Hexad Preons and Emergent Gravity in 3-dimensional Complex Spacetime

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We suggest that at high energy each space dimension has their own time dimension, forming a 3-dimensional complex spacetime. Based on this hypothesis, we propose that the primordial universe is made of six fundamental fermions and their complex conjugate states. These fermions are called Hexad Preons which carry hypercolor degree of freedom transforming under $U(3,3)$ gauge group. The Hermitian metric emerges upon the breakdown of the gauge group from $U(3,3)$ to its maximal compact subgroup $U(3)\otimes U(3)$. Leptons, quarks, as well as other matter states may be formed from the subsequent condensate of Hexad Preons. Strong and electroweak forces are manifestations of the hypercolor interaction in the corresponding cases. Our framework sheds light on many problems in cosmology and particle physics.

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I. INTRODUCTION

It is known that there are four fundamental forces, namely, strong, weak, electromagnetic forces and gravity. Gravity is described by the Einstein's general relativity while the other three forces are incorporated in the Standard Model (SM) of particle physics. Though general relativity and SM have been confirmed to very high accuracy[1], many problems remain unresolved. The nature of gravity at small distance is unknown. Current cosmological models based on general relativity must answer such questions like dark matter, dark energy, baryon and CP asymmetries. In SM, the Quantum Chromodynamics(QCD) and Quantum electroweak theory(GWS) are not genuinely unified. Other questions include: Why gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$? Why replication of leptons and quarks? What are the mechanism for electroweak symmetry breaking, fermion mass generation and mixing?

Many attempts, including extra dimensions[[2]], supersymmetry[3], grand unified theories(GUTs)[4], technicolor [5] and compositeness [6], have been made to resolve these problems. However, the solutions remain obscure.

Spacetime plays the fundamental role in physics. The evolution of concept about spacetime often results in breakthrough in physics theories. Poincaré invariance in four dimensional spacetime (three space dimensions plus one time dimension) is the foundation of modern theoretical physics. Moreover, the great success of general relativity and the SM has now made Poincaré invariance the guiding principle for further development of physics, e. g., in supersymmetry and string theory[7].

However, we'd better to keep an open mind on the nature of spacetime at very high energy. Here we would point out two aspects about the Poincaré invariance in four dimensional spacetime. One is about the general coordinate covariance. In order to consider general relativity in a unified theoretical framework, we must require general coordinate covariance rather than Poincaré invariance. The other is the obvious asymmetry between space dimensions and time dimension in four dimensional spacetime. Though at low energy it is sufficient to describe the world in a four dimensional spacetime continuum, it is not necessary that the three space dimensions should share the same time dimension at all scales.

The idea of time-like extra dimensions is not new. The early works on this subject are mainly concerned with the conformal aspects of field theory[8–10]. Most time-like extra dimension models are Kaluza-Klein theory with compact extra time dimensions [11–21]. The constrains from unitarity and causality on the maximum radius of the internal time-like directions have been investigated [22–25]. Gravitation and electrodynamics, Electroweak interaction were reconsidered in the six-dimensional spacetime with three times[26–28]. Other developments include time-like extra dimensions in the brane world scenarios[29–36], F-theory with one time-like extra dimension[37], supergravity and string theory involving a holographic principle and extra time dimensions[38–40], as well as quantum theory in spacetime with one time-like extra dimension[41, 42]. In addition, the so called "two-time physics (2T-physics)" where the phase space is gauged, has established the unitarity and causality in 4+2 dimensions with one extra space and one extra time dimensions and have obtained many interesting results [43].

In this paper, we suggest that at high energy the space dimension remains three and each space dimension has their own time dimension, forming a 3-dimensional complex spacetime. We will show that this simple assumption has many important consequences. In the following we will first give a mathematical account for complex spacetime within the framework of Generalized Complex Geometry[44] and then, in a subsequent section, we will explore the relation between this mathematical constructure and the emergent fundamental forces.

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II. GENERALIZED COMPLEX SPACETIME MANIFOLD

Suppose M is a six dimensional smooth manifold. Let $\{x^i, y^i, i = 1, 2, 3\}$ be the real spacetime coordinates in a chart $\{U, \phi\}$ around $p \in M$. At every spacetime point p there is a tangent space $T_p(M)$ and a cotangent space $T_p^*(M)$. If every point p is assigned a vector from tangent space $T_p(M)$, we have a tangent vector field. The space of tangent vector fields are denoted by TM. Similar for the space of cotangent vector fields T^*M. In terms of the coordinate basis, tangent vector fields are spanned by $\{\partial/\partial x^i, \partial/\partial y^i, i = 1, 2, 3\}$ and the cotangent vector fields by $\{dx^i, dy^i, i = 1, 2, 3\}$. T^*M is dual to TM and the coordinate basis are dual to each other:

$$dx^i(\frac{\partial}{\partial x^j}) = dy^i(\frac{\partial}{\partial y^j}) = \delta^i_j,\ i,j = 1,2,3,$$
$$dx^i(\frac{\partial}{\partial y^j}) = dy^i(\frac{\partial}{\partial x^j}) = 0,\ i,j = 1,2,3.$$

Consider the space $TM \oplus T^*M = \{X + \xi, X \in TM, \xi \in T^*M\}$. There is an *inner product* $<,>$ defined by

$$< X + \xi, Y + \eta >= \frac{1}{2}(\xi(Y) + \eta(X)), \qquad (1)$$

where $X, Y \in TM$ and $\xi, \eta \in T^*M$. This inner product is symmetric and non-degenerate.

The linear mappings from $TM \oplus T^*M$ to itself which preserve the inner product $<,>$ are called orthogonal transformations. All orthogonal transformations form a group $O(TM \oplus T^*M)$ called the orthogonal group. The inner product $<,>$ has signature $(6, 6)$, so we have non-compact orthogonal group $O(TM \oplus T^*M) \cong O(6, 6)$.

One important class of orthogonal transformations is B- field transformations. B field is a smooth 2-form, a bilinear mapping with the property that for all $X, Y \in TM, B(X, Y) = -B(Y, X)$. B may be viewed as a map $TM \to T^*M$ in the sense that for every $X \in TM, B(X)(Y) = B(X, Y)$. In the form of block matrix, B may be written as

$$B \sim \begin{pmatrix} 0 & 0 \\ B & 0 \end{pmatrix}$$

Taking the usual matrix exponential yields the B- field transformation

$$\exp(B) = \begin{pmatrix} 1 & 0 \\ B & 1 \end{pmatrix} \qquad (2)$$

that is, B- field transformations send $X \oplus \xi \in TM \oplus T^*M$ to $X \oplus (B(X) + \xi) \in TM \oplus T^*M$. Obviously the B- field transformations preserve the inner product $<,>$.

In order to reduce the orthogonal group $O(6, 6)$ to $U(3, 3)$, we introduce a generalized complex structure J on $TM \oplus T^*M$ as follows.

$$J(\frac{\partial}{\partial x^i}) = \frac{\partial}{\partial y^i}, \quad J(\frac{\partial}{\partial y^i}) = -\frac{\partial}{\partial x^i},$$
$$J(dx^i) = dy^i, \quad J(dy^i) = -dx^i.$$

This is a linear map $J : TM \oplus T^*M \to TM \oplus T^*M$ such that $J^2 = -1$ and J preserves the inner product

$$< Jv_1, Jv_2 >=< v_1, v_2 >, \quad v_1, v_2 \in TM \oplus T^*M.$$

Note that our choice of generalized complex structure J is different from the usual mathematical texts and from the literature[44].

The structure J can be extended to the complexified space $(TM \oplus T^*M) \otimes C$. Let's rewrite the spacetime coordinates as

$$z^i = x^i + \sqrt{-1}y^i,\ \overline{z^i} = x^i - \sqrt{-1}y^i.$$

In the following we will use the convention that $\{\overline{z^i} = \bar{z}_i, i = 1, 2, 3\}$. The coordinate basis of complex space $(TM \oplus T^*M) \otimes C$ may be taken as $\{\partial_i, d\bar{z}_i, dz^i, \bar{\partial}^i, i = 1, 2, 3\}$, where

$$\partial_i \equiv \frac{\partial}{\partial z^i} = \frac{1}{2}(\frac{\partial}{\partial x^i} - \sqrt{-1}\frac{\partial}{\partial y^i}),$$
$$\bar{\partial}^i \equiv \frac{\partial}{\partial \bar{z}_i} = \frac{1}{2}(\frac{\partial}{\partial x^i} + \sqrt{-1}\frac{\partial}{\partial y^i}),$$
$$dz^i = dx^i + \sqrt{-1}dy^i,$$
$$d\bar{z}_i = dx^i - \sqrt{-1}dy^i. \qquad (3)$$

that satisfy

$$J(\partial_i) = \sqrt{-1}\partial_i,\ J(d\bar{z}_i) = \sqrt{-1}d\bar{z}_i,$$
$$J(\bar{\partial}^i) = -\sqrt{-1}\bar{\partial}^i,\ J(dz^i) = -\sqrt{-1}dz^i.$$

Extending the inner product $<,>$ to the complex space $(TM \oplus T^*M) \otimes C$, we have

$$< \partial_i, \partial_j >=< \partial_i, d\bar{z}_j >=< d\bar{z}_i, d\bar{z}_j >= 0,$$
$$< \bar{\partial}^i, \bar{\partial}^j >=< \bar{\partial}^i, dz^j >=< dz^i, dz^j >= 0,$$
$$< \partial_i, \bar{\partial}^j >=< d\bar{z}_i, dz^j >= 0,$$
$$< \partial_i, dz^j >=< d\bar{z}_i, \bar{\partial}^j >= \frac{1}{2}\delta^j_i. \qquad (4)$$

Therefore we can devide the space $(TM \oplus T^*M) \otimes C$ into two parts:

$$(TM \oplus T^*M) \otimes C = V_J^+ \oplus V_J^-.$$

The V_J^+ is spanned by the basis $\{\partial_i, d\bar{z}_i\}$, while the V_J^- is spanned by the basis $\{dz^i, \bar{\partial}^i\}$.

Rotate the basis $\{\partial_i, d\bar{z}_i, i = 1, 2, 3\}$ as follows:

$$(\theta_a,\ \phi_a) = (\partial_i,\ d\bar{z}_i)M^i_a,\ a, i = 1, 2, 3 \qquad (5)$$
$$M = \frac{1}{\sqrt{2}}\begin{pmatrix} 1_{3\times3} & 1_{3\times3} \\ 1_{3\times3} & -1_{3\times3} \end{pmatrix}. \qquad (6)$$

where $1_{3\times3}$ stands for 3×3 identity matrix and $M^\dagger = M^{-1} = M$. We have

$$< \theta_a, \bar{\theta}^b >= \frac{1}{2}\delta^b_a, \quad < \phi_a, \bar{\phi}^b >= -\frac{1}{2}\delta^b_a,$$

and other pairing are equal to zero. This shows that the space V_J^+ with the basis $\{\theta_a, \phi_a\}$ has a inner symmetry $U(3, 3)$, while the space V_J^- with the basis $\{\bar{\theta}^a, \bar{\phi}^a\}$ has a inner symmetry $\overline{U(3, 3)}$.

III. HEXAD PREONS AND EMERGENT FORCES

Given the inner product $<,>$, the space $(TM \oplus T^*M) \otimes C$ may be turned into a Clifford algebra by the ralations

$$v_1 v_2 + v_2 v_1 = 2 < v_1, v_2 >, \quad (7)$$

where $v_1, v_2 \in (TM \oplus T^*M) \otimes C$. In particular, we have following anticommutative ralations$(i, j = 1, 2, 3)$

$$\partial_i^2 = (d\bar{z}_i)^2 = 0, \ \ \partial_i d\bar{z}_j + d\bar{z}_j \partial_i = 0, \quad (8)$$
$$\partial_i dz^i + dz^i \partial_i = 1, \quad (9)$$

and their complex conjugate relations. Vector field $\Psi \in V_J^+$ can be expressed as

$$\Psi = \psi^i \partial_i + \bar{\chi}^i d\bar{z}_i \ , i = 1, 2, 3.$$

Following physical terminology, we have obtained six fundamental fermions $\{\psi^i, \bar{\chi}^i, i = 1, 2, 3\}$. These fermions are called Hexad Preons. In the form of components they can be grouped into one multiplet:

$$\Psi = \begin{pmatrix} \psi^i \\ \bar{\chi}^i \end{pmatrix} \ , i = 1, 2, 3. \quad (10)$$

The fields $\{\psi^i, \bar{\chi}^i, i = 1, 2, 3\}$ are called mass eigenstates of Hexad Preons. After the rotation defined in Eqn.(5), we obtain the interaction eigenstates. The component form of the interaction eigenstates can be written as

$$\Psi^a = M_i^a \Psi^i \ , a, i = 1, 2, 3, \quad (11)$$

where matrix M is the rotating matrix in (6). The above discussion shows that Hexad Preons may carry hypercolor degree of freedom transforming under $U(3,3)$ gauge group.

At this stage there is no metric on the tangent space and no difference between the spacetime coordinates $x^i, y^i, i = 1, 2, 3$. However, by reducing the gauge group $U(3,3)$ to its maximal compact subgroup $U(3) \otimes U(3)$, we can introduce a generalized metric on $TM \otimes T^*M$ which is compatible with the natural pairing (1). It is shown[44] that this generalized metric is equivalent to a choice of metric G on TM and B-field transformation. We now identify this datum (G, B) with a Hermitian metric $g = g_{ij} dz^i \otimes dz^j, i, j = 1, 2, 3$. g_{ij} is given by

$$g_{ij} = G_{ij} + \sqrt{-1} B_{ij}.$$

In the case of "flat" spacetime where $g_{ij} = \delta_{ij}$, the spacetime interval is

$$\begin{aligned} ds^2 &= (dz^1)^2 + (dz^2)^2 + (dz^3)^2 \\ &= (dx^1)^2 + (dx^2)^2 + (dx^3)^2 - (dy^1)^2 - (dy^2)^2 - (dy^3)^2 \\ &\quad +2i(dx^1 dy^1 + dx^2 dy^2 + dx^3 dy^3). \end{aligned} \quad (12)$$

At low energy we require the imaginary part of (12) disappear, which implies that the time "direction" is orthogonal to the space "direction". Compared with the spacetime interval in special relativity,

$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 - (dt)^2,$$

we reach the conclusion that at low energy the time we measured is the "length" of the time "vector"

$$dt^2 = (dy^1)^2 + (dy^2)^2 + (dy^3)^2.$$

This also shows that the difference between the space coordinates $x^i, i = 1, 2, 3$, and time coordinates $y^i, i = 1, 2, 3$, is the consequence of symmetry breaking $U(3,3) \to U(3) \otimes U(3)$.

After the symmetry breaking, the Hexad Preons can also condensate to form subsequent matter states. The symmetry group, $U(3) \otimes U(3)$, means that they can accommodate particle contents of SM easily.

It should be mentioned that our treatment of complex spacetime is intrinsically different from previous works[45–47], where spacetime has four complex dimensions and eventually only the real slice is considered.

IV. DISCUSSION AND CONCLUSIONS

It is a dream of theoretical physics society to unify the fundamental forces into one entity and consequently resolve the problems we confront. In this paper, it is postulated that at high energy we have three dimensional complex spacetime and at low energy only the "length" of the three time "vector" plus the three space dimensions can be felt. Based on this hypothesis, we can interpret the generalized complex geometry as a mathematical framework to incorporate the four fundamental forces.

The emphasis in this paper is mainly put on the mathematical aspects of the Hexad Preons and emergent gravity in three dimensional complex spacetime. The dynamical aspects will be treated in subsequent papers. However, we could give some comments on problems mentioned at the beginning of the paper.

We propose that the primordial universe is made of six fundamental fermions, Hexad Preons Ψ in (10)(or Ψ^a in (11)), and their complex conjugate states

$$\bar{\Psi} = \Psi^\dagger \begin{pmatrix} 0 & 1_{3\times3} \\ 1_{3\times3} & 0 \end{pmatrix} = \begin{pmatrix} \chi_i, & \bar{\psi}_i \end{pmatrix} \ , i = 1, 2, 3. \quad (13)$$

Hexad Preons carry hypercolor degree of freedom transforming under $U(3,3)$ gauge group. At this stage the universe has the biggest internal and spacetime symmetry.

As the gauge group $U(3,3)$ breaks down to its maximal compact subgroup $U(3) \otimes U(3)$, the Hermitian metric emerges so that the gravity may be viewed as a symmetry-breaking effect in quantum field theory. People has been pursued in this direction since 1960's [48] .

However, the situation here is more involved because the gauge group is non-compact.

Upon the appearance of metric, complex conjugate operation on the spacetime coordinates $z^i = x^i + \sqrt{-1}y^i$ means time reversal. V_J^- contains the complex conjugate states of Hexad Preons in V_J^+. If the states in V_J^- condensate forming "vacua", the time reversal symmetry is broken and we have a time arrow. This is closely related to the problems of baryon and CP asymmetries. At the same time, this "negative time" vacua may be the source of (or at least part of) dark energy.

The subsequent condensate of Hexad Preons, now transforming under $U(3) \otimes U(3)$, may form leptons, quarks, dark matter, as well as other matter states. The hypercolor interaction here manifests itself as strong and electroweak forces in the corresponding cases. Therefore QCD and GWS theory have the same origin. The SM gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, as well as replication of leptons and quarks can be easily obtained. However, the correct identification of leptons and quarks needs the knowledge of the hypercolor dynamics and will give new clues to mechanism for electroweak symmetry breaking. The mixing of fermions may result partly from the rotation (6).

Usually it is assumed that the properties of preons are similar to that of QCD [49]:

(a) The preons are relativistic (Dirac or Majorana) fermions in four dimensional spacetime.

(b) The dypercolor dynamics is described by an unbroken non-abelian gauge symmetry under which preons are nonsinglets.

(c) Leptons, quarks and Higgs are singlets under the dypercolor gauge symmetry.

Therefore our Hexad Preons model is different from other preon models at least at the points (a) and (c). Chirality is important concept in SM. Since we are considering Hexad Preons in three dimensional complex spacetime, the Hexad Preons are not necessary to be Chiarl. However, we must be able to recover the Chiral structure at low energy.

In conclusion, we have shown that in three dimensional complex spacetime, the four fundamental forces may have a unique origin. In addition, the picture of universe evolution in this paper is in accord with Y. Nambu's opinion that [50] "As the universe expands and cools down, it may undergo one or more SSB phase transitions from states of higher symmetries to lower ones, which change the governing laws of physics".

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