respectively, and, in this notation, $\nabla\chi := d\chi + \omega\chi - A \otimes J\chi$. The curvature $\mathbb{F}$ satisfies the Bianchi identity $D^{\mathbb{A}}\mathbb{F} = d\mathbb{F} + [\mathbb{A}, \mathbb{F}] \equiv 0$, where $D^{\mathbb{A}}$ denotes the covariant derivative with respect to the full $\mathbb{A}$, here adapted to ad-tensorial forms.

The $\mathcal{OS}p(2|2)$-transformations of the connection is $\mathbb{A}^g = g^{-1}\mathbb{A}g + g^{-1}dg$, which infinitesimally restricts as $\delta_\lambda \mathbb{A} = D^{\mathbb{A}}\lambda := d\lambda + [\mathbb{A}, \lambda]$. In matrix form,

$$\delta_\lambda \mathbb{A} = \begin{pmatrix} \delta_\lambda \omega & \frac{i}{\sqrt{\ell}}\delta_\lambda(\gamma\chi) \\ \frac{i}{\sqrt{\ell}}\delta_\lambda(\bar{\chi}\gamma) & \delta_\lambda A \otimes J \end{pmatrix} \tag{6}$$
$$= \begin{pmatrix} d\beta + [\omega, \beta] - \frac{i}{\ell}(\gamma\chi\,\bar{\varepsilon} + \varepsilon\,\bar{\chi}\gamma) & \frac{1}{\sqrt{\ell}}\left[\nabla\varepsilon - (\beta - \alpha\otimes J)i\gamma\chi\right] \\ * & (d\alpha + \frac{i}{2\ell}\mathrm{Tr}(\gamma\chi\,\bar{\varepsilon} + \varepsilon\,\bar{\chi}\gamma))\otimes J \end{pmatrix}.$$

The curvature transforms tensorially: $\mathbb{F}^g = g^{-1}\mathbb{F}g$, so $\delta_\lambda \mathbb{F} = [\mathbb{F}, \lambda]$.

B. Dynamics of the model

The Lagrangian of the theory is a CS 3-form, which is the simplest Lagrangian one can think of for the connection $\mathbb{A}$ in $2+1$ dimensions. It reads

$$L(\mathbb{A}) = \mathrm{sTr}\left(\mathbb{A}d\mathbb{A} + \frac{2}{3}\mathbb{A}^3\right) = \mathrm{sTr}\left(\mathbb{A}\mathbb{F} - \frac{1}{3}\mathbb{A}^3\right). \tag{7}$$

It is *not* $\mathcal{OS}p(2|2)$-invariant, *nor* quasi-invariant (CS forms never are under gauge group transformations, see [30]), and is merely quasi-invariant (transforms via a d-exact term) under Lie$\mathcal{OS}p(2|2)$. The latter still suffices to ensure that the field equations are $\mathcal{OS}p(2|2)$-covariant: indeed they are, since – as is well-known in CS theories – given by the flatness condition $\mathbb{F} = 0$, which unfolds in an obvious way as field equations for ω, A and χ by (5). In components they read, explicitly:

$$\begin{aligned} \Omega^{ab} &:= R^{ab} + \frac{2}{\ell}\bar{\chi}_A\chi_A e^a \wedge e^b = 0, \\ \tilde{F} &:= dA - \frac{i}{2\ell}\bar{\chi}_A\gamma^c\chi_B\epsilon_{AB}\epsilon_{abc}e^a \wedge e^b = 0, \\ &\gamma_a T^a\chi_A - \gamma\nabla\chi_A = 0 \\ \Rightarrow\quad &\gamma_a T^a{}_{\mu\nu}\chi_A - \gamma_{[\mu}\nabla_{\nu]}\chi_A = 0. \end{aligned} \tag{8}$$

Contracting the last equation with $\gamma^{\mu\nu} := \frac{1}{2}[\gamma^\mu, \gamma^\nu]$, we get the massive Dirac equation

$$\not{\nabla}\chi_A = (T^a{}_a - 3i\tau)\,\chi_A, \tag{9}$$

where we used $\gamma^{abc} = i\epsilon^{abc}$ and defined $T^a{}_a := \gamma^b T^a{}_{ab}$ and $\tau := \frac{1}{3!}\epsilon^{abc}T_{abc}$. One could further simplify (9) by expressing ω in terms of the torsion.

What makes the AVZ model "unconventional", from a supersymmetric standpoint, is that the spinor-valued 1-form ψ, gauge potential of supersymmetry that should feature (in the most general case) in $\mathbb{A}$ as $\bar{\mathbb{Q}}\psi - \bar{\psi}\mathbb{Q}$, is replaced by a composite $i\gamma\chi = i\gamma_a e^a\chi$ of the soldering form e^a and a spin-1/2 (Dirac) field χ. This substitution $\psi \to i\gamma\chi$ was dubbed the "*matter ansatz*" in [11]. The latter was declared to be a gauge fixing in [28] (see also [43]), implemented via a BRST-covariant procedure which, in turn, implies the introduction of scalar ghost and anti-ghost fields. We show here that it is actually a special case of the DFM, which we describe next.

III. AVZ MATTER ANSATZ VIA THE DFM

We first briefly describe the Dressing Field Method, and then show that the usual AVZ matter ansatz is a special application.

A. The DFM and its infinitesimal version

The DFM [29–32, 35–37] is a systematic tool to produce gauge-invariants out of the field space $\Phi = \{\phi\}$ of a gauge theory with gauge group $\mathcal{H}$ whose action on Φ defines gauge transformations: $\phi \mapsto \phi^g$.

Suppose $\mathcal{K} \subseteq \mathcal{H}$ is a gauge subgroup, corresponding to the rigid subgroup $K \subseteq H$. The DFM relies on identifying a *Φ-dependent $\mathcal{K}$-dressing field*, i.e. a map

$$\begin{aligned} u : \Phi &\to \mathcal{D}r[K, \mathcal{K}], \\ \phi &\mapsto u = u[\phi], \\ \phi^k &\mapsto u^k := u[\phi^k] = k^{-1}u[\phi], \quad \forall k \in \mathcal{K}, \end{aligned} \tag{10}$$

where $\mathcal{D}r[K, \mathcal{K}] := \{u : U \subset M \to K \,|\, u^k = k^{-1}u\}$ is the space of (Φ-independent) dressing fields. From it, one can systematically build the *$\mathcal{K}$-invariant dressed fields* by the surjective map

$$\begin{aligned} \Phi &\to \Phi^u, \\ \phi &\mapsto \phi^u = \phi^{u[\phi]}, \\ \phi^k &\mapsto (\phi^k)^{u^k} := (\phi^k)^{k^{-1}u[\phi]} = \phi^{u[\phi]}. \end{aligned} \tag{11}$$

If one is interested, as we shall be in this work, in invariance at first order, i.e. under infinitesimal gauge transformations Lie$\mathcal{H}$, one may linearise the above: Defining a Lie$\mathcal{K}$-dressing field as

$$\begin{aligned} &\upsilon = \upsilon[\phi] : U \subset M \to \mathfrak{K} = \mathrm{Lie}K, \\ &\text{s.t.}\quad \delta_\lambda\upsilon = \upsilon[\delta_\lambda\phi] \approx -\lambda, \quad \forall\lambda \in \mathrm{Lie}\mathcal{K}, \end{aligned} \tag{12}$$

where in the defining transformation law one is to neglect higher-order terms polynomial in λ and υ. Then, one defines the *perturbatively dressed fields*

$$\phi^\upsilon := \phi + \bar{\delta}_\upsilon\phi, \tag{13}$$

where $\bar{\delta}_\upsilon\phi$ mimics the functional expression of the Lie$\mathcal{H}$ gauge transformation $\delta_\lambda\phi$, substituting the gauge parameter by the infinitesimal dressing, $\lambda \to \upsilon$. The perturbatively dressed fields are $\mathcal{K}$-invariant at first order:

$$\begin{aligned} \delta_\lambda(\phi^\upsilon) &= \delta_\lambda\phi + \bar{\delta}_{(\delta_\lambda\upsilon)}\phi = \delta_\lambda\phi + \bar{\delta}_{-\lambda}\phi \\ &= \delta_\lambda\phi - \delta_\lambda\phi \equiv 0, \end{aligned} \tag{14}$$

neglecting higher-order terms in λ and υ.

Considering a quasi-invariant Lagrangian, i.e. such that $\delta_\lambda L(\phi) = d\beta(\phi;\lambda)$, its perturbative dressing is

$$L(\phi^\upsilon) := L(\phi) + d\beta(\phi;\upsilon). \tag{15}$$

The field equations $\boldsymbol{E}(\phi^\upsilon) = 0$ are thus $\mathcal{K}$-invariant at first order (in both λ and υ).

We stress that, despite a superficial formal similarity, dressed fields ϕ^u are *not* gauge transformed fields ϕ^g. This is clear from the definition of a dressing field, which implies $u \notin \mathcal{H}$. The dressing operation is not a mapping from field space Φ to itself, but a mapping from field space to another mathematical space: the space of dressed fields Φ^u. A fortiori, a dressing operation *is not* a gauge fixing. See [34] and [37] for more on this point.

B. Matter ansatz via dressing

The DFM was shown to be foundational to supersymmetric gauge field theories in [35], where the Rarita-Schwinger and the gravitino 1-form fields were proved to be dressed fields. We now establish that the DFM is at the core of ususy too, showing that the spin-1/2 field appearing in the matter ansatz (1) is actually a partially invariant variable obtained via dressing.

To this aim, we start with the most general $\mathfrak{osp}(2|2)$ superconnection $\mathbb{A}_{\mathfrak{osp}} = \frac{1}{2}\omega^{ab}\mathbb{J}_{ab} + \bar{\mathbb{Q}}\psi - \bar{\psi}\mathbb{Q} + A\,\mathbb{T}$, with curvature $\mathbb{F}_{\mathfrak{osp}} = d\mathbb{A}_{\mathfrak{osp}} + \frac{1}{2}[\mathbb{A}_{\mathfrak{osp}}, \mathbb{A}_{\mathfrak{osp}}] = d\mathbb{A}_{\mathfrak{osp}} + \mathbb{A}_{\mathfrak{osp}}^2$. In our compact notation it reads

$$\mathbb{A}_{\mathfrak{osp}} = \begin{pmatrix} \omega & \frac{1}{\sqrt{\ell}}\,\psi \\ -\frac{1}{\sqrt{\ell}}\,\bar{\psi} & A\otimes J \end{pmatrix}, \tag{16}$$

and it gauge-transforms as

$$\delta_\lambda \mathbb{A}_{\mathfrak{osp}} = \begin{pmatrix} \delta_\lambda\omega & \frac{1}{\sqrt{\ell}}\delta_\lambda\psi \\ -\frac{1}{\sqrt{\ell}}\delta_\lambda\bar{\psi} & \delta_\lambda A\otimes J \end{pmatrix} = D^{\mathbb{A}}\lambda \tag{17}$$

$$= \begin{pmatrix} d\beta + [\omega,\beta] - \frac{1}{\ell}(\psi\,\bar{\varepsilon} - \varepsilon\,\bar{\psi}) & \frac{1}{\sqrt{\ell}}\,[\nabla\varepsilon - (\beta - \alpha\otimes J)\psi\,] \\ * & (d\alpha + \frac{1}{2\ell}\mathrm{Tr}(\psi\,\bar{\varepsilon} - \varepsilon\,\bar{\psi}))\otimes J \end{pmatrix}.$$

We first notice that the components of the general 1-form field $\psi = \psi_\mu dx^\mu$ have the following (reducible) "gamma-trace" decomposition:

$$\psi_\mu(\rho,\zeta) = \rho_\mu + i\gamma_\mu\zeta, \tag{18}$$

where ρ_μ, satisfying $\gamma^\mu\rho_\mu = 0$, contains both a "longitudinal" (divergence-free) mode and a "transverse" mode, and $\zeta := -\frac{i}{3}\gamma^\mu\psi_\mu$ is a spin-1/2 field. This decomposition generalises to n spacetime dimensions as $\zeta := -\frac{i}{n}\gamma^\mu\psi_\mu$. By (17), the susy transformation of ψ is $\delta_\varepsilon\psi = \nabla\varepsilon$, in components $\delta_\varepsilon\psi_\mu = \nabla_\mu\varepsilon$, and given (18) it splits as

$$\begin{aligned} \delta_\varepsilon\zeta &= -\frac{i}{3}\not{\nabla}\varepsilon, \\ \delta_\varepsilon\rho_\mu &= \frac{i}{3}\gamma_\mu\not{\nabla}\varepsilon + \nabla_\mu\varepsilon =: -b_\mu(\varepsilon), \end{aligned} \tag{19}$$

where we define the ($\mathfrak{spin}(1,2)\oplus\mathfrak{u}(1)$)-covariant operator $b_\mu := -\frac{i}{3}\gamma_\mu\not{\nabla} - \nabla_\mu$, whose formal left inverse is $[b^{-1}]^\mu$. We now aim to build the susy-invariant dressed field $\psi^\upsilon_\mu := \psi_\mu + \nabla_\mu\upsilon$, where $\upsilon = \upsilon[\psi]$ is a spinorial susy dressing field. Its gamma-trace decomposition is a priori

$$\begin{aligned} \psi^\upsilon_\mu &= \rho^\upsilon_\mu + i\gamma_\mu\zeta^\upsilon \\ &:= (\rho - b_\mu(\upsilon)) + i\gamma_\mu\left(\zeta - \tfrac{1}{3}\not{\nabla}\upsilon\right). \end{aligned} \tag{20}$$

The dressing field is found by solving explicitly the constraint $\rho^\upsilon_\mu = 0$, so that, indeed,

$$\begin{aligned} &\upsilon[\psi] := [b^{-1}]^\mu(\rho_\mu) \quad \text{and} \\ &\delta_\epsilon\upsilon := \upsilon[\delta_\epsilon\psi] \approx [b^{-1}]^\mu(\delta_\epsilon\rho_\mu) = [b^{-1}]^\mu(-b_\mu(\varepsilon)) = -\varepsilon, \end{aligned} \tag{21}$$

as expected by definition (12) of an infinitesimal dressing field. We therefore end up, as intended, with the susy-invariant dressed field

$$\psi^\upsilon_\mu = i\gamma_\mu\zeta^\upsilon =: i\gamma_\mu\chi, \tag{22}$$

where we defined the susy-invariant dressed spinor $\chi := \zeta^\upsilon := \zeta - \frac{1}{3}\not{\nabla}\upsilon$. We thus obtain the matter ansatz (1) of the AVZ model via the DFM (22), producing its spin-1/2 field χ as a susy-invariant dressed variable.

Let us now make a few important observations.

First, since $\upsilon = \upsilon[\psi]$ in (21) is non-local – involving the formal inverse of a differential operator – so is the dressed field (22). In the terminology of [37, 44], we might thus say that susy is a "substantive" symmetry, as it is reduced via dressing at the cost of locality.

Second, the condition $\rho^\upsilon_\mu = 0$ is susy-invariant, and is *not* a gauge fixing of ψ_μ. As per the usual caveat expressed in the DFM, ψ^υ is not a gauge-fixed version of ψ: being susy-invariant, the former does not even belong to the same mathematical space as the latter.

Finally, and relatedly, we stress that for field-dependent dressing fields $\upsilon = \upsilon[\phi]$, dressed fields ϕ^υ have a natural interpretation as *relational variables*: they encode the gauge-invariant *relations* among physical degrees of freedom. Relationality, such understood, is the conceptual core of general-relativistic gauge field theory, and thus of supersymmetric field theory. See [31, 35–37]. The dressed spinor $\chi = \zeta^\upsilon$ is such a relational variable, encoding the susy-invariant physical relations among the d.o.f. embedded in the bare field ψ.

To fully exploit the power of the DFM, we can now dress the full $\mathfrak{osp}(2|2)$ superconnection (16). Writing the dressing field in matrix form as

$$\boldsymbol{\upsilon} = \begin{pmatrix} 0 & \frac{1}{\sqrt{\ell}}\upsilon \\ -\frac{1}{\sqrt{\ell}}\bar{\upsilon} & 0 \end{pmatrix}, \tag{23}$$

by (13), the susy-invariant dressing of $\mathbb{A}_{\mathfrak{osp}}$ is

$$\begin{aligned}\mathbb{A}^{\upsilon}_{\mathfrak{osp}} &:= \mathbb{A}_{\mathfrak{osp}} + \bar{\delta}_{\boldsymbol{\upsilon}}\mathbb{A}_{\mathfrak{osp}} \\ &= \mathbb{A}_{\mathfrak{osp}} + D^{\mathbb{A}_{\mathfrak{osp}}}\boldsymbol{\upsilon} \\ &= \begin{pmatrix} \omega^{\upsilon} & \frac{1}{\sqrt{\ell}}\,\psi^{\upsilon} \\ -\frac{1}{\sqrt{\ell}}\,\bar{\psi}^{\upsilon} & A^{\upsilon}\otimes J \end{pmatrix} \\ &= \begin{pmatrix} \omega^{\upsilon} & \frac{i}{\sqrt{\ell}}\,\gamma_\mu\chi \\ \frac{i}{\sqrt{\ell}}\,\bar{\chi}\gamma & A^{\upsilon}\otimes J \end{pmatrix} \\ &= \begin{pmatrix} \omega - \frac{1}{\ell}(\psi\,\bar{\upsilon} - \upsilon\,\bar{\psi}) & \frac{1}{\sqrt{\ell}}\,[\psi + \nabla\upsilon] \\ * & (A + \frac{1}{2\ell}\mathrm{Tr}(\psi\,\bar{\upsilon} - \upsilon\,\bar{\psi}))\otimes J \end{pmatrix},\end{aligned} \tag{24}$$

which is none other than a susy-invariant version of the AVZ connection (4). The associated invariant dressed curvature is $\mathbb{F}^{\upsilon}_{\mathfrak{osp}} := \mathbb{F}_{\mathfrak{osp}} + \bar{\delta}_{\upsilon}\mathbb{F}_{\mathfrak{osp}} = \mathbb{F}_{\mathfrak{osp}} + [\mathbb{F}_{\mathfrak{osp}}, \boldsymbol{\upsilon}]$. Observe how the invariant $\mathfrak{spin}(1,2)$-connection ω^{υ} and $\mathfrak{u}(1)$-connection A^{υ} are dressed with d.o.f. from ψ via υ, and therefore encode the relational d.o.f. between the bare fields ω, A, and ψ.

The dressing of the CS Lagrangian $L(\mathbb{A}_{\mathfrak{osp}})$ for $\mathbb{A}_{\mathfrak{osp}}$ is, by (15),

$$\begin{aligned}L(\mathbb{A}^{\upsilon}_{\mathfrak{osp}}) &= L(\mathbb{A}_{\mathfrak{osp}}) + d\beta(\mathbb{A}_{\mathfrak{osp}};\upsilon) \\ &= \mathrm{sTr}\left(\mathbb{A}^{\upsilon}_{\mathfrak{osp}}\mathbb{F}^{\upsilon}_{\mathfrak{osp}} - \frac{1}{3}(\mathbb{A}^{\upsilon}_{\mathfrak{osp}})^3\right).\end{aligned} \tag{25}$$

It is the (u)susy-invariant version of the AVZ Lagrangian (7). The (u)susy-invariant dressed field equations are then $\mathbb{F}^{\upsilon}_{\mathfrak{osp}} = 0$, which unfold as dressed versions of (8).

We stress again that, since a dressing is not a gauge fixing, the dressed Lagrangian (25) is not a gauge-fixed version of $L(\mathbb{A}_{\mathfrak{osp}})$. Rather, $L(\mathbb{A}^{\upsilon}_{\mathfrak{osp}})$ is the *relational* rewriting of the theory, its field equations $\mathbb{F}^{\upsilon}_{\mathfrak{osp}} = 0$ are the relational version of the bare ones $\mathbb{F}_{\mathfrak{osp}} = 0$, and they differ only by a boundary term – a general feature proven in full generality in [37].

IV. RESIDUAL SYMMETRIES OF THE MODEL

We analyse the gauge transformations remaining after dressing. First, the we focus on the natural residual Lorentz and $\mathcal{U}(1)$ symmetries of the model. Then we comment on the so-called Nieh-Yan-Weyl symmetry [38–42] that has been associated to the matter ansatz.

A. Residual Lorentz and $\mathcal{U}(1)$ transformations

Since the odd, susy, part of the gauge superalgebra Lie$\mathcal{OS}p(2|2)$ has been reduced via dressing, one expects that the dressed fields $\mathbb{A}^{\upsilon}_{\mathfrak{osp}}$, and $\mathbb{F}^{\upsilon}_{\mathfrak{osp}}$, exhibit residual transformations under the remaining even gauge subalgebra Lie$\mathcal{S}pin(1,2) \oplus$ Lie$\mathcal{U}(1)$, with parameter

$$\theta = \begin{pmatrix} \beta & 0 \\ 0 & \alpha\otimes J \end{pmatrix}. \tag{26}$$

A basic result of the DFM is that if one proves

$$\delta_\theta \boldsymbol{\upsilon} = [\boldsymbol{\upsilon}, \theta], \tag{27}$$

$$\begin{pmatrix} 0 & \frac{1}{\sqrt{\ell}}\delta_\theta\upsilon \\ -\frac{1}{\sqrt{\ell}}\delta_\theta\bar{\upsilon} & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{\ell}}(\beta - \alpha\otimes J)\upsilon \\ * & 0 \end{pmatrix},$$

then it follows that

$$\begin{aligned}\delta_\theta\mathbb{A}^{\upsilon}_{\mathfrak{osp}} &= D^{\mathbb{A}^{\upsilon}_{\mathfrak{osp}}}\theta = d\theta + [\mathbb{A}^{\upsilon}_{\mathfrak{osp}}, \theta], \\ \text{and}\quad \delta_\theta\mathbb{F}^{\upsilon}_{\mathfrak{osp}} &= [\mathbb{F}^{\upsilon}_{\mathfrak{osp}}, \theta].\end{aligned} \tag{28}$$

That is, the susy-invariant fields have standard residual Lorentz and $\mathcal{U}(1)$ gauge transformations. Which implies that the field equations have the required residual covariance. To secure the result, one needs only to prove (27): The dressing has been found by solving $\rho^{\upsilon}_{\mu} = 0$, i.e. $\rho_\mu = b_\mu(\upsilon)$. And by (17)-(18) we have that $\delta_\theta\rho_\mu = -(\beta - \alpha\otimes J)\rho_\mu$. Now, since b_μ is a ($\mathfrak{spin}(1,2)\oplus\mathfrak{u}(1)$)-covariant operator, the latter holds if υ has the same covariance as ρ_μ; so $\delta_\theta\upsilon = -(\beta - \alpha\otimes J)\upsilon$, as required in (27). So, indeed, (28) holds.

B. NYW invariance as an artificial symmetry

The AVZ matter ansatz (1) is invariant under local scale transformations associated with the so-called NYW symmetry [38–42]. In n spacetime dimensions, in terms of the scaling parameter $\Lambda(x)$, under the NYW transformation we have:

$$\begin{aligned}e^a_{\ \mu} &\mapsto e^{\Lambda(x)}e^a_{\ \mu}, \\ \chi &\mapsto e^{-\frac{n-1}{2}\Lambda(x)}\chi.\end{aligned} \tag{29}$$

In the AVZ model of ususy, the NYW symmetry is identified as the Weyl symmetry associated with a conformal rescaling of the metric on the base space. In $n = 3$,

$$\chi \mapsto e^{-\Lambda(x)}\chi, \tag{30}$$

and the NYW transformation (29)-(30) clearly leaves ψ in (1) invariant, or ψ^{υ} in (22). So, since ω and A are NYW singlets, $\mathbb{A}$ and $\mathbb{A}^{\upsilon}_{\mathfrak{osp}}$ are NYW invariant, and therefore so is the model.

Yet, the transformations (29)-(30) are external to the $OSp(2|2)$ supergeometry underlying the model, they are imposed on it by fiat and therefore carries little relevant information about its kinematics. Furthermore, since it is a symmetry that can be "reduced" without losing the locality of the theory – via dressing, considering ${e^a}_\mu$ as a dressing field for χ – in the terminology of [37, 44] it is an "artificial" symmetry, or "fake" symmetry in the terms of [45]. So, if the NYW transformations (29)-(30) can be thought of as a residual symmetry of the dressed theory $L(\mathbb{A}^{\upsilon}_{\mathfrak{osp}})$, it is and artificial residual symmetry.

V. CONCLUSIONS

In this letter, we have presented a susy-invariant version of the $\mathcal{OS}p(2|2)$ AVZ model of unconventional supersymmetry, obtained through the *Dressing Field Method* (DFM) of gauge symmetry reduction. To do so efficiently, we have first clarified the structure of the gauge supergroup and superalgebra, relying on the supergeometry underlying the model, and then exploited a new compact matrix notation, greatly streamlining the presentation of the model and clarifying the conceptual picture. In particular, it allows a straightforward analysis of the residual gauge Lorentz and $\mathcal{U}(1)$ transformations of the susy-invariant fields. We have also commented that the NYW transformations, externally imposed on the fields, are to be understood as an "artificial" residual symmetry of the model, not part of the underlying geometry.

Crucially, the so-called matter ansatz has been shown to be the result, not of a gauge fixing as claimed in antecedent literature, e.g. [28], but of a dressing operation, implemented via the DFM. The mathematical difference between gauge fixing and dressing as been argued at length in [34, 37], where many instances of popular "gauges" (Coulomb, Lorentz gauges in electromagnetism, the unitary gauges in the electroweak model) have been shown to actually be cases of dressings [46].

An essential conceptual aspect of the DFM, distinguishing gauge-fixed fields from invariant dressed fields, is that only the latter have a *relational* interpretation: i.e. they encode the invariant physical relations among bare field-theoretic degrees of freedom. Our findings thus entail that the spin-1/2 field of the AVZ ususy matter ansatz (1) is actually a (non-local) (u)susy-invariant relational field variable.

This adds to, and reinforces, the results of [35], where the DFM was shown, for the first time, to be pivotal to supersymmetric field theory, as the Rarita-Schwinger and gravitino field were obtained as dressed, relational variables. Together, the present work and [35] establish the DFM as foundational to both supersymmetric field theories and unconventional supersymmetry.

All of this was unknown to, or essentially overlooked by, the susy community and becomes of fundamental importance either if one wants to maintain a clear phenomenological interpretation of supersymmetry, in high energy physics, or if one wants to exploit the supergeometric setup to describe particles of half-integer spin within a superconnection. In both cases, the DFM allows to systematically identify the physical, relational, d.o.f. postulated in the models, which is essential their development and comparison with experiments. Indeed, it should be reminded that what is measured and confronted to experiments are the dressed variables, or "relational observables", of a theory [37].

Regarding the second scenario just mentioned, which goes back to one of the original motivations for the introduction of supersymmetry [2], the present work is ground for further developing a (*principal*) super-bundle geometric description of spinorial Dirac matter fields as arising as part of a superconnection after (super)symmetry reduction via the DFM, unifying matter and gauge fields (possibly including also gravity). At the dynamical level, this also allows to use spinor-valued 1-form fields, and higher half-integer spin fields as well, to write down Lagrangians that actually describe propagating spin-1/2 particles, in the spirit of, e.g., [47]. This would build a bridge with non-commutative geometric approaches to Yang-Mills-Higgs models [48–51], where the gauge and Higgs fields are unified in a single connection.

The technical and conceptual lens of the DFM applied in such a supergeometric framework thus opens a new intriguing research path for applications of supersymmetric field theory to established (high energy and/or condensed matter) physics.

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