

Faster Than Light?

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Abstract. It is argued that special relativity remains a viable physical theory even when there is permitted signals traveling faster than light.

1 Introduction

The view is widely held that the speed of light, c , plays a fundamental role in physics: It is the universal upper limit to all signal speeds that can be achieved, by any process whatever, in the physical world. This view comes to us, of course, from special relativity. Indeed, it is normally taken as a fundamental tenant of that theory — be it as an “axiom”, on which the theory is based; or as a consequence of other ingredients of the theory. But our confidence in this special role for the speed c is based on more than merely its status within special relativity. There are also solid physical arguments to support this position.

One such argument is the following. Try to concoct a physical mechanism that could be used to generate a superluminal signal. For example, one might, by applying various forces to a particle, attempt to accelerate it up to a speed exceeding c . Such a particle, once so accelerated, could then be used to transmit signals. But such mechanisms seem invariably to fail. In this example of a particle, for instance, we must contend with the formula, $m = m_o/\sqrt{1 - v^2/c^2}$, for the mass-increase of a particle with its speed v .

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This formula guarantees that no application of any finite force for any finite amount of time will ever achieve $v > c$. It appears, then, that Nature conspires to prevent the manufacture of superluminal signals.

A very different kind of argument is that of the so-called grandfather paradox. Suppose that one had produced a mechanism for sending a signal faster than light. For example, suppose that one had constructed a long pipe containing a fluid whose sound-speed exceeds c . This means that the world-line of a sound pulse sent down this pipe would be spacelike. Assuming that Lorentz invariance holds for such fluid-filled pipes, it follows: For a pipe moving, along its length, with a speed comparable with (but less than) c , the world-line of a sound pulse going down that pipe in the direction of its motion would actually go backward in time (as measured by the observer with respect to which the pipe is moving). Now let our observer arrange two such pipes, parallel to each other and close together; with one at rest and the other moving by at high speed along its length. Send a sound pulse down the rest pipe, and, after that signal has traveled for a while, have it initiate a second sound pulse sent back to the observer along the moving pipe. The result of this arrangement would be to send a signal into this observer's own past. By using a sequence of such arrangements, our observer could send a signal very far into his own past. Finally, this observer could use such a signal to change something in the past — e.g., to have his own grandfather killed while still a child. But now we have a paradox: If the observer's grandfather was killed as a child, then the observer would never have come into being, and so would never have been able to construct this machine for the murder of his grandfather! It appears to be difficult to resurrect “physics” (at least in any form that we are familiar with) in the presence of superluminal signals.

We shall argue here that, all this evidence notwithstanding, special relativity need *not* be construed as prohibiting superluminal signals. Relativity theory with such signals permitted, we shall argue, is as viable and physically acceptable as relativity theory without. We suggest that a universal limitation on signal speeds need not be taken as any fundamental principle of physics. Rather, the whole idea of such a limit has more to do with history and with the types of interactions to which we are commonly exposed. We emphasize that we are *not* suggesting here that some new theory be introduced to replace special relativity; nor, indeed, that any of the basic structural components of the theory of relativity be changed. What is to be changed is merely our perspective on relativity theory.

This claim is based on some features of a class of partial differential

equations called symmetric hyperbolic systems. This subject is reviewed briefly in Sect. 2. We return to special relativity in Sect. 3.

2 Symmetric Hyperbolic Systems

In this section, we review the properties of a certain class of systems of partial differential equations [9], [2], [6], [7]. Our treatment follows [4].

Denote by M the 4-dimensional manifold of space-time events. Consider a first-order, quasilinear system of partial differential equations on this manifold. That is, consider a system of equations of the form

$$k^{Aa}{}_{\alpha} (\nabla_a \phi^\alpha) = j^A. \quad (1)$$

Here, ϕ^α represents the fields of the system, where the index “ α ” runs through field-space¹. Any algebraic conditions on the fields — be they equalities or inequalities — are to be reflected in the construction of the ϕ^α : There are no “algebraic constraints” on the fields in this formulation. The expression “ $(\nabla_a \phi^\alpha)$ ” in (1) is the system of first derivatives of these fields², where the index “ a ” labels tensors in M . Eqn. (1), then, requires that certain expressions linear in these derivatives vanish. The coefficients in these linear combinations, $k^{Aa}{}_{\alpha}$ and j^A , are any fixed smooth functions of the fields ϕ^α (but not of their derivatives) and the point of the underlying manifold M . Eqn. (1) is to be imposed at each point of M . The free index, “ A ”, in (1) lives in the vector space of equations, i.e., there is one equation for each “value” of A . This system is called first-order, since it involves at most first derivatives of the fields ϕ^α ; and quasilinear, since the equations themselves are linear in those first derivatives. We demand at the outset that *all* first-order equations on the fields — even any that might be derived from the other equations by taking derivatives — have been included in (1).

¹A more elegant formulation of this scheme is the following. Introduce, at each point of M , the manifold of “all possible field values” at that point. Then the set of all such field-values at all points of M becomes a fibre bundle over M . The field-configuration of the system, ϕ , is then represented as a smooth cross-section of this bundle. This formulation, among other things, makes it clear that how the fields are represented — e.g., as densities, or contravariant tensors, or whatever — plays no role in what follows.

²In the more elegant formulation, we fix any smooth connection on the field-bundle, and then use this connection to take the derivative, in (1), of the cross-section. Which particular connection is chosen plays no essential role in what follows.

The vast majority of systems of equations that describe physical systems can be cast into this form. For electromagnetism, for example, the field ϕ is a skew, second-rank tensor field F_{ab} . And Eqn. (1), of course, is Maxwell's equations:

$$\nabla^b F_{ab} = 0, \tag{2}$$

$$\nabla_{[a} F_{bc]} = 0. \tag{3}$$

Here, the “field index”, α , runs over the six dimensions of field-space, while the “equation index”, A , runs over the 8-dimensional vector space of equations (2)-(3). A second example is that of a simple perfect fluid. The fields in this case consist of a unit timelike vector u^a (the fluid 4-velocity) and a positive scalar ρ (the mass density); and Eqn. (1) is the system of fluid equations:

$$(\rho + p)u^m \nabla_m u^a + (g^{ab} + u^a u^b) \nabla_b p = 0, \tag{4}$$

$$u^a \nabla_a \rho + (\rho + p) \nabla_a u^a = 0. \tag{5}$$

In these equations, p is some fixed nonnegative function of ρ , the function of state, which characterizes the fluid under consideration. Here, index “ α ” runs over the 4-dimensional space of pairs (u^a, ρ) with u^a unit and $\rho > 0$; while “ A ” runs over the 4-dimensional vector space of pairs (w^a, w) , where w^a is orthogonal to u^a , and w is a number. The coefficients k^{Aa}_α and j^A in these examples are read off immediately from the equations. Similar remarks apply to other systems, for example, those of neutrino or Dirac fields, of spin- s fields, of stressed solids, of more complex fluids (e.g., those with a higher-dimensional manifold of local fluid states), of fluids manifesting dissipative effects (viscosity, thermal conductivity), etc. Second- and higher-order equations are reduced to first-order form by introducing new fields to represent the lower derivatives of the original fields. Thus, for example, the wave equation is represented by a pair of fields, (ψ, v_a) , with equations $\nabla_a \psi = v_a$, $\nabla_{[a} v_b] = 0$, and $\nabla^a v_a = 0$. [Note that we include in our system the second of these equations, even though it follows from the first by taking the curl.] And, finally, Einstein's equation can be cast into the form of Eqn. (1). The fields in this case are the space-time metric g_{ab} and the derivative operator ∇_a ; and the equations are $\nabla_a g_{bc} = 0$ and $G_{ab} = 0$, where G_{ab} is the Einstein tensor. Note that these equations are indeed first-order and quasilinear.

Consider two systems — say those of the electromagnetic field and of a simple fluid. We can imagine a world in which these two systems coexist independently of each other. This situation is described by fields that merely

combine the fields of the individual systems; and equations that combine the equations of the individual systems. Thus, in our example, ϕ^α for the electromagnetism-fluid system would consist of a skew tensor F_{ab} , a unit timelike u^a , and a positive scalar ρ (10 dimensions of fields); and the system of equations would be the total system (2)-(5) (12 dimensions of equations). Next, let there be turned on an interaction between these two systems. Nature, apparently, always “turns on interactions” in a very special way: It is only the j^A of the combined system that is modified to reflect the interaction, while the $k^{Aa}{}_\alpha$ remains the same. Consider, in the example above, the interaction that results from allowing the fluid to carry charge. First introduce the charge-current vector, $J^a = \mu u^a$ of the fluid, where μ is a certain function of ρ so designed that $\nabla_a J^a = 0$ follows already from Eqn. (5). Then modify the system (2)-(5) by adding the term J_a on the right in (2) (The charge-current creates electromagnetic field.), and adding the term $F^{ab} J_b$ on the right in (4) (The fluid is subject to the Lorentz force.) But note that the terms that have been added to our equations are algebraic in the fields of the combined system, i.e., that they involve no derivatives of those fields. In other words, we have merely modified the j^A of that system. To take another example, let us turn on the interaction between a mass- m Klein-Gordon field and gravity. This is done by inserting the field stress-energy, $(v^a v^b - (1/2)g^{ab} v^s v_s - m^2 g^{ab} \psi^2/2)$, on the right side of Einstein’s equation. Again, what is changed is only the j^A of the combined system, and not $k^{Aa}{}_\alpha$. [Note that we are here making use of the fact that $v_a = \nabla_a \psi$ has been included among the “fields” for the Klein-Gordon system.]

The view here is that the fields ϕ , and their equation (1), is all there is. In particular, there is no need for a separate chapter explaining how each field is to be interpreted. We “interpret” a field exclusively through the physical effects that it produces, i.e., by making observations on it. But an observation on a field, in turn, consists of nothing more than the result of turning on an interaction, in Eqn. (1), between that field and various observer-fields. Thus, we think of Eqn. (1) as a kind of “theory of everything” (at least, everything non-quantum-mechanical).

We now turn to the initial-value formulation for a first-order, quasilinear, system of partial differential equations. For purposes of the present treatment, we shall ignore two features of these systems — the possible presence of diffeomorphism freedom and of constraints. These two features certainly play a significant role in the mathematics of the general first-order, quasilinear system. But by ignoring these two features we shall greatly simplify

the present discussion, while losing nothing of significance. That is, we are ignoring these features merely for convenience.

A *hyperbolization* of the system (1) is a tensor $h_{A\beta}$ (depending, in general, on field-values ϕ^α and point of M) such that i) the combination $h_{A\beta}k^{Aa}{}_\alpha$ is symmetric in the indices α and β ; and ii) for some covector n_a in M , the symmetric tensor $n_a h_{A\beta}k^{Aa}{}_\alpha$ is positive-definite. The equations for the standard systems of physics – electromagnetism, the wave equation, spin- s fields, fluids (simple, complex, or even dissipative ([8], [5]), stressed solids, gravitation³ ([1], [3]), etc. – all have hyperbolizations⁴. Note that the existence of a hyperbolization depends only on the $k^{Aa}{}_\alpha$ of the system, and not on its j^A .

Fix a first-order, quasilinear, system of partial differential equations, (1), and a hyperbolization, $h_{A\beta}$, for that system. Fix also a point p of M , and a value for the fields, ϕ^α , at that point. By the *causal cone*, \mathcal{C} , of this system (at this point and for this field-value) we mean the set of all tangent vectors ξ^a at $p \in M$, such that $\xi^a n_a > 0$ for every n_a for which $n_a h_{A\beta}k^{Aa}{}_\alpha$ positive-definite. We note that, quite generally, \mathcal{C} is a nonempty, open, convex cone of tangent vectors at the point p of M . Note that $k^{Aa}{}_\alpha$ and $h_{A\beta}$ in general depend on the choice of field-value at $p \in M$; and so, therefore, does the causal cone at p .

For many examples — Maxwell’s equations, the Klein-Gordon equation, the neutrino or Dirac equations, the spin- s field equation and Einstein’s equation — the causal cone is precisely the future light cone. [A choice of “future” was singled out by the choice of sign in the hyperbolization $h_{A\beta}$.] For the case

³The situation for the gravitational case is more complicated than suggested here, because of the diffeomorphism freedom.

⁴There is one example of a system that seems “physically reasonable”, and yet does not admit a hyperbolization — the conducting fluid. The fields are (F_{ab}, u^a, ρ, μ) , where F_{ab} is the (skew) Maxwell field, u^a is the (unit) 4-velocity, ρ is the (positive) mass density, and μ is the charge density. The equations are Eqns. (2)-(5), with the Lorentz-force term $F^a{}_b J^b$ inserted on the right in (5) and the current-source term J_a inserted on the right in (2); together with charge-conservation, $\nabla^a J_a = 0$. Here, the charge-current is given by $J_a = \mu u_a + \sigma F_{ab} u^b$ (these two terms representing, respectively, the charge-current due to bulk motion of the charges in the fluid and the conductivity-current), where σ is some fixed constant (the electrical conductivity). In fact, it does not seem to be possible to achieve an hyperbolization for this system by any obvious modifications, e.g., including the twist of the velocity as an additional field. I do not know whether this system satisfies the conclusion of the initial-value formulation (later); nor, if it does, what the causal cones are. It would be of interest to understand what is happening with this system.

of a simple fluid, the situation is a little more complicated. First, there is no hyperbolization at all unless $dp/d\rho > 0$. When this inequality is satisfied, there is a hyperbolization, and the corresponding causal cone \mathcal{C} is given by a cone of vectors ξ^a satisfying $u_a \xi^a < 0$ and $(g_{ab} + (1 - dp/d\rho)u_a u_b)\xi^a \xi^b > 0$. This will be recognized as the “sound cone” of the fluid — i.e., the set of directions in space-time whose speed, measured with respect to the fluid 4-velocity u^a , is less than the sound speed v , given by $(v/c)^2 = dp/d\rho$. Note that when $v < c$, then all the vectors in \mathcal{C} are timelike; whereas when $v > c$ there are necessarily some spacelike vectors in \mathcal{C} . We emphasize that all of these remarks about fluids — the existence of a hyperbolization and of the causal cone — apply equally well in the case of a subluminal and a superluminal sound speed. The case of a stressed solid is very similar to that of a fluid. There are functions of state that must be fixed to specify equations for this system, and these functions give rise to acoustical-wave speeds associated with the solid. For appropriate functions of state, there exists a hyperbolization. These acoustical-wave speeds may be greater than or less than the speed of light c ; and the corresponding causal cones may lie outside or inside the light-cones, respectively.

Note that all of the above — the existence of a hyperbolization, and the corresponding causal cone — depend only on the coefficient $k^{Aa}{}_{\alpha}$ in the system (1) of equations, and not on the j^A . This observation has important implications. For instance, it follows immediately (from the corresponding facts for the wave equation) that the mass- m Klein-Gordon equation has a hyperbolization, with causal cone the light cone. Note that this holds even for the Klein-Gordon equation with the “wrong” sign for the term $m^2\psi$. It follows further that the act of turning on an interaction between two systems does not change the hyperbolization nor the causal cones of the system.

The causal cones deserve their name. Roughly speaking, any first-order, quasilinear system is capable of sending signals only within its causal cones. This remark is made precise by the initial-value formulation for such systems, which we now describe.

Fix a first-order, quasilinear system of partial differential equations, together with a hyperbolization, $h_{A\beta}$, for that system. Let S be a 3-dimensional submanifold of the manifold M , and let there be given fields ϕ_o^α on S . We call this (S, ϕ_o) *initial data* for our system provided that, at each point of S , the closure of the causal cone at that point lies entirely on one side of S . [This means, in other words, that a normal n_a to S is such that $n_a h_{A\beta} k^{Aa}{}_{\alpha}$ is positive-definite.] Note that these causal cones in general *depend* on the

field ϕ_o on S : Thus, changing only ϕ_o , keeping the submanifold S fixed, may render (S, ϕ_o) no longer initial data. We now have:

Initial-Value Formulation. Fix a first-order, quasilinear system of partial differential equations, a hyperbolization $h_{A\beta}$ for this system, and initial data (S, ϕ_o) for this system. Then

1. There exists a neighborhood U of S in M , together with fields ϕ^α in U , such that i) $\phi|_S = \phi_o$, and ii) ϕ satisfies⁵ Eqn. (1) in U .
2. The field ϕ at a point p of U depends only on the initial data in a region A of S such that p is in the domain of dependence of A .

Item 1 guarantees a solution ϕ of our equation, in some neighborhood of S , that reproduces the given initial data. [Note that we do not guarantee a solution in *all* of M : The fields may, for example, so evolve to become singular.] In item 2, p lying in the *domain of dependence* of A means that every curve γ in U that ends at p and that has its tangent vector at each of its points lying within the causal cone at that point, meets A once and only once. By “depends on” in item 2, we mean that two sets of data on S , provided they agree within $A \subset S$ yield the same field ϕ at p . Thus, item 2 asserts, in essence, that solutions of Eqn. (1) send signals that always lie within the causal cones: If region $A \subset S$ records every such signal that reaches point $p \in U$, then that record, stored on A , determines completely what is happening at p . Note that the domain of dependence in general depends on the field-values, since the causal cones do.

The assertion above, in particular, guarantees an initial-value formulation even for a fluid with superluminal sound speed.

Here is an example of these ideas. Let us combine two systems, turning on some interaction between them. Let the respective coefficients in their differential equations be k^{Aa}_α and $k'^{A'a}_{\alpha'}$, and let their respective hyperbolizations be $h_{A\beta}$ and $h'_{A'\beta'}$. Then $(h_{A\beta}, h'_{A'\beta'})$ is our candidate for an hyperbolization of the combined system. It automatically satisfies the symmetry condition; and it satisfies the positive-definiteness condition provided that, at each point, there is some covector n_a such that both $n_a h_{A\beta} k^{Aa}_\alpha$ and $n_a h'_{A'\beta'} k'^{A'a}_{\alpha'}$ are positive-definite. Suppose that this condition holds, so we indeed have a

⁵We are ignoring constraints in this discussion. The actual theorem guarantees only that the ϕ satisfy the “ $h_{A\beta}$ ”-part of Eqn. (1). The rest of that equation is also, ultimately, satisfied, but it is handled in a different manner.

hyperbolization for the combined system. Then the set of covectors n_a that give positive-definiteness for this hyperbolization is precisely the set of n_a for which $n_a h_{A\beta} k^{Aa}{}_{\alpha}$ and $n_a h'_{A'\beta'} k'^{A'a}{}_{\alpha'}$ are both positive-definite. It follows that the causal cone of the combined system is the convex hull of the causal cones of the two individual systems — i.e., the set of all sums of the form $\xi^a + \xi'^a$, with ξ^a in \mathcal{C} and ξ'^a in \mathcal{C}' . This is what, in light of the statement above, we would expect physically. Signal propagation, for the combined system, is in those space-time directions obtained by taking sums of the signal-propagation directions for the two systems separately.

To summarize, each first-order, quasilinear system of partial differential equations — provided that system has a hyperbolization — carries within itself its own initial-value formulation. And, as a part of that formulation, the system carries its own causal cones for signal propagation. These cones are inherent in the structure of the equation itself, i.e., they do not necessarily require that there be fixed any outside fields. We may combine such systems — and turn on interactions between systems — and when we do so the causal cones also combine, in the way we expect physically.

This formulation manifests what might be called a democracy of causal cones. All systems, and their cones, are on an equal footing: No one set of fields, or one set of causal cones, has priority over any others.

3 Special Relativity

We now turn to the special theory of relativity. This theory involves, of course, a flat metric, g_{ab} , on the space-time manifold M .

We first note that “flat metric” can be restated in terms of a first-order, quasilinear system of partial differential equations. The fields consist of the metric g_{ab} and the derivative operator ∇_a ; and the equations are

$$\nabla_a g_{bc} = 0, \quad R_{abcd} = 0, \tag{6}$$

where R_{abcd} is the Riemann tensor. Note that this system is indeed first-order and quasilinear. It is true that the solutions of this system are rather uninteresting dynamically, e.g., because they are all “locally identical”. Indeed, this is probably the reason why one does not normally think of special relativity in terms of two fields satisfying a system of partial differential equations.

We claim, next, that the system (6) admits a hyperbolization⁶! Indeed, this is a consequence of the existence of a hyperbolization for Einstein’s equation, in light of the fact that the Einstein system is merely a subset of the system (6). The causal cones for the system of special relativity are, of course, the null cones of the metric g_{ab} .

So, we may adopt the perspective that “special relativity” is merely one more first-order, quasilinear system of partial differential equations admitting a hyperbolization. It is just one more physical theory, not dissimilar from the theory of electromagnetism or the theory of a simple fluid. Like all such systems, special relativity carries with it, by virtue of the structure of its equations, causal cones. Some systems, such as that of electromagnetism, share those cones with special relativity; while other systems, such as that of a fluid, do not. But each system — special relativity included — looks to its own causal cones — to its own system of partial differential equations — for the propagation of signals within that theory.

In short, the causal cones of special relativity, from this perspective, have no special place over and above the cones of any other system. This is democracy of causal cones with a vengeance. This, of course, is not the traditional view. That view — that the special-relativity causal cones have a preferred role in physics — arises, I suspect, from the fact that a number of other systems — electromagnetism, the spin- s fields, etc — employ precisely those same cones as their own. And, indeed, it may be the case that the physical world is organized around such a commonality of cones. On the other hand, it is entirely possible that there exist any number of other systems — not yet observed (or maybe they have been!) — that employ quite different sets of causal cones. And the cones of these “other systems” could very well lie outside the null cones of special relativity, i.e., these systems could very well manifest superluminal signals. None of this would contradict our fundamental ideas about how physics is structured: An initial-value formulation, causal cones governing signals, etc.

To illustrate these points, let us return to the example of the system consisting of special relativity together with a fluid manifesting superluminal sound signals. This is a completely viable system of partial differential equations. It has, in particular, an initial-value formulation. Initial data must be specified on a 3-dimensional surface S that is spacelike (as determined by

⁶ Here, again, we are ignoring the diffeomorphism freedom, which, again, does not materially impact the present considerations.

the metric g_{ab} of special relativity), *and* is such that each fluid sound-cone lies on one side of S . These data then evolve, producing fields g_{ab}, ∇_a, u^a , and ρ within the domain of dependence of those initial data, as determined by the causal cones of the combined system. This system does not differ, in any essential way, from the system of special-relativity-electromagnetism. In particular, this system is “Lorentz-invariant”, at least in the sense that any g_{ab} -preserving diffeomorphism on M sends solutions of the fluid equations to new solutions. There is nothing peculiar happening here.

We discussed at the beginning two “concrete” arguments in support of the idea that it is appropriate to take the nonexistence of superluminal signals as a fundamental principle of physics. It is instructive to return to those arguments, in light of the discussion above.

The first involved the difficulty in generating superluminal signals. One example of this was the problem of accelerating a particle to a superluminal speed, in light of the mass-increase formula, $m = m_o/\sqrt{1 - v^2/c^2}$. From the present perspective, this “difficulty in generating superluminal signals” merely reflects the fact that we are trying to create such signals using fields (such as electromagnetism, etc) that take as their causal cones the null cones of special relativity. The “mass-increase formula” is now seen, not as a general property of *all* particles, but rather as a property only of particles constructed from such fields. Were there other fields — with other causal cones — and were we able to construct particles from these fields, then those particles would manifest “mass increase” by quite a different formula. These newly constructed particles could then be used to transmit superluminal signals. Of course, such particles could not be used to achieve a signal-speed greater than that dictated by the causal cones of those new fields.

The second argument involved the grandfather paradox. Let us first consider the arrangement in which the two pipes have already been set up, with one moving rapidly past the other. This is, presumably, a solution of the special-relativity-superluminal-fluid system. But it has closed causal curves (via, of course, the causal cones of this combined system). It follows that this arrangement cannot be in the domain of dependence of any surface, i.e., it cannot be “predicted”, via the initial-value formulation, from any initial data. Our observer will object at this point, claiming that he can “build” precisely this arrangement: First lay out the two pipes of fluid parallel to each other and at rest, and then accelerate one of those pipes along its length. Our response to this objection is the following. We grant that the observer can

set up those initial conditions (the two pipes at rest). But the issue of what happens from those initial conditions must be determined by evolving, from those initial data, the differential equations for the system. [Presumably, we would include also within this system the fields describing the observer, and the initial conditions would reflect that observer's resolve to get the one pipe moving.] Whatever results from these data and these equations is what results. But we know that — whatever it turns out to be — the result of this evolution will *not* consist of the two pipes moving in the prescribed manner. Probably, it will be difficult to include in the system interactions that will allow the observer to move the fluid around in the manner he wishes — for example, the fluid may interact back with the observer, preventing him from manipulating that fluid in the desired manner; or, because of its equations, the fluid might respond to such manipulation in an unexpected manner. It is also possible that the fluid solutions themselves might become singular when the fluid is pushed too hard.

This circumstance is not as strange as it might seem at first glance. Indeed, it arises all the time in physics. Suppose, for example, that an observer reported that he planned to build, and then use as a signalling device, certain electromagnetic fields specified on a timelike surface. We would certainly insist on knowing the details of how this is to be done. We would grant this observer the power to set up some initial conditions for the electromagnetic field (on a *spacelike* surface). But we would then insist that the final field configuration is, not what the observer wills it to be, but rather what follows, evolving these data via Maxwell's equations. If the observer can achieve the desired field-configuration in this way, we will accept it; if not, we will not. And, in a similar vein, there exist solutions of Einstein's equation in general relativity that manifest closed causal curves. But we do *not*, in light of this circumstance, allow observers to build time-machines at their pleasure. Instead, we permit observers to construct initial conditions — and then we require that they live with the consequences of those conditions. It turns out that a “time-machine” is never a consequence, in this sense, of the equations of general relativity, in close analogy with the situation in the special-relativity-superluminal-fluid example above.

To summarize, from the present viewpoint the problems associated with superluminal signals do not seem nearly as severe as they did at first glance.

Here, in any case, is another perspective on special relativity. The theory emerges as just one more physical system. It consists, just like the others, of certain fields subject to a certain system of first-order, quasilinear partial

differential equations. The causal cones of special relativity (which reflect the speed of light) have no special significance over the causal cones of any of the many other such systems in physics. I am not sure that this is the right perspective — or even whether “right” makes much sense in this context. But I would suggest that this viewpoint has something to offer.

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